Neural Networks and Biological Modeling

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Question set 9

Exercise 1: Diffusive noise (stochastic spike arrival)

Consider a passive membrane receiving stochastic synaptic input $S(t) = \sum_f \delta(t - t_k^f)$, where the index f runs over the firing times of a presynaptic neuron. The spike train starts only at t = 0, so that $t_k^f > 0$ for all firing times. The membrane potential obeys the equation:

$$\frac{du}{dt} = -\frac{u - u_{\text{rest}}}{\tau} + \frac{q}{C} S(t)\,. \tag{1}$$

where q is the charge brought by each spike and C is the capacitance of the membrane. The solution to this equation writes:

$$u(t) = u_{rest} + \frac{qR}{\tau} \int_0^t \exp(-s/\tau) S(t-s) ds \tag{2}$$

1.1 Calculate the expected voltage $\langle u(t) \rangle$ as a function of t for a constant presynaptic rate $\langle S(t) \rangle = \nu$ for $t \geq 0$ ($\nu = 0$ for $t < 0$). Where $\langle\ \rangle$ is the average over multiple repetitions or over a population of neurons having the same dynamics and inputs.

1.2 Calculate $\langle u(t)^2 \rangle$. Assume that the spike times of the presynaptic neuron are uncorrelated, i.e., $\langle S(t)S(t') \rangle = \nu\delta(t - t') + \nu^2$, and use Eq. 2.

1.3 Calculate the variance of the potential across multiple repetitions: $Var[u](t) = \langle [u(t) - \langle u(t) \rangle]^2 \rangle$.

Homework:

1.4 Calculate the autocorrelation of the voltage $\langle u(t)u(t') \rangle$ in the steady state regime (replace the upper bound of the integral by ∞ in equation 2).

1.5 Suppose that there are two presynaptic neurons which fire independently with rates $\nu_1 = \langle S_1(t) \rangle$ and $\nu_2 = \langle S_2(t) \rangle$, such that the input to the postynaptic neuron is given by $w_1 S_1(t) + w_2 S_2(t)$ where w_i denote the synaptic weights. Calculate again the mean and autocorrelation of the voltage.

1.6 Redo question 1.5 with correlated spike trains $S_1(t) = S_2(t)$.

Exercise 2: Firing statistics

Consider a stochastic spike generation process in discrete time. The probability of generating a spike in a time Δt is $P_{\Delta t} = \nu \Delta t$. Hence when we take the limit of Δt to 0 the expected value of the quantity $S(t) = \sum_f \delta(t - t_k^{(f)})$ is:

$$\langle\, S(t) \,\rangle = \lim_{\Delta t \to 0} \frac{P_{\Delta t}(t)}{\Delta t} = \nu \;;\; \text{for}\, t > 0.$$

Consider the probability of having two spikes in different time bins around t and t'. Define $\langle\, S(t)S(t') \,\rangle$ in a similar fashion, and show that it is equal to $\nu\delta(t - t') + \nu^2$.

Exercise 3: Coding by spikes

Imagine we are injecting a step current in a neuron that receives no other input. In this exercise we investigate how the time of the first spike T codes for the amplitude of the step current I_0. You should interpret your results in terms of coding efficiency: How is the time of the first spike coding for the current amplitude in each case? How could you make a precise measurement of the current amplitude with each model?

3.1 Determine the timing of the first spike as a function of I_0 for a leaky integrate-and-fire model.

3.2 Consider a poisson neuron with a firing rate proportional to current: $\rho(t) = kI(t)$. Can you determine the time of the first spike? Make a qualitative sketch of the probability distribution of the time of the first spike. Calculate the expected $T(I_0)$.

3.3 Consider leaky integrator neuron model:

$$\frac{du}{dt} = \frac{-u}{\tau} + \frac{I(t)}{C}$$

where u is the membrane potential above rest. Consider further that firing is stochastic and occurs via escape noise proportional to the potential. Precisely, the instantaneous probability to fire is given by:

$$\rho(t) = \begin{cases} u(t) - a & \text{for } u(t) > a \\ 0 & \text{for } u(t) < a. \end{cases}$$

where a is some threshold potential above which the neuron has a non-zero probability to spike. Can you determine the time of the first spike? Make a qualitative sketch of the probability distribution of the time of the first spike. Calculate the expected $T(I_0)$ in the limit of very large τ using the concepts from renewal theory (lecture 8).