Neural Networks and Biological Modeling

Lecture 5 – Networks of Neurons and **Associative Memory**

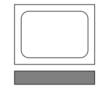
-Introduction
-Associative memory and Classification by similarity
-Detour: magnetic materials
-Associative Memory
-Hopfield Model
-Memory Capacity

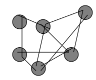
Wulfram Gerstner, EPFL

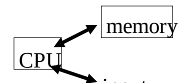
Systems for computing and information processing

Brain

Computer





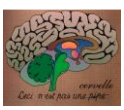


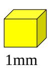
Distributed architecture

(10^{10} proc. Elements/neurons)

No separation of
processing and memory

Von Neumann architecture
1 CPU
(10^{10} transistors)

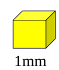




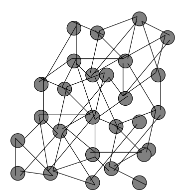
10 000 neurons
3 km wires

Systems for computing and information processing

Brain



10 000 neurons
3 km wires



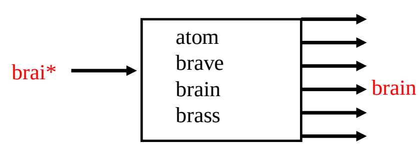
Distributed architecture
10^{10} neurons
10^{4} connections/neurons

No separation of processing and memory

Associations, Associative Memory

Read this text ***NOW!***

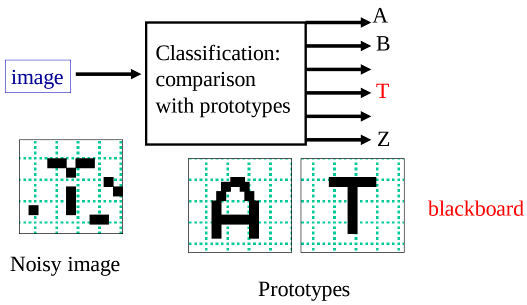
pattern completion/word recognition



Noisy word | List of words | Output the closest one

Your brain fills in missing information: 'associative memory'

- Classification by similarity:
pattern recognition

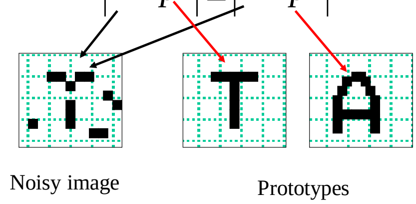


- recognize/understand images:
pattern recognition

Blackboard:

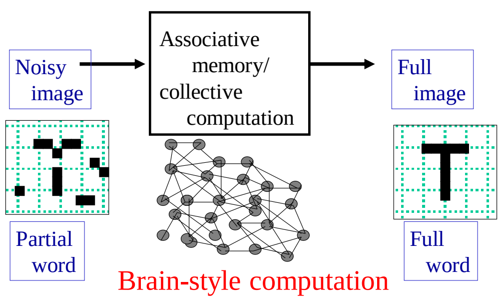
Classification by closest prototype

$$|x - p^T| \le |x - p^A|$$



Aim: Understand Associative Memory

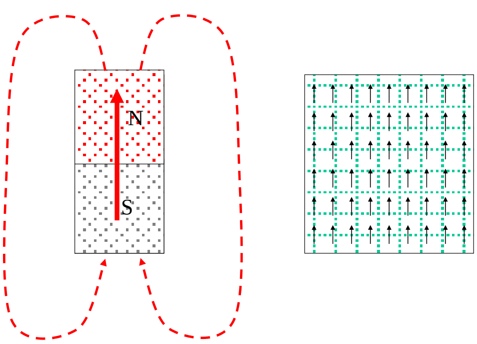
Pattern recognition/Pattern completion



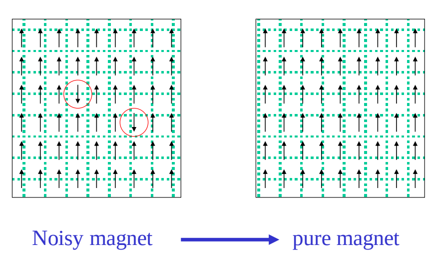
Lecture 5 – Network of neurons
and associative memory

- -Introduction
- - Associative Memory and Classification
- -Detour: magnetic materials
- -Associative Memory
- -Hopfield Model
- -Dense networks (mean-field)

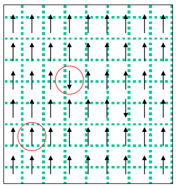
Detour: magnetism



Detour: magnetism



Detour: magnetism



Elementary magnet

↑ $S_i = +1$

↓ $S_i = -1$

Blackboard: example

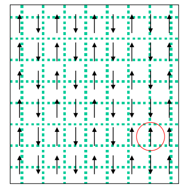
dynamics

$$S_i(t+1) = \text{sgn}\left(\sum_j S_j\right)$$

Sum over all interactions with i

Detour: magnetism

Anti-ferromagnet



Elementary magnet

↑ $S_i = +1$ ↑↑ $w_{ij} = +1$

↓ $S_i = -1$ ↑↓ $w_{ij} = -1$

dynamics

$$S_i(t+1) = \text{sgn}\left(\sum_j w_{ij} S_j\right)$$

blackboard

Sum over all interactions with i

Lecture 5 – Network of neurons and associative memory

- -Introduction
- √ -Associative Memory and Classification by similarity
- √ -Detour: magnetic materials
- → -Associative Memory
- -Hopfield Model
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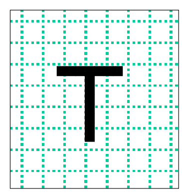
Associative memory



Elementary pixel

■ $S_i = +1$ ■↔■ $w_{ij} = +1$

□ $S_i = -1$ □↔□ $w_{ij} = +1$

□↔■ $w_{ij} = -1$

dynamics

$$S_i(t+1) = \text{sgn}\left(\sum_j w_{ij} S_j\right)$$

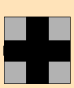
blackboard

Sum over all interactions with i

Hopfield model

Exercise 1: Associative memory (1 pattern)

Next lecture at 10h15



Elementary pixel

■ $S_i = +1$ ■↔■ $w_{ij} = +1$

□ $S_i = -1$ □↔□ $w_{ij} = +1$

□↔■ $w_{ij} = -1$

dynamics

$$S_i(t+1) = \text{sgn}\left(\sum_j w_{ij} S_j\right)$$

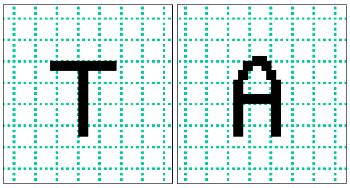
Sum over all interactions with i

9 neurons
- define appropriate weights
- what happens if one neuron wrong?
- what happens if *n* neurons wrong?

Associative memory – many patterns

Hopfield Model

Associative memory – many patterns



Prototype $\vec{p}^1$ Prototype $\vec{p}^2$

interactions

$$w_{ij} = \sum_{\mu} p_i^{\mu} p_j^{\mu}$$

Sum over all prototypes

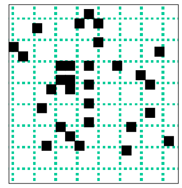
dynamics

$$S_i(t+1) = \text{sgn}\left[\sum_j w_{ij} S_j\right]$$

Sum over all interactions with i

Hopfield model

Associative memory – many patterns



Prototype $\vec{p}^1$

interactions

$$w_{ij} = \sum_{\mu} p_i^{\mu} p_j^{\mu}$$

Sum over all prototypes

This rule is very good for **random** patterns

It does not work well for correlated patters

dynamics

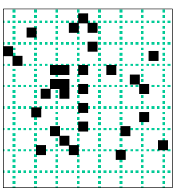
$$S_i(t+1) = \text{sgn}\left[\sum_j w_{ij} S_j\right]$$

Sum over all interactions with i

DEMO

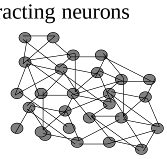
Random patterns, fully connected:
Hopfield model

Associative memory – many patterns



Prototype $\vec{p}^1$

Interacting neurons



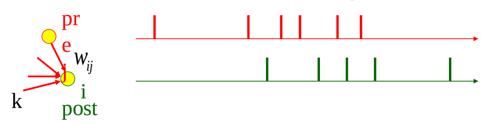
Finds the closest prototype i.e. maximal overlap (similarity) m^{μ}

Computation
- without CPU,
- without explicit memory unit

Hopfield model

Where do the connections come from?

Hebbian Learning

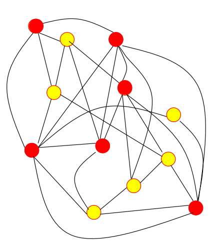


When an axon of cell **j** repeatedly or persistently takes part in firing cell **i**, then j's efficiency as one of the cells firing i is increased

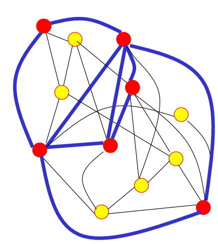
Hebb, 1949

- local rule
- simultaneously active (correlations)

Hebbian Learning



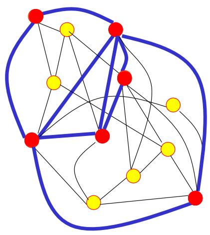
Hebbian Learning



item memorized

Hebbian Learning – Associative Recall

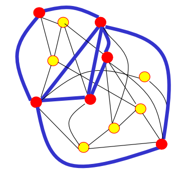
Recall:
Partial info

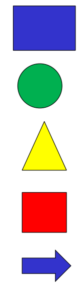


item recalled

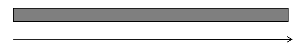
Associative Recall

Tell me the object **shape**
Tell me the **color**
the following list of 5 items:
the following list of 5 items:





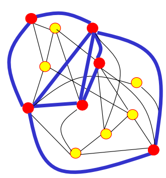
be as fast as possible:

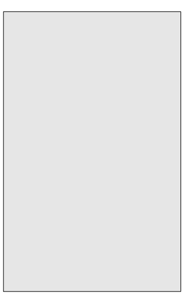


time

Associative Recall

Tell me the **color**
the following list of 5 items:



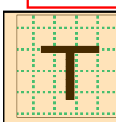


be as fast as possible:

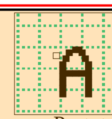
time

Stroop effect:
Slow response: hard to work
Against natural associations

Exercises 2+3 now: learning of prototypes



Prototype $\vec{p}^1$



Prototype $\vec{p}^2$

interactions

$$(1) \quad w_{ij} = \sum_{\mu} p_i^{\mu} p_j^{\mu}$$

Sum over all prototypes

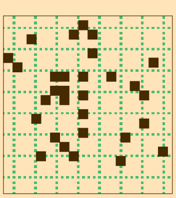
a) Show that (1) corresponds to a rate learning rule

$$(2) \quad \frac{d}{dt} w_{ij} = a_2^{corr} (\nu_j^{pre} - \vartheta)(\nu_i^{post} - \vartheta)$$

Assume that weights are zero at the beginning;
Each pattern is presented (enforced) during 0.5 sec (One after the other).
note that $p_j^{\mu} = \pm 1$ *but* $\nu_j \geq 0$

b) Compare with: $\frac{d}{dt} w_{ij} = a_0 + a_1^{pre} \nu_j^{pre} + a_1^{post} \nu_i^{post} + a_2^{corr} \nu_j^{pre} \nu_i^{post} + \ldots$

Exercise 2+3 (start now, rest homework)



Prototype $\vec{p}^1$

Next lecture at
11h15

$$w_{ij} = \frac{1}{N} \sum_{\mu} p_i^{\mu} p_j^{\mu}$$

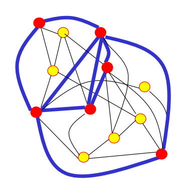
$$S_i(t+1) = \mathrm{sgn}\left[\sum_j w_{ij} S_j\right]$$

Sum over all interactions with i

Assume 4 patterns. At time t=0, overlap with Pattern 3, no overlap with other patterns.
discuss temporal evolution
(assume that patterns are orthogonal)

Associative Recall

Hierarchical organization of
Associative memory



animals

birds fish

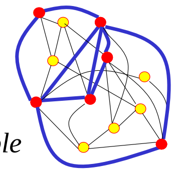
Name as fast as possible
an example of a bird

swan (or goose or raven or …)

Write down first letter: *s* for *swan* or *r* for *raven* …

Associative Recall

Nommez au plus vite possible un exemple d'un /d'une



name as fast as possible an example of a

outil	**tool**
couleur	**color**
fruit	**fruit**
instrument de musique	**music instrument**

Lecture 5 – Network of neurons and associative memory

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- -Detour: magnetic materials
- -Associative Memory
- -Hopfield model
- -How many patterns?

Memory Capacity

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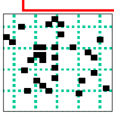
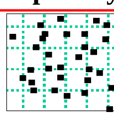
learning of prototypes



Prototype $\vec{p}^1$



Prototype $\vec{p}^2$

interactions

(1) $w_{ij} = \frac{1}{N}\sum_\mu p_i^\mu p_j^\mu$

Sum over all prototypes

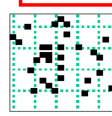
Q; How many prototypes can be stored?

dynamics $S_i(t+1) = \mathrm{sgn}\left[\sum_j w_{ij} S_j\right]$

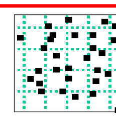
Sum over all interactions with i

Q; How many prototypes can be stored?

blackboard



Prototype $\vec{p}^1$



Prototype $\vec{p}^2$

Random patterns

Interactions (1) $w_{ij} = \sum_\mu p_i^\mu p_j^\mu$

Dynamics (2) $S_i(t+1) = \mathrm{sgn}\left[\sum_j w_{ij} S_j\right]$

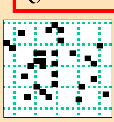
Minimal condition: pattern is fixed point of dynamics

-Assume we start directly in one pattern
-Pattern stays

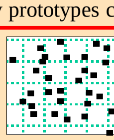
Attention: Retrieval requires more (pattern completion)

Exercise 4 now: Associative memory

Q; How many prototypes can be stored?



Prototype $\vec{p}^1$



Prototype $\vec{p}^2$

End of lecture, exercise+ Computer exercise : 12:00

Random patterns

Interactions (1) $w_{ij} = \sum_\mu p_i^\mu p_j^\mu$

Dynamics (2) $S_i(t+1) = \mathrm{sgn}\left[\sum_j w_{ij} S_j\right]$

Random patterns → random walk

a) show relation to erf function: importance of p/N
b) network of 1000 neurons – allow at most 1 wrong pixel?
c) network of N neurons – at most 1 promille wrong pixels?

The end