3: Interaction of ionizing radiation with tissue

1. What is the basis of contrast for x-ray imaging ?
2. By which mechanisms does ionizing radiation interact with matter ?
   - Rayleigh scattering
   - Compton scattering
   - Photoelectric effect
   - Pair production
3. How does this interaction depend on the tissue ?
   - Energy dependence and effective atomic number Z_{eff}
4. How can we protect ourselves against the biological effects of ionizing radiation ?
   - A radiation protection primer

After this course you

1. Know the definition of linear and mass attenuation coefficient
2. Understand the major mechanism of x-ray absorption in tissue
3. Understand the dependence of these mechanisms on photon energy and tissue composition
4. Are able to perform contrast-to-noise calculations using effective Z
5. Understand and are able to apply the basic principles of radiation protection

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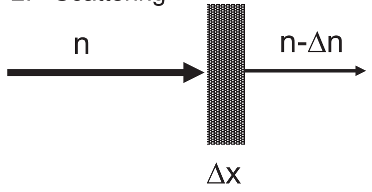
3-1. How can we describe attenuation of x-rays?

Linear attenuation coefficient μ

Fates of the photon (other than transmission):

1. Absorption (transfer of $h\nu$ to lattice)
2. Scattering



Photons are removed according to probability law:

The number of absorbed/scattered photons Δn in a layer with thickness of Δx

$$\Delta n \equiv -\mu n \Delta x$$

μ : linear attenuation coefficient
Unit: [cm^{-1}]

$\mu = f(E_\nu, Z, \rho)$

Consider situation where $\Delta x \rightarrow 0$, and n=f(x)

$$dn(x) = -\mu n(x) dx$$

$$\frac{dn(x)}{dx} = -\mu n(x)$$

Solution ?

$$\boxed{n(x) = N_0 e^{-\mu x}}$$

(provided μ is constant in x)

Definition

Half value layer (HVL) ≡ The thickness of a material allowing to pass one half of photons:

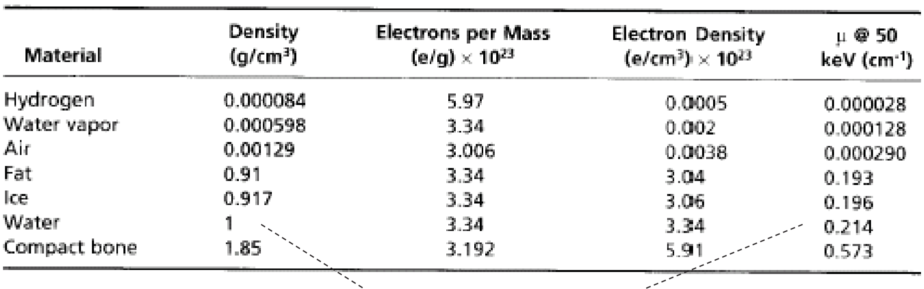
$n(x_{HVL}) = N_0/2$

$$N_0 / 2 = N_0 \mathrm{e}^{-\mu(\mathrm{HVL})}$$

$$\mathrm{HVL} = 0.693/\mu$$

Typical HVL values: several cm for tissues, 1-2 cm for aluminum, 0.3 cm for lead

What are typical attenuation coefficients ?



Material	Density (g/cm³)	Electrons per Mass (e/g) × 10^{23}	Electron Density (e/cm³) × 10^{23}	μ @ 50 keV (cm⁻¹)
Hydrogen	0.000084	5.97	0.0005	0.000028
Water vapor	0.000598	3.34	0.002	0.000128
Air	0.00129	3.006	0.0038	0.000290
Fat	0.91	3.34	3.04	0.193
Ice	0.917	3.34	3.06	0.196
Water	1	3.34	3.34	0.214
Compact bone	1.85	3.192	5.91	0.573

μ/ρ (water) = μ/ρ (ice)

Definition

Mass attenuation coefficient μ/ρ

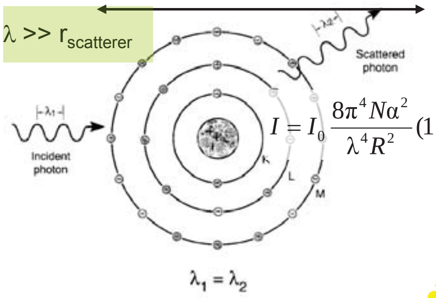
Unit : [cm^2/g]

(constant for all forms of the same chemical substance, e.g. water)

3-2. What are the 4 basic interactions of x-rays with biological tissue ?

I. Rayleigh scattering & Compton

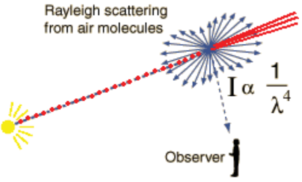
$\lambda >> r_{scatterer}$



$$I = I_0 \frac{8\pi^4 N\alpha^2}{\lambda^4 R^2}(1+\cos^2\theta)$$

$$I \propto \frac{I_0\rho}{\lambda^4}(1+\cos^2\theta)$$

Strong λ dependence: short λ more scattered



$I \alpha \frac{1}{\lambda^4}$

Elastic scattering of light or other electromagnetic radiation by particles **much smaller than the wavelength** of the light, $x = 2\pi r/\lambda$; $x << 1$.

Elastic: photon energy = constant

No ionization occurs ("classical" scattering)

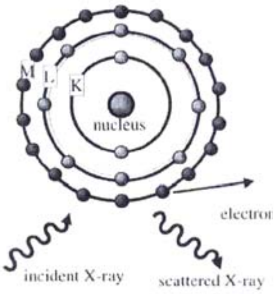
Scattered X-rays ⇒ deleterious effect on image quality

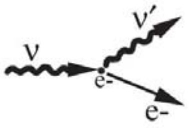
very low probability (< 5%) in x-ray imaging

E(keV)=1.2/λ(nm)

λ=1.2/E = 0.012nm (@100keV)

II. Compton scattering





$\nu' < \nu$

Electron is initially at rest

e- gains energy

E binding energy is neglected (why?)

Arthur Holly Compton

Physics, 1927

Occurs at the outer shell electrons

⇒ ionization

Scattered photon: subject to subsequent interactions
(Rayleigh, Compton scattering or photoelectric effect)

Probability increases with

photon energy

electron density

Electron/mass density in tissues ~ constant (independent of Z)
→ proportional to the **density of the material**

Relativistic linear momentum

a brief tour back to 1st year physics

From the definition p≡mv (which is true at any velocity) it follows

$$\boxed{\vec{p} = m(v)\vec{v} = m_0 \frac{\vec{v}}{\sqrt{1-v^2/c^2}}}$$

$$\vec{p}\cdot\vec{p} = m_0{}^2 \frac{\vec{v}\cdot\vec{v}}{1-v^2/c^2}$$

The value of p is

Relativistic kinetic energy E=m(v)c²

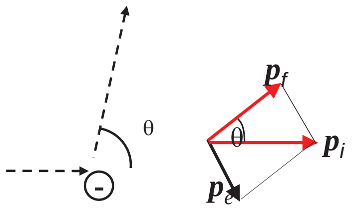
$$E^2 - m_0{}^2c^4 = \underbrace{\overbrace{m_0{}^2\frac{c^4}{1-v^2/c^2}}^{} - m_0{}^2\frac{c^4\left(1-v^2/c^2\right)}{1-v^2/c^2}}_{m_0{}^2\frac{v^2}{1-v^2/c^2}c^2 \,=\, (pc)^2}$$

$$\boxed{\vec{p}\cdot\vec{p} = \frac{E^2}{c^2} - m_0{}^2c^2}$$

NB. Light carries energy, but moves at the speed of light (!) c.

Photon with energy E:
particle with rest mass (m_0=0)
(otherwise its energy would be infinite, since v=c) :

$$\boxed{\left|\vec{p}\right| = \frac{E}{c}}$$



Compton scattering

The basic Equations

A simple elastic collision:

Conservation of energy

$$\boxed{E_i + m_e c^2 = E_f + E_e}$$

before after

$E_e = (E_i - E_f) + m_e c^2$

$$p_e^{\ 2} = \frac{E_e^{\ 2}}{c^2} - m_e^{\ 2} c^2$$

$$\boxed{E_f = \frac{E_i}{(1-\cos\theta)E_i / m_e c^2 + 1}}$$

Conservation of linear momentum:

$$\boxed{\vec{p}_i = \vec{p}_f + \vec{p}_e}$$

before after

$$(\vec{p}_e)^2 = (\vec{p}_i - \vec{p}_f)^2$$

With λ=c/ν E=hc/λ it follows:

$$\boxed{\lambda_f - \lambda_i = \frac{h}{m_e c}(1-\cos\theta)}$$

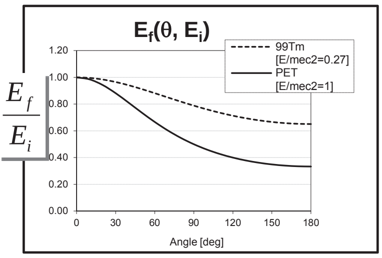
E_f is

Maximal if θ=0: $E_f^{max} = E_i$

Minimal, if θ=π: $E_f^{\ \min} = \frac{E_i}{2E_i / m_e c^2 + 1}$

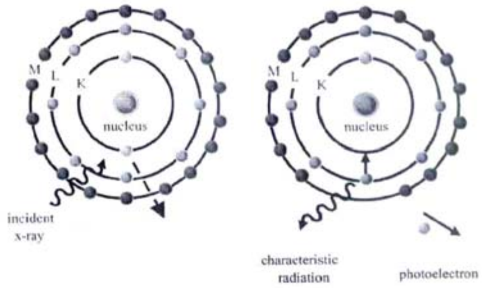
$E_i = m_e c^2$:

$$E_f = \frac{m_e c^2}{(2-\cos\theta)}$$



3-3. Interaction of photons with tissue II

Photoelectric effect, pair production & summary



Photoelectric absorption effect:

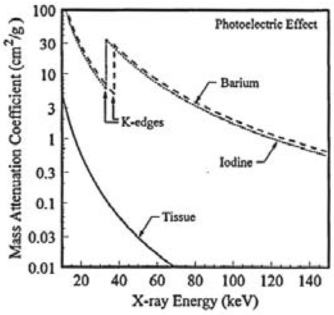
Abruptly increases when *E* is slightly above I_K – **absorption edges**

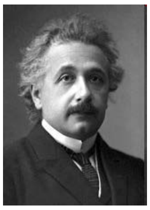
Absorption edge energy increases with *Z* (very low for H, C, N, O)

Inner shell e^- is removed

⇒ energy of the incident X-ray quantum E_i > ionization energy of an electron I_K

$$E_i = E_e^K + I_K$$





Albert Einstein

Physics, 1921

Vacancy in K-shell: filled with outer shell e

⇒ cascade of emitting *characteristic* X-ray quanta

(or *Auger electrons*, but not so frequent in diagnostic imaging of soft tissues with low Z)

What is the relative contribution of x-ray scattering mechanisms ?

in soft tissue

IV. Pair production

When the photon energy > $2m_ec^2$ = 1.02MeV

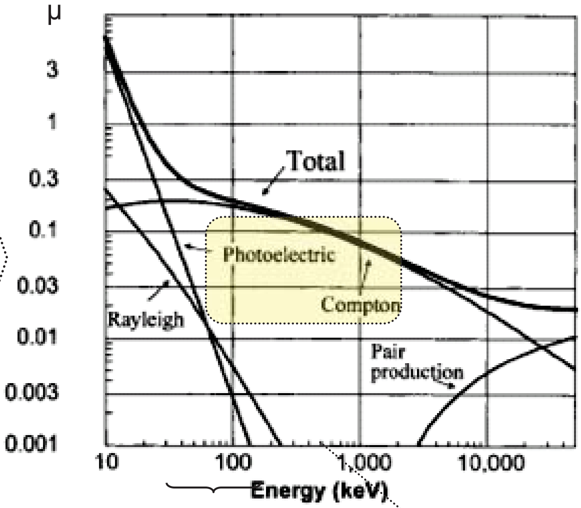
It interacts with the electric field of the nucleus of an atom

$\nu \rightarrow e^- + e^+$

The kinetic energy of the produced particles is

$$E = E_\nu - 2m_ec^2$$

Not important in Bio-imaging

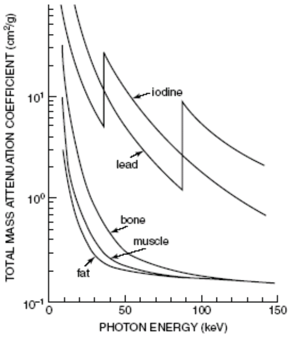


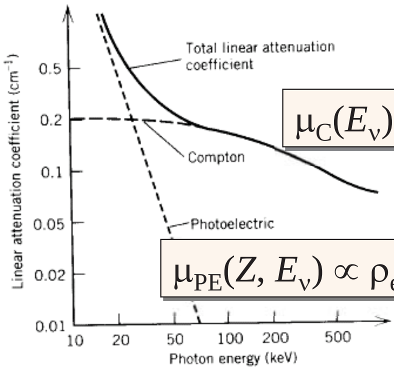
x-ray (CT, SPECT)

PET

3-4. What affects the linear attenuation coefficient ?

effective Z, Z_{eff}





$$\mu_C(E_\nu) \propto \rho_e \, f(E_\nu)$$

$$\mu_{PE}(Z, E_\nu) \propto \rho_e(E_\nu) \, Z_{eff}^{3.4}/E_\nu^{3.1}$$

Linear attenuation coefficient of water and the contribution of each interaction to the total attenuation of X-rays as a function of energy.

Compton:

- depends mainly on the e density ρ_e
- modestly on energy of x-rays E_ν

Photoelectric effect:

- depends strongly on atomic number Z

Biology:

empirical Z_{eff}

What is the effective atomic number Z_{eff} of biological tissue ?

Necessary for estimating μ of the photoelectric effect

Empirical relationship for compound materials such as biological tissue:

$$Z_{eff} = \left(\sum_{\substack{\text{all tissue} \\ \text{components } i}} \lambda_i Z_i^{3.4} \right)^{1/3.4}$$

$$\lambda_i = \frac{P_i Z_i / A_i}{\sum_{\substack{\text{all tissue} \\ \text{components } j}} P_j Z_j / A_j}$$

P : percentage weight

Z : atomic number

A : atomic weight

Example: Estimation of Z_{eff} for water

^{1}H: Z=A=1, P=11%,

^{16}O: Z=8, A=16, P=89%

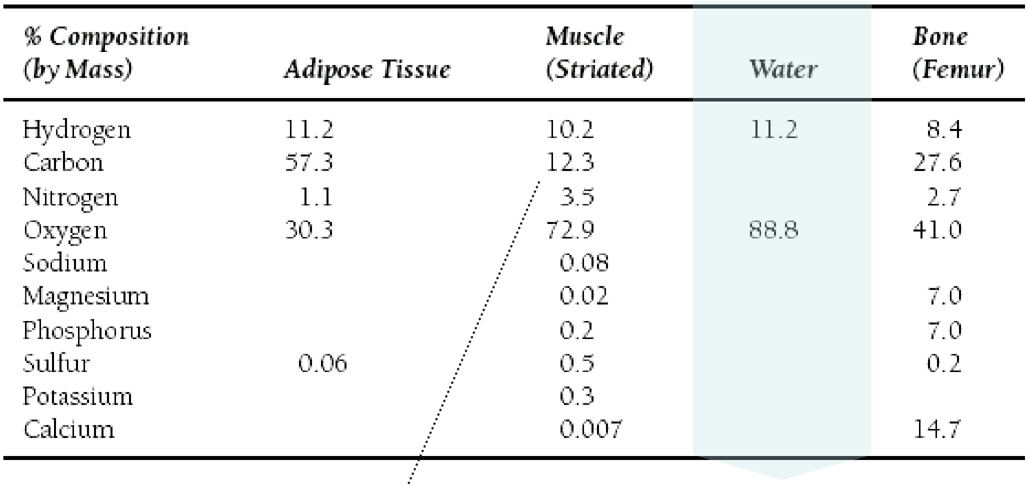
Denominator of λ : $\sum P_j \frac{Z_j}{A_j} = 11\frac{1}{1} + 89\frac{8}{16} = 55.5$

Protons: λ = 11/55.5=0.20
Oxygen: λ = 44.5/55.5=0.80.

$Z_{eff}=(0.2 \cdot 1^{3.4}+0.8 \cdot 8^{3.4})^{1/3.4}$
$=(0.2+1180 \cdot 0.8)^{1/3.4}$
$=944^{1/3.4}=7.5$

How good were we ?

What is the % mass composition P_i of select biological tissues ?



% Composition (by Mass)	Adipose Tissue	Muscle (Striated)	Water	Bone (Femur)
Hydrogen	11.2	10.2	11.2	8.4
Carbon	57.3	12.3		27.6
Nitrogen	1.1	3.5		2.7
Oxygen	30.3	72.9	88.8	41.0
Sodium		0.08		
Magnesium		0.02		7.0
Phosphorus		0.2		7.0
Sulfur	0.06	0.5		0.2
Potassium		0.3		
Calcium		0.007		14.7

Z_{eff} ~ 6 7.4 12

Carbon:

Z/A=6/12
(same for O, 8/16)

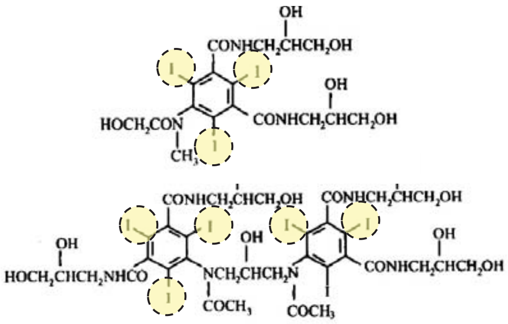
What are X-ray contrast agents ?

Exogenously administered substance (by infusion/ingestion)

modifying Z_{eff}

$\Rightarrow$ use high Z compounds

e.g., compounds with multiple iodine atoms, lanthanides etc.



Z_{eff} of (water+10 mmol/kg iodine) = ?

Iodine:

P_I = 10[mmol/kg]×127[mg/mmol] = 0.127%

Z_I = 53

A_I = 127

Calculation of Z_{eff}:

$$Z_{eff}=\left(\sum_{\substack{\text{all tissue}\\ \text{components } i}}\lambda_i Z_i^{3.4}\right)^{1/3.4} \qquad \lambda_i=\frac{P_i Z_i/A_i}{\sum\limits_{\substack{\text{all tissue}\\ \text{components } j}} P_j Z_j/A_j}$$

~ denominator λ of pure H_2O

Denominator of λ :

$$\sum P_i\frac{Z_i}{A_i}=11\frac{1}{1}+89\frac{8}{16}+0.13\frac{53}{127}=55.6$$

→ λ of H and O are as for water:

λ_H=0.2

λ_O=0.8

$$\lambda_I=\frac{0.127\cdot 53/127}{55.6}=\frac{0.053}{55.6}=9.5\times 10^{-4}$$

$$Z_{eff}^{3.4}=\underbrace{0.2\cdot 1^{3.4}+0.8\cdot 8^{3.4}}_{944}+\underbrace{9.5\cdot 10^{-4}\cdot 53^{3.4}}_{690}$$

$$Z_{eff}=8.8$$

$\mu_{PE}\propto Z_{eff}^{3.4}$

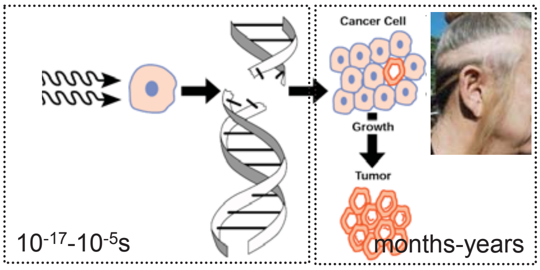
$\mu_{PE}(H_2O)\propto 944$

$\mu_{PE}(H_2O+I)\propto 1650$

3-5. What are the biological effects of ionizing radiation ?

Ionization effects:
instantaneous (10^{-17}-10^{-5}s)

1. Produce free radicals
2. Break chemical bonds
3. Produce new chemical bonds and cross-linkage between macromolecules
4. Damage molecules that regulate vital cell processes (e.g. DNA, RNA, proteins)



Radicals (unpaired valence e⁻) and reactive oxygen species, e.g. 2 OH· –> H_2O_2

produced by the body

(e.g. oxygen consumption)

Biological effects are delayed:

Cataract (months to years)

Cancer (years-decades)

Tissue sensitivity to radiation

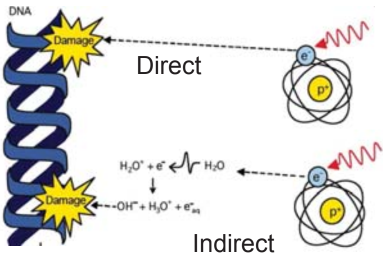
proportional: rate of cell proliferation
inversely prop.: degree of cell differentiation

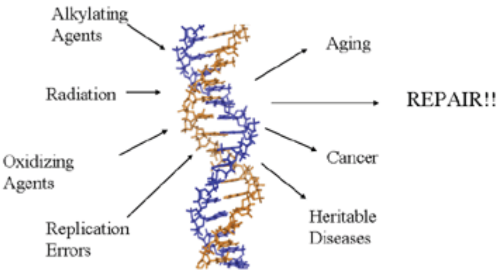
Pregnancy vs. old age …

Blood-forming organs
Reproductive organs
Skin
Bone and teeth
Muscle
Nervous system

How does the tissue defend itself against radiation damage ?

DNA repair





Healing

Repair	Regeneration
Replace damaged cells with different cell type (fibrosis)	Replace damaged cells by same
Organ not returned to original state	Organ returned to original state
Radioresistant tissues (muscle, brain): only repair possible	Radiosensitive tissues (skin, digestive system, bone marrow)

How are the effects of ionizing radiation quantified ?

Three forms of radiation dose

Absorbed dose D:

energy deposited by ionizing radiation per unit mass of material:

D = Energy/mass

Units: [Gray[Gy]=1J/kg]

Equivalent dose *H*:

= D corrected for effectiveness of radiation to produce biological damage (w_R = radiation weighting factor)

$H = Dw_R$

Units: [Sievert[Sv] = 1J/kg]

Effective dose *E* :

H corrected for sensitivity of tissue T (w_T = tissue weighting factor)

$$E(\mathrm{Sv}) = \sum w_T H_T(\mathrm{Sv})$$

Absorbed dose D depends on

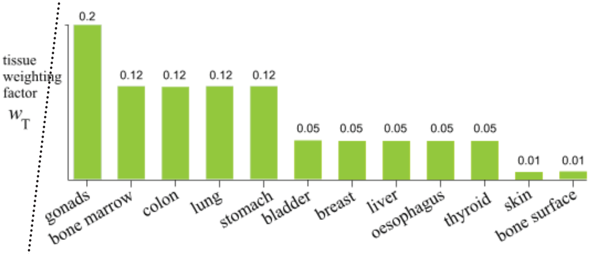
1. Intensity of incident x-ray
2. Duration of exposure

w_R = 1 (x-rays, β particles)

[20: for α particles]

Bio-imaging:

Equivalent dose H = Absorbed dose D



3.6 How can we protect ourselves against x-rays ?

Exposure time, distance and HVL

Absorbed dose depends on intensity I_0 of radiation (activity in Bq for radioactive materials)

1 Becquerel [(Bq) = 1 decay/s]

NB. β-emitters (^{14}C, ^{3}H):

- e^- do not penetrate low *Z* material (plastic, glass, avoids Bremsstrahlung)
- high energy β-emitters (^{32}P): low *Z* shielding + lead (attenuate Bremsstrahlung)

Three basic elements of radioprotection:

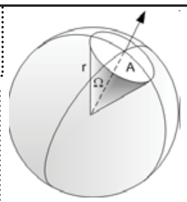
1. **Time**
2. **Distance**
3. **Shielding**

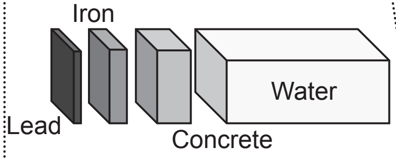
Occupancy:

fraction of time of working day (8h), human is in area exposed to radiation

Received radiation = Fraction of **solid angle** occupied by human surface A = A/4πd²

~ I_0/d^2 (*d*=distance to source)





10-fold reduction:
5 10 20 100 cm

Depends on energy and type of radiation

HVL	Iron or steel	Brick	Concrete	Dirt	Ice	Wood (soft)	Snow
(cm)	1.8	5.1	5.8	8.4	17	22	52

Half-value layers

What are typical radiation exposures ?

Natural, artificial and some examples

Natural	mrem/yr	mSv/yr	%total
Radon	200	2.0	55%
Cosmic	27	0.27	8%
Terrestrial	28	0.28	8%
Internal	39	0.39	11%
Total	**300**	**3**	**82%**

Other			
Occupational	0. 9	<0.01	<0.3
Nuclear Fuel Cycle	<1	<0.01	<0.03
Fallout	<1	<0.01	<0.03
Misc.	<1	<0.01	<0.03

Artificial			
Medical X ray	39	0.39	11%
Nuclear Med.	14	0.14	4%
Consumer products	10	0.1	3%
Total	**63**	**0.63**	**18%**

Source of Exposure	
X-rays from TV set (3 cm)	0.500 mrem/hour
Airplane ride (12km)	0.500 mrem/hour
Radionuclides in the body (i.e., K)	39 mrem/year
Building materials (concrete)	3 mrem/year
Drinking Water	5 mrem/year
Pocket watch (radium dial)	6 mrem/year
Eyeglasses (containing thorium)	6 - 11 mrem/year
Transatlantic Airplane roundtrip	5 mrem

http://newnet.lanl.gov/info/dosecalc.asp

http://www.epa.gov/radiation/understand/calculate.html

Chest x-ray, dental x-ray, head & neck	5-20 mrem
Lumbar spinal x-rays	130 mrem
Pelvis x-ray	44 mrem
Hip x-ray	83 mrem
CT (head and body)	1,100 mrem

Useful to know:

100 Röntgen equivalent man (REM) = 1 Sievert

Appendix: Derivation of the Compton Relationship

Situation: A photon with energy E_i collides with e of mass m_e at rest. One wants to know the energy of the photon after the collision.

Conservation of energy $E_e = (E_i - E_f) + m_ec^2$

$$E_e^2 = (E_i - E_f)^2 + (m_ec^2)^2 + 2(E_i - E_f)\, m_ec^2$$

$$E_e^2 = p_e^2c^2 + m_e^2c^4 \longleftarrow p_e^2 = \frac{E_e^2}{c^2} - m_e^2c^2$$

$$p_e^2c^2 + (m_ec^2)^2 = (E_i - E_f)^2 + (m_ec^2)^2 + 2(E_i - E_f)\, m_ec^2$$

$$p_e^2c^2 = (E_i - E_f)^2 + 2(E_i - E_f)\, m_ec^2$$

$$p_e^2c^2 = E_i^2 + E_f^2 - 2E_iE_f + 2(E_i - E_f)\, m_ec^2$$

Conservation of momentum:

$$(\vec{p}_e)^2 = (\vec{p}_i - \vec{p}_f)^2$$

$$(\vec{p}_e)^2 = p_i^2 + p_f^2 - 2p_ip_f\cos\theta$$

$$p_i = \frac{E_i}{c} \quad p_f = \frac{E_f}{c}$$

$$(\vec{p}_e)^2 = \left(\frac{E_i}{c}\right)^2 + \left(\frac{E_f}{c}\right)^2 - 2\frac{E_iE_f}{c^2}\cos\theta$$

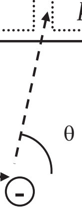
$$p_e^2c^2 = E_i^2 + E_f^2 - 2E_iE_f\cos\theta$$

$$-2E_iE_f + 2(E_i - E_f)\, m_ec^2 = -2E_iE_f\cos\theta$$

$$E_iE_f - (E_i - E_f)\, m_ec^2 = E_iE_f\cos\theta$$

$$E_iE_f + E_fm_ec^2 - E_iE_f\cos\theta = E_i\, m_ec^2$$

$$E_f\,(E_i + m_ec^2 - E_i\cos\theta) = E_i\, m_ec^2$$



$$\Rightarrow \quad E_f = \frac{E_im_ec^2}{E_i(1-\cos\theta)+m_ec^2}$$