Neural Networks and Biological Modeling

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Question set 11

Exercise 1: Reverse correlation, optimal stimulus

Assume a linear Poisson neuron. The probability to fire a spike in Δt is

$$P(t; t + \Delta t) = \rho(t)\Delta t$$

where $\rho(t) = h(t) = \int_0^\infty \kappa(s)I(t-s)ds = \sum_n K_n I_{i-n}$ is the instantaneous rate, and $K_i = \kappa(i\Delta t)\Delta t$ makes a linear filter.

Show that the optimal stimulus to generate a spike at $t = 0$ under a constraint $\int_{-\infty}^{\infty} I^2(t)dt = \text{const}$ is directly proportional to the filter in reverse order of time:

$$I(t) \propto \kappa(-t).$$

Hint 1: Easier to solve in discrete time.

Reminder: The standard approach to constrained optimization problems is to use *Lagrange multipliers*. A necessary condition that a function $f(x)$ has an extremum at x^* under the constraint that $g(x^*) = 0$ is that the gradient of the function $\mathcal{L}(x) = f(x) - \lambda g(x)$ with respect to x vanishes for $x = x^*$. Here, $\lambda \in \mathbb{R}$ is a *Lagrange multiplier*, which has to be chosen such that the constraint is fulfilled.

Exercise 2: Reverse correlation, PSTH and population response

Consider a spiking neuron with the following features:

- An input spike evokes at time t_f a current
$$\tau_s \frac{d}{dt} i(t) = -i(t) + \delta(t - t_f).$$
- Synaptic current $i(t)$ evokes a potential
$$\tau \frac{d}{dt} h(t) = -h(t) + i(t).$$
- Probability to fire a spike in Δt is
$$P(t; t + \Delta t) = \rho(t)\Delta t$$
with $\rho(t) = h(t)$.

1. Calculate the PSTH in response to a single input spike at time t_0.
2. Calculate the Reverse Correlation under the assumption of random spike arrival.
3. Calculate the response of a population of these neurons to a change in the spike arrival rate from r_0 to r_1.

Exercise 3: Population of noise-free SRM neurons

Convince yourself that for a population of SRM_0 neurons, the population activity can be written

$$A(t) = \left[1 + \frac{h'}{\eta'}\right] A(t - T_b(t)) \tag{1}$$

where $T_b(t)$ is the *preceding* firing time of the neurons that fire at t.

Hint 1: Start with the population equation and the deterministic interspike interval distribution $P(t|\hat{t}) = \delta(t - t^f(\hat{t}))$. Write the time of the next spike as $t^f = \hat{t} + T(\hat{t})$ and use the threshold condition $u(t^f) = \vartheta$.

Hint 2: $\int_a^b \delta[f(x)]g(x)dx$ is integrated with the help of a change of variable.

Exercise 4: Wilson-Cowan model

The Wilson-Cowan model for a population of neurons with absolute refractory period Δ is

$$A(t) = f[h(t)] \left[1 - \int_{t-\Delta}^{t} A(s)ds\right] \tag{2}$$

where $f[h(t)]$ is the firing rate of a neuron that is *not* in the refractory period (during the refractory period, the firing rate is zero). The potential h is defined by

$$h(t) = J_0 \int_0^\infty \epsilon(s)A(t-s)ds + \int_0^\infty \kappa(s)I(t-s)ds \tag{3}$$

4.1 Try to interpret this equation intuitively (no calculations).

4.2 Show that the Wilson-Cowan formula is a special case of the population equation

$$A(t) = \int P_I(t \mid \hat{t})A(\hat{t})d\hat{t} \tag{4}$$

if $f[h(t)]$ is regarded as an escape rate $\rho(t) = \begin{cases} 0 & t - \hat{t} < \Delta \\ f[h(t)] & t - \hat{t} \geq \Delta \end{cases}$.

Hints:

1. Start with the population equation (4) and use $P_I(t \mid \hat{t}) = \rho(t)S_I(t \mid \hat{t})$.
2. Use $1 = \int S_I(t \mid s)A(s)ds$
3. Use $S_I(t \mid s) = 1$ for $0 \leq t - s < \Delta$.