Lecture 4: *Wulfram Gerstner, EPFL*

-Complement to 2-dim. Neuron models

-separation of time scales

-Hopf bifurcation

-Saddle node

BOOK: Spiking Neuron Models,
W. Gerstner and W. Kistler
Cambridge University Press, 2002
Chapter 3

-Introduction to Hebbian Plasticity

-Hebb rule

-BCM rule

BOOK: Spiking Neuron Models,
W. Gerstner and W. Kistler
Cambridge University Press, 2002
Chapter 10.1+10.2+11.1

-Receptive Fields

-development

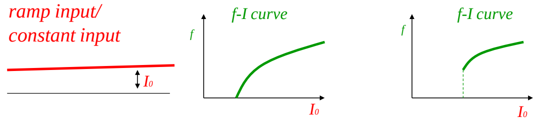
Can we understand the dynamics of the Hodgkin-Huxley model?

- *mathematical principle of Action Potential generation?*
- *Types of neuron model (type I and II)?*
- *threshold behavior?*

→ yes, we can after reduction to 2 dimensions

TODAY: SEPARATION OF TIME SCALES
BIFURCATIONS AND NEURONAL BEHAVIOR

Type I and type II models

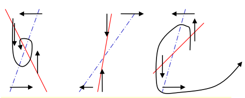


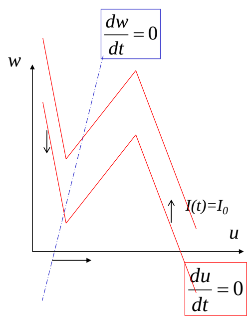
Phase plane analysis- solution of Exercise

stimulus

$$\tau \frac{du}{dt} = au - w + RI_0$$

$$\tau_w \frac{dw}{dt} = cu - w$$





FitzHugh Nagumo Model – limit cycle

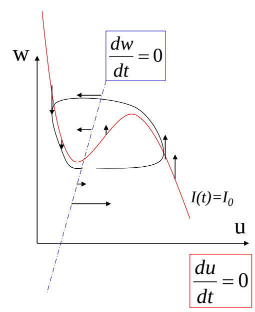
stimulus

$$\tau \frac{du}{dt} = F(u,w) + RI(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$

-unstable fixed point
-closed boundary
with arrows pointing inside

→ limit cycle



limit cycle

Separation of time scales

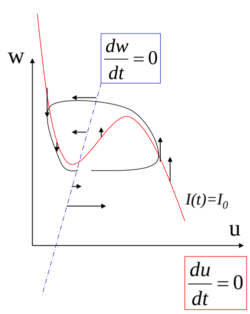
stimulus

$$\tau_u \frac{du}{dt} = F(u,w) + RI(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$

$$\tau_w >> \tau_u$$

Blackboard:
separation of time scales



limit cycle

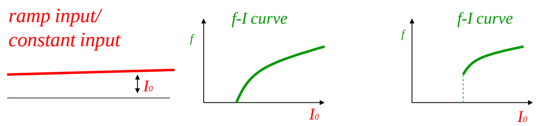
Can we understand the dynamics of the HH model?

- *mathematical principle of Action Potential generation?*
- *Types of neuron model (type I and II)?*
- *threshold behavior?*

→ yes, we can after reduction to 2 dimensions

Now: BIFURCATIONS AND NEURONAL BEHAVIOR

Type I and type II models



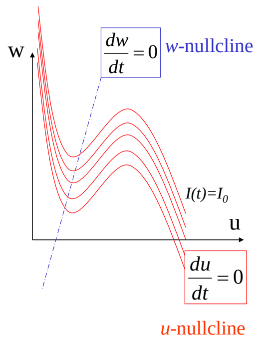
Reduction of Hodgkin-Huxley Model: 2 dimensional Neuron Models

stimulus

$$\tau \frac{du}{dt} = F(u,w) + I(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$

apply constant stimulus I_0



FitzHugh Nagumo Model – limit cycle

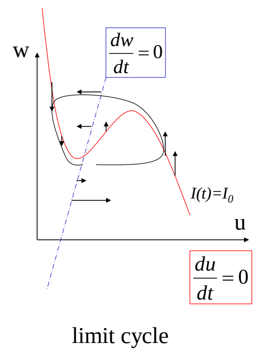
stimulus

$$\tau \frac{du}{dt} = F(u,w) + I(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$

-unstable fixed point
-closed boundary
with arrows pointing inside

→ limit cycle



limit cycle

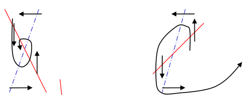
Type II Model constant input

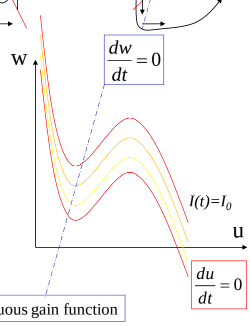
stimulus

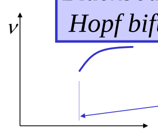
$$\tau \frac{du}{dt} = F(u,w) + I(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$

Blackboard: Hopf bifurcation







Discontinuous gain function

Stability lost → oscillation with finite frequency

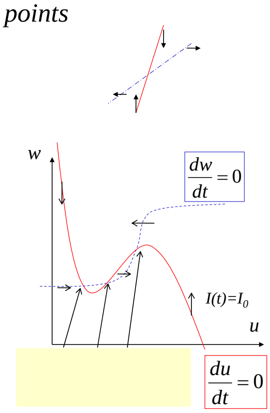
type I Model- 3 fixed points

stimulus

$$\tau \frac{du}{dt} = F(u,w) + I(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$

Blackboard

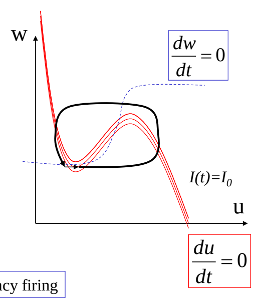


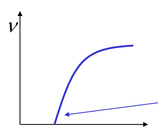
type I Model – constant input

stimulus

$$\tau \frac{du}{dt} = F(u,w) + I(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$

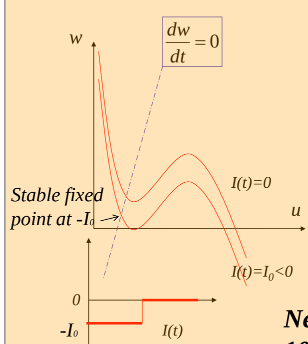




Low-frequency firing

Exercise 1: NOW!

inhibitory rebound



Exercise 2 at home

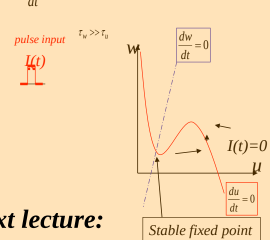
Separation of time scales

stimulus

$$\tau \frac{du}{dt} = F(u,w) + I(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$

pulse input $\tau_w >> \tau_u$



Next lecture: 10:15

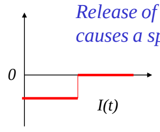
Discussion of Exercise 1 – inhibitory rebound

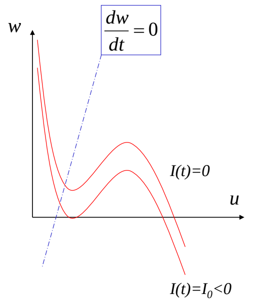
stimulus

$$\tau \frac{du}{dt} = F(u,w) + I(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$

Inhibitory rebound:
Release of inhibition
causes a spike





Show this in phase plane!

Lecture 4: *Wulfram Gerstner, EPFL*

-Complement to 2-dim. Neuron models
- -type I and type II
- -constant input
- ➔ -pulse input

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-Introduction to Hebbian Plasticity
- -Hebb rule
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Cambridge University Press, 2002
Chapter 10.1+10.2+11.1

-Receptive Fields
- -development

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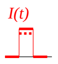
type I Model-pulse input

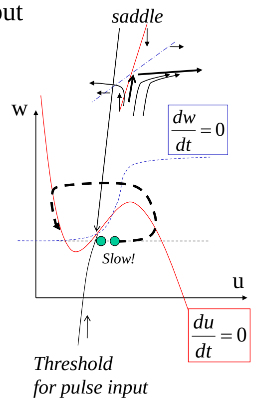
stimulus

$$\tau \frac{du}{dt} = F(u,w) + I(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$

pulse input

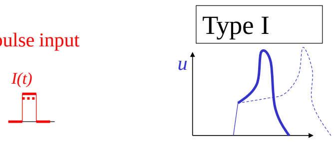




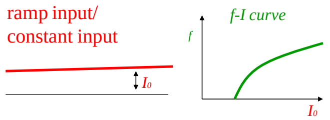
Type I and type II models

Response at firing threshold?

pulse input



ramp input/
constant input



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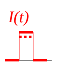
type II Model - pulse input

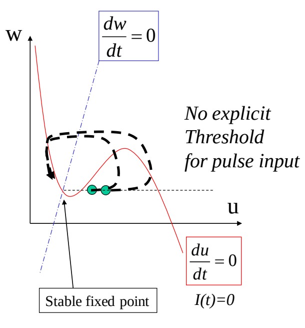
stimulus

$$\tau \frac{du}{dt} = F(u,w) + I(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$

pulse input

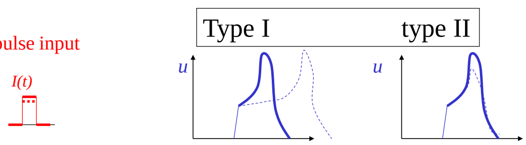




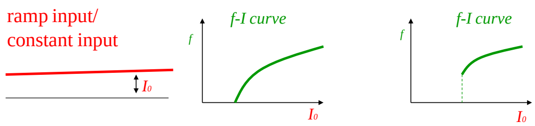
Type I and type II models

Response at firing threshold?

pulse input



ramp input/
constant input



type II Model - pulse input threshold?

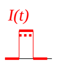
Separation of time scales

stimulus

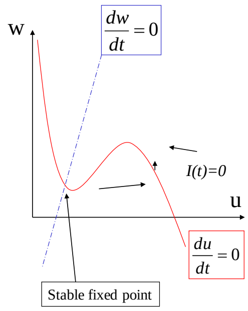
$$\tau \frac{du}{dt} = F(u,w) + I(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$

pulse input $\quad \tau_w >> \tau_u$



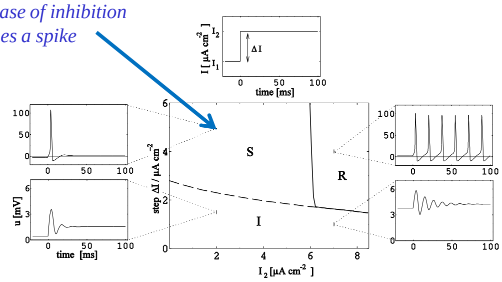
Blackboard



Inhibitory rebound:Hodgkin-Huxley Model

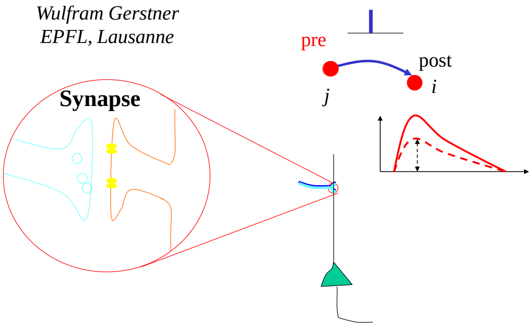
Step current input

Inhibitory rebound:
Release of inhibition
causes a spike

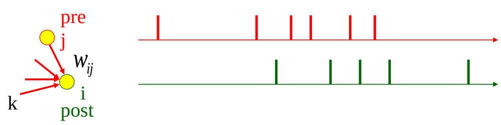


Models of synaptic Plasticity

Wulfram Gerstner
EPFL, Lausanne



Hebbian Learning

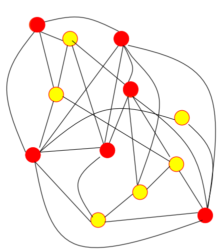


*When an axon of cell **j** repeatedly or persistently takes part in firing cell **i**, then j's efficiency as one of the cells firing i is increased*

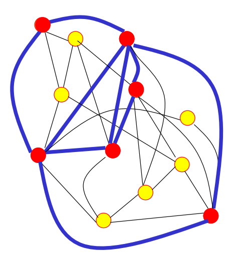
Hebb, 1949

- local rule
- simultaneously active (correlations)

Hebbian Learning



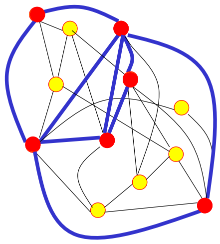
Hebbian Learning



item memorized

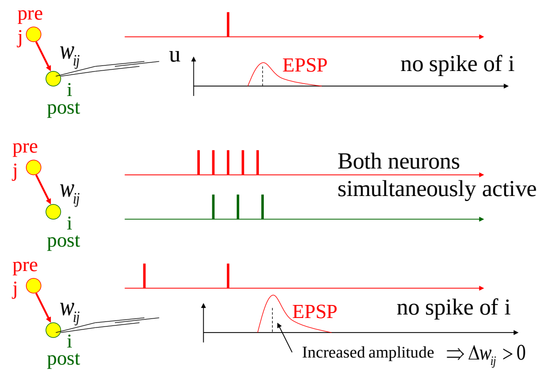
Hebbian Learning

Recall:
Partial info



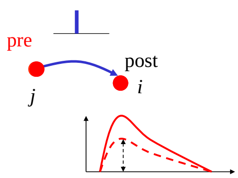
item recalled

Hebbian Learning in experiments (schematic)



Synaptic Dynamics

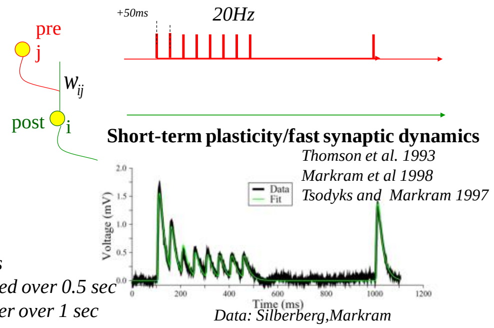
Induction of changes
- *fast (if stimulated appropriately)*
- *slow (homeostasis)*



Persistence of changes
- *long (LTP/LTD)*
- *short (short-term plasticity)*

Functionality
- *useful for learning a new behavior*
- *useful for development (e.g., wiring for receptive field development)*
- *useful for activity control in network*
- *useful for coding*

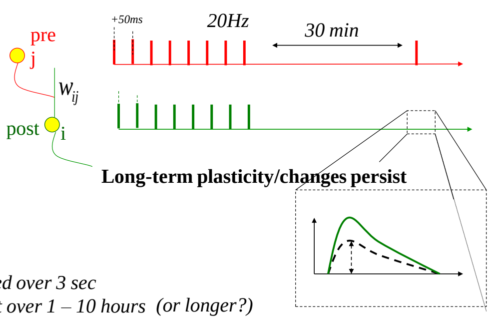
Classification of plasticity: short-term vs. Long-term



Changes
- *induced over 0.5 sec*
- *recover over 1 sec*

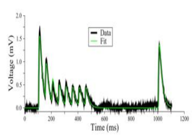
Data: Silberberg,Markram
Fit: Richardson (Tsodyks-Markram model)

Classification of plasticity: short-term vs. Long-term



Changes
- *induced over 3 sec*
- *persist over 1 – 10 hours (or longer?)*

Classification of plasticity: short-term vs. Long-term



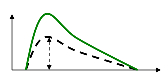
Changes
- *induced over 0.1-0.5 sec*
- *recover over 1 sec*

Protocol
- *presynaptic spikes*

Model
- *well established*

(Tosdyks, Senn, Markram)

LTP/LTD/Hebb

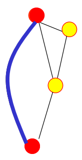


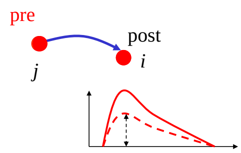
Changes
- *induced over 0.5-5sec*
- *remains over hours*

Protocol
- *presynaptic spikes + …*

Model
- *we will see*

Hebbian Learning
= unsupervised learning

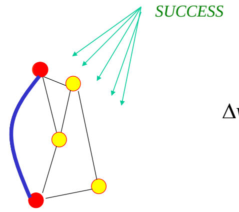




$$w_{ij}\,\varepsilon\left(t - t_j^f\right)$$

$$\Delta w_{ij} \propto F(\,pre,\,post\,)$$

Reinforcement Learning
= reward + Hebb



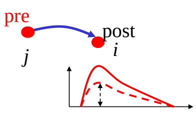
$$\Delta w_{ij} \propto F(\,pre,\,post,\,SUCCESS\,)$$

local global

Classification of plasticity: unsupervised vs reinforcement

LTP/LTD/Hebb

Theoretical concept
- *passive changes*
- *exploit statistical correlations*

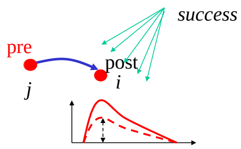


Functionality

-useful for development
(wiring for receptive fields)

Reinforcement Learning

Theoretical concept
- *conditioned changes*
- *maximise reward*



Functionality
- *useful for learning a new behavior*

Models of synaptic Plasticity

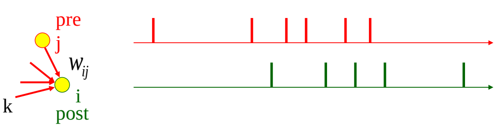
√ 0. Introduction

I. Hebbian Learning (unsupervised): review of rate-based theory

II. Spike-Timing Dependent theory

Swiss Federal Institute of Technology Lausanne, EPFL
Laboratory of Computational Neuroscience, LCN, CH 1015 Lausanne

Hebbian Learning (rate models)



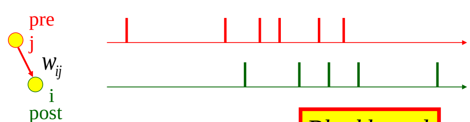
*When an axon of cell **j** repeatedly or persistently takes part in firing cell **i**, then j's efficiency as one of the cells firing i is increased*

Hebb, 1949

- local rule
- simultaneously active (correlations)

Rate model:
active = high rate = many spikes per second

Rate-based Hebbian Learning

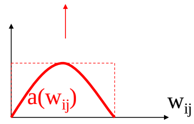


Blackboard

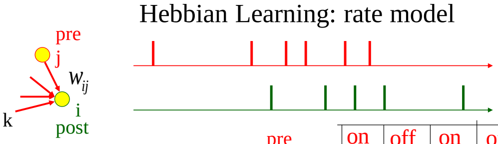
$$\frac{d}{dt} w_{ij} = F(w_{ij};\nu_j^{pre},\nu_i^{post})$$

$$\frac{d}{dt} w_{ij} = a_0 + a_1^{pre}\nu_j^{pre} + a_1^{post}\nu_i^{post} + a_2^{corr}\nu_j^{pre}\nu_i^{post} + \ldots$$

$a = a(w_{ij})$



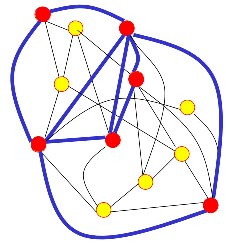
Hebbian Learning: rate model



pre	on	off	on	off
post	on	on	off	off
$\frac{d}{dt}w_{ij} = a_2^{corr}\nu_j^{pre}\nu_i^{post}$	+	0	0	0

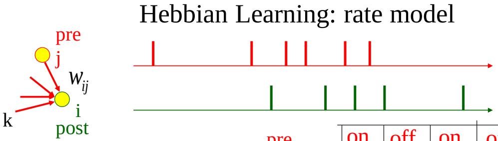
Hebbian Learning

Recall:
Partial info



item recalled

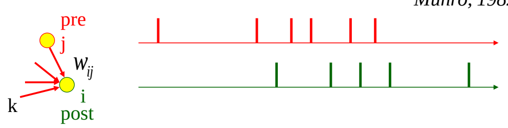
Hebbian Learning: rate model



pre	on	off	on	off
post	on	on	off	off
$\frac{d}{dt}w_{ij} = a_2^{corr}\nu_j^{pre}\nu_i^{post}$	+	0	0	0
$\frac{d}{dt}w_{ij} = a_2^{corr}\nu_j^{pre}\nu_i^{post} - c$	+	-	-	-
$\frac{d}{dt}w_{ij} = a_2^{corr}\nu_j^{pre}(\nu_i^{post} - \vartheta)$	+	0	-	0
$\frac{d}{dt}w_{ij} = a_2^{corr}(\nu_j^{pre} - \vartheta)(\nu_i^{post} - \vartheta)$	+	-	-	+

Rate-based Hebbian Learning: BCM

Bienenstock, Cooper Munro, 1982



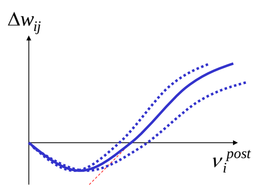
presynaptically gated

$$\frac{d}{dt}w_{ij} = a_2^{corr}(\nu_i^{post} - \vartheta)\nu_j^{pre}$$

BCM

$$\frac{d}{dt}w_{ij} = a_2^{corr}\Phi(\nu_i^{post} - \vartheta)\nu_j^{pre}$$

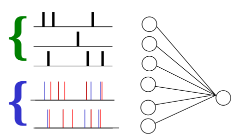
$$\vartheta = f(\bar{\nu}_i^{post})$$



Functional consequences of Hebbian Learning

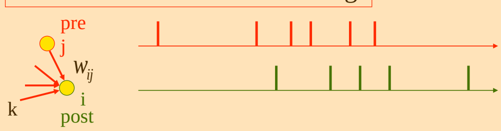
Fixed rate

Jointly variing rate



Detects correlations in the input

Exercise now: Hebbian Learning:



Show that

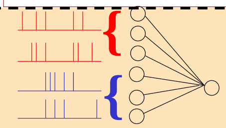
$$\frac{d}{dt}w_{ij} = a_2^{corr}(\nu_i^{post} - \vartheta)\nu_j^{pre}$$

Is a special case of

$$\frac{d}{dt}w_{ij} = a_0 + a_1^{pre}\nu_j^{pre} + a_1^{post}\nu_i^{post} + a_2^{corr}\nu_j^{pre}\nu_i^{post} + \ldots$$

Next lecture at 11:15

Exercise 3 and 4 now: Hebbian Learning:



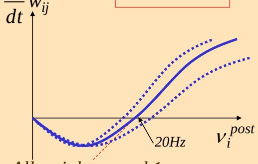
$$\nu_i^{post} = g(I_i) = \sum_j w_{ij}\, \nu_j^{pre}$$

Next lecture: 11:20

BCM rule

$$\frac{d}{dt} w_{ij} = a_2^{corr}\, \Phi(\nu_i^{post} - \vartheta)\, \nu_j^{pre}$$

20Hz



Next lecture at 11:20

Assume 2 groups of 10 neurons each. All weights equal 1.
a)Group 1 fires at 3 Hz, then group 2 at 1 Hz. What happens?
b)Group 1 fires at 3 Hz, then group 2 at 2.5 Hz. What happens?
c) As in b, but make theta a function of the averaged rate. What happens?

Lecture 4: *Wulfram Gerstner, EPFL*

-Complement to 2-dim. Neuron models

-Introduction to Hebbian Plasticity
- -Hebb rule
- -BCM rule
- - functional consequences of Hebb rule

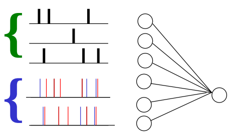
BOOK: Spiking Neuron Models,
W. Gerstner and W. Kistler
Cambridge University Press, 2002
Chapter 10.1+10.2+11.1

-Receptive Fields
- -orientation selectivity
- -development

Functional consequences of Hebbian Learning

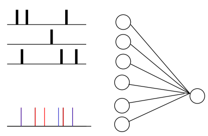
Fixed rate

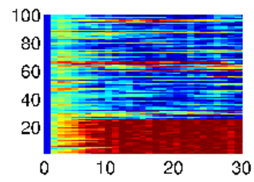
Jointly variing rate



Detects correlations in the input

Example: BCM

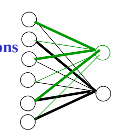




Plasticity rule detects where the 'interesting' input occurs
-- some synapses strengthened at the expense of others

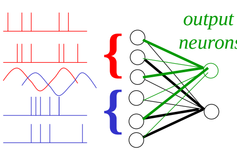
Synaptic changes for development

Initial: random connections

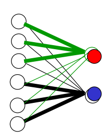


unselective neurons

BCM leads to specialized Neurons (developmental learning);
BUT: ***rate model***

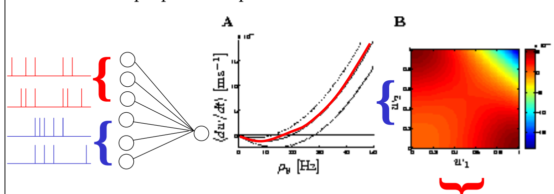


Correlated input



output neurons specialize

Functional properties: specialisation

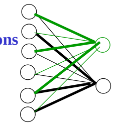


Mean weight in first group

BCM leads to specialized Neurons (developmental learning);
BUT: ***rate model***

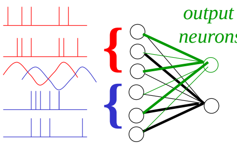
Detour: Receptive field development

Initial: random connections



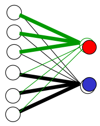
unselective neurons

Hebbian learning leads to specialized Neurons (developmental learning);



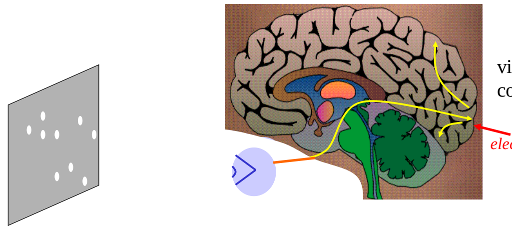
output neurons

Correlated input



output neurons specialize

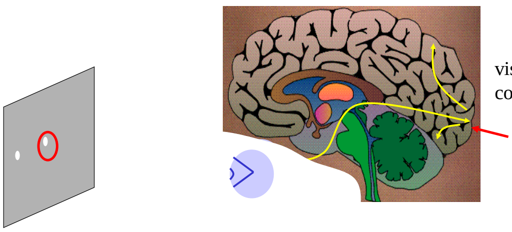
Detour: Receptive field development



visual cortex

electrode

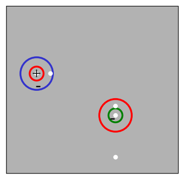
Detour: Receptive field development



visual cortex

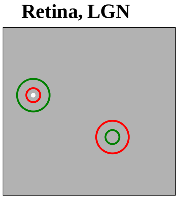
Detour: Receptive field development

Receptive fields:
Retina, LGN

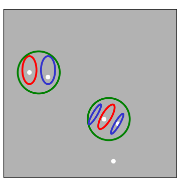


Detour: Receptive field development

Receptive fields:
Retina, LGN



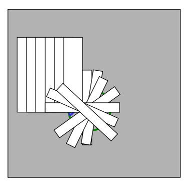
Receptive fields:
visual cortex V1



Orientation selective

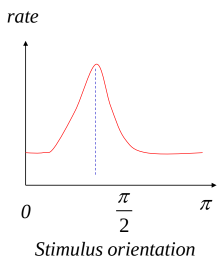
Detour: Receptive field development

Receptive fields:
visual cortex V1

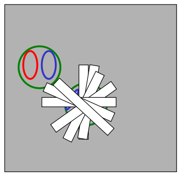


Orientation selective

Detour: Receptive field development



Receptive fields:
visual cortex V1

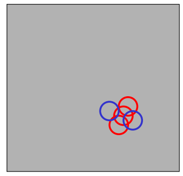


Orientation selective

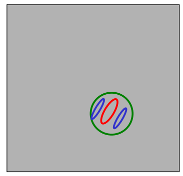
Detour: Receptive field development

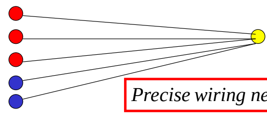
What makes cells Orientation selective? - wiring

Receptive fields:
in LGN



Receptive fields:
visual cortex V1

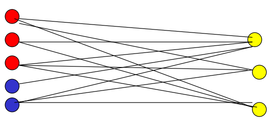




Precise wiring necessary – how done?

Detour: Receptive field development

Precise wiring necessary – how done?



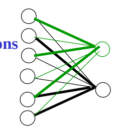
Make conetions plastic:

Hebbian (unsupervised) learning rule

Detour: Receptive field development

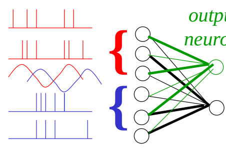
Hebbian leanring leads to specialized
Neurons (developmental learning);

Initial:
random
connections

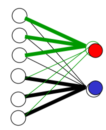


unselective
neurons

output
neurons



Correlated input

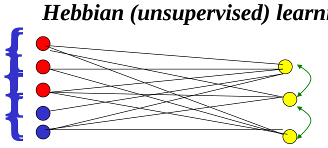


output neurons specialize

Detour: Receptive field development

Precise wiring necessary – how done?

Hebbian (unsupervised) learning rule



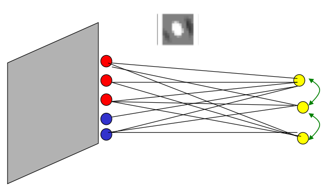
LGN neurons

cortical neurons

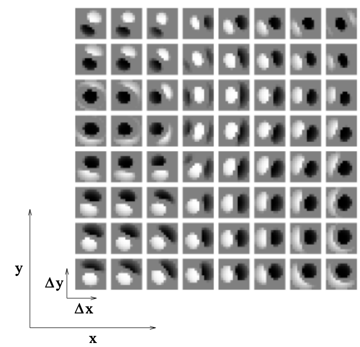
Stimulation with
Locally correlated inputs

Wiring develops=
receptive fields develop

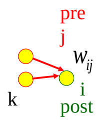
Detour: Receptive field development - model results



Detour: Receptive field development - model results



Neuron with two inputs:



Receptive field development: neurons specialize

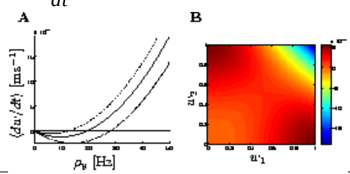
2 patterns $\nu_1^{pre} = 0.1$ $\nu_1^{pre} = 1$

$\nu_2^{pre} = 1$ $\nu_2^{pre} = 0.1$

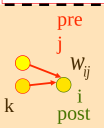
$$\nu^{post} = \sum_k w_k \nu_k^{pre}$$

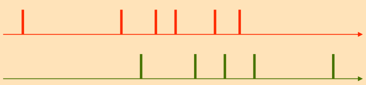
$$\frac{d}{dt} w_{ij} = a_2^{corr} \Phi(\nu_i^{post} - \vartheta)\nu_j^{pre}$$

See exercise 4



Computer exercise now: Hebbian Learning:





Oja's rule

$$\frac{d}{dt} w_{ij} = a_2^{corr} \nu_j^{pre} \nu_i^{post} - w_{ij}(\nu_i^{post})^2$$

BCM rule

$$\frac{d}{dt} w_{ij} = a_2^{corr} \Phi(\nu_i^{post} - \vartheta)\nu_j^{pre}$$

Computer 12:00-12:45

The end