Neural Networks and Biological Modeling

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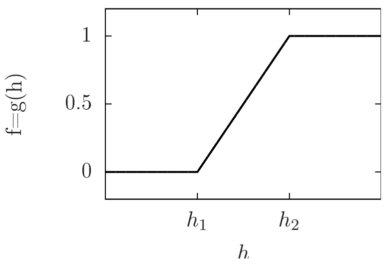
Question set 12

Exercise 1: Mean field model

Consider a network of N neurons with all-to-all connectivity and scaled synaptic weights $w_{ij} = J_0/N$. The transfer function (rate as a function of input potential) of the neurons is piecewise linear:

$$f = g(h) = \begin{cases} 0 & , \quad h < h_1 \\ \frac{h-h_1}{h_2-h_1} & , \quad h_1 \le h \le h_2 \\ 1 & , \quad h_2 < h \end{cases} \tag{1}$$



The dynamics of the input potential for neuron i is:

$$\tau \frac{dh_i}{dt} = -h_i + RI_i(t) \tag{2}$$

where

$$I_i(t) = I^{\text{ext}}(t) + \sum_j \sum_f w_{ij}\alpha(t - t_j^f) \tag{3}$$

where α denotes the shape of a the input current caused by a single spike. We are interested in the stationary solutions.

1.1 Find graphically the fixed points of the activity.

1.2 Find the solutions analytically.

Exercise 2: Continuous population model

We study the system with lateral connection $w(x-y)$ given by:

$$\tau \frac{\partial u(x,t)}{\partial t} = -u(x,t) + \int w(x-y) f[u(y,t)]dy + I_{ext}(x,t), \tag{4}$$

where $f[u(x,t)] = A(x,t)$ is the population's activation at the point x at time t.

2.1 Show that for a constant current I_{ext}, the homogeneous stationary solution $u(x,t) = u_0$ leads to a constant activity A_0 given by:

$$A_0 = f(u_0) = \frac{u_0 - I_{ext}}{\bar{w}},$$

with $\bar{w} = \int w(y)dy$.

2.2 We set $u(x,t) = u_0 + \Delta u(x,t)$ where Δu is a small perturbation. Linearize the equation (4) about u_0 and solve the Fourier transformed equation and obtain $\Delta u = \int g(k)dk$ where

$$g(k) = u_1 e^{ikx} e^{-\kappa(k)t}.$$

Identify the function κ. For which values of k do we get $\kappa < 0$?

2.3 Consider:

$$w(z) = \frac{\sigma_2 e^{-z^2/(2\sigma_1^2)} - \sigma_1 e^{-z^2/(2\sigma_2^2)}}{\sigma_2 - \sigma_1},$$

with $\sigma_1 = 1$ and $\sigma_2 = 10$. Sketch the qualitative behaviour of $w(z)$ and

$$\int w(z)\cos(kz)dz.$$

Determine graphically the stability condition.

Exercise 3: Stationary state in a network with lateral connections

Consider a neural network with the lateral connections represented in the figure shown below. σ corre-

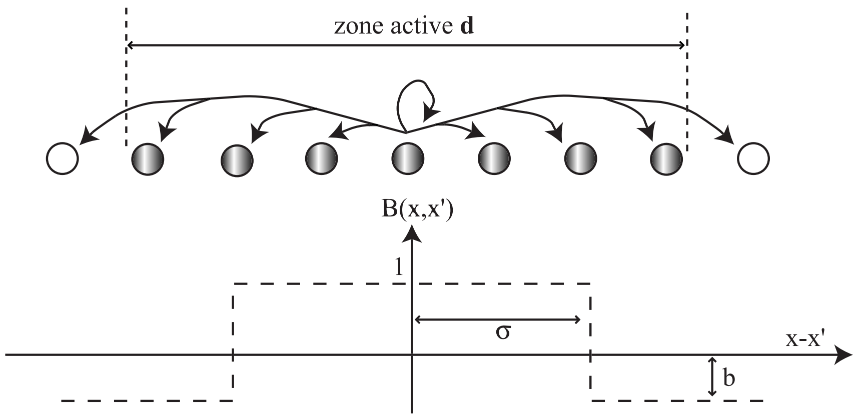


Figure 1: Spatial structure of the network (exercise 3).

sponds to the range of the excitatory connections. On the figure 1 we have $\sigma = 2$. The activity a of a neuron at position x is given by:

$$a(x) = g[h(x)], \tag{5}$$

where $h(x)$ total potential of the neuron at position x. It is described by:

$$h(x) = \sum_{x'} B(x, x')a(x') + h_{ext}(x). \tag{6}$$

The interaction is locally excitatory and long range surround inhibitory connections:

$$B(x, x') = \begin{cases} 1 & |x - x'| \leq \sigma \\ -b & |x - x'| > \sigma \end{cases} \tag{7}$$

The transfer function $g(h)$ is a simple threshold function:

$$g(h) = \begin{cases} 1 & h > \theta \\ 0 & h \leq \theta \end{cases} \tag{8}$$

In this exercise we do not add any external input i.e. $h_{ext}(x) = 0$. We claim that a group of neighbour neurons of active neurons of size d can be a solution of the system. The aim of the exercise is to derive the possible sizes d of the group.

3.1 Show that we need $d > \sigma$. To do so, suppose $d \leq \sigma$ and calculate the potential $h(x)$ on both sides of the boundary of the active neurons group.

3.2 Use the same argument in the case where $d > \sigma$ to derive the maximal minimal values of $(d - \sigma)$ (do not forget the connection of a neuron to itself).

3.3 What are the conditions that give one single active neuron as the system's solution?

Exercise 4: Dynamics of a network with lateral connections

We keep the same type of network as before. The dynamics is described by the following equations:

$$\begin{aligned} A(x,t) &= g[h(x,t)], \qquad (9) \\ \tau \frac{d}{dt} h(x,t) &= -h(x,t) + \sum_{x'} B(x,x')\, A(x',t), \qquad (10) \end{aligned}$$

with the initial condition $A(x,t) \equiv 0$ for all x.

4.1 Discretize time putting $\Delta t = \tau$.

4.2 Suppose all neurones are inactive. At time $t > 0$ we apply a stimulus on neuron at position $x = 0$:

$$h_{ext}(x) = I_0 \quad \text{pour } x = 0 \qquad (11)$$

and 0 otherwise. How will the activity of the network evolve in the case where $I_0 < \theta$ and $I_0 > \theta$? Compute $A(x, t + \Delta t) = g[h(x,t)]$ for three iterations. Explore the case $\theta = 0.9$ and $\theta = 1.1$. Use the condition $\sigma/(\sigma + 1) < b \leq 1$ and show that we get a stable state! What is the size of the blob of activity? Can we rule out oscillations in the size for any values of b?