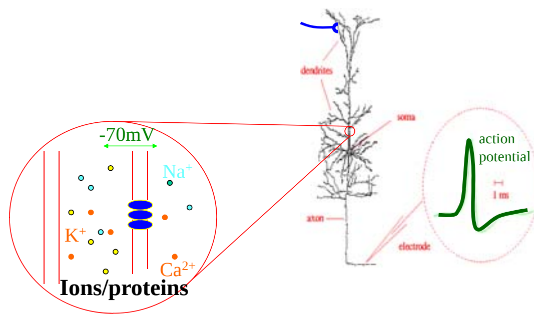
Lecture 3: Two-dimensional neuron models

BOOK: Spiking Neuron Models,
W. Gerstner and W. Kistler
Cambridge University Press, 2002

Chapter 3

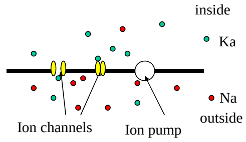
Biophysics of neurons

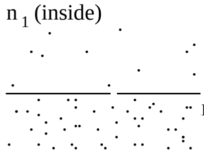


Hodgkin-Huxley Model

Ion density

$$n \propto e^{-\frac{E}{kT}}$$



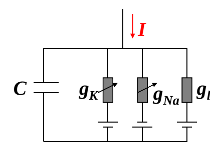


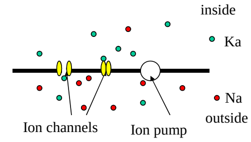
$$\Delta u = u_1 - u_2 = \frac{kT}{q} \ln \frac{n(u_1)}{n(u_2)}$$

Reversal potential

Concentration difference ⇔ voltage difference

Hodgkin-Huxley Model

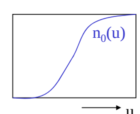


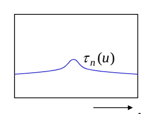


stimulus

$$C\frac{du}{dt} = \underbrace{-g_{Na}m^3h(u-E_{Na})}_{I_{Na}} \underbrace{- g_K n^4 (u-E_K)}_{I_K} \underbrace{- g_l (u-E_l)}_{I_{leak}} + I(t)$$

$$\frac{dm}{dt} = -\frac{m - m_0(u)}{\tau_m(u)}$$



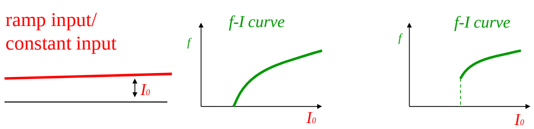


Can we understand the dynamics of the HH model?
- mathematical principle of Action Potential generation?
- Types of neuron model (type I and II)?
- threshold behavior?

→ Reduce from 4 to 2 dimensions

Type I and type II models

ramp input/
constant input



Two-dimensional neuron models

-Reduction of Hodgkin-Huxley
-step 1: separation of time scales

Reduction of Hodgkin-Huxley Model

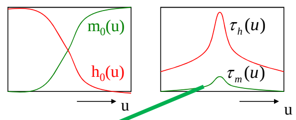
stimulus

$$C\frac{du}{dt} = \overbrace{g_{Na}m^3h(u-E_{Na})}^{I_{Na}} + \overbrace{g_K n^4(u-E_K)}^{I_K} + \overbrace{g_l(u-E_l)}^{I_{leak}} + I(t)$$

$$\frac{dm}{dt} = -\frac{m-m_0(u)}{\tau_m(u)}$$

$$\frac{dh}{dt} = -\frac{h-h_0(u)}{\tau_h(u)}$$

$$\frac{dn}{dt} = -\frac{n-n_0(u)}{\tau_n(u)}$$



1) dynamics of m is fast $\longrightarrow$ $m(t) = m_0(u(t))$

Reduction of Hodgkin-Huxley Model

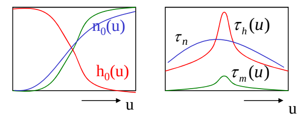
stimulus

$$C\frac{du}{dt} = \overbrace{g_{Na}m^3h(u-E_{Na})}^{I_{Na}} + \overbrace{g_K n^4(u-E_K)}^{I_K} + \overbrace{g_l(u-E_l)}^{I_{leak}} + I(t)$$

$$\frac{dm}{dt} = -\frac{m-m_0(u)}{\tau_m(u)}$$

$$\frac{dh}{dt} = -\frac{h-h_0(u)}{\tau_h(u)}$$

$$\frac{dn}{dt} = -\frac{n-n_0(u)}{\tau_n(u)}$$



1) dynamics of m is fast $\longrightarrow$ $m(t) = m_0(u(t))$

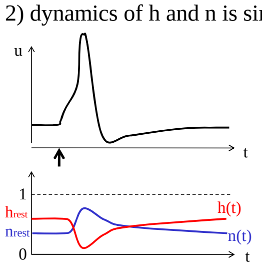
2) dynamics of h and n is similar

Reduction of Hodgkin-Huxley Model

stimulus

$$C\frac{du}{dt} = g_{Na}m^3h(u-E_{Na}) + g_K n^4(u-E_K) + g_l(u-E_l) + I(t)$$

2) dynamics of h and n is similar $\longrightarrow$ $1-h(t) = a\,n(t)$



Blackboard

Reduction of Hodgkin-Huxley Model

stimulus

$$C\frac{du}{dt} = \overbrace{g_{Na}m_0(u)^3(1-w)(u-E_{Na})}^{I_{Na}} + \overbrace{g_K(\frac{w}{a})^4(u-E_K)}^{I_K} + \overbrace{g_l(u-E_l)}^{I_{leak}} + I(t)$$

$$\frac{dw}{dt} = -\frac{w-w_0(u)}{\tau_w(u)}$$

1) dynamics of m is fast $\longrightarrow$ $m(t) = m_0(u(t))$

2) dynamics of h and n is similar $\longrightarrow$ $\underbrace{1-h(t)}_{w} = \underbrace{a\,n(t)}_{w}$

NOW Exercise 1: separation of time scales

stimulus

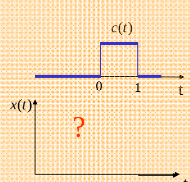
$$C\frac{du}{dt} = \overbrace{g_{Na}m^3h(u-E_{Na})}^{I_{Na}} + \overbrace{g_K n^4(u-E_K)}^{I_K} + \overbrace{g_l(u-E_l)}^{I_{leak}} + I(t)$$

$$\frac{dm}{dt} = -\frac{m-m_0(u)}{\tau_m(u)}$$

Exercises:
1.1-1.4 ***now!***
1.5 *homework*

Next lecture:
10h15

$$\frac{dx}{dt} = -\frac{x-c(t)}{\tau}$$



$$\frac{dm}{dt} = -\frac{m-c(u)}{\tau_m}$$

$$\frac{du}{dt} = f(u) - m$$

Two-dimensional neuron models

-Reduction of Hodgkin-Huxley
  -step 1: separation of time scales
  -step 2: effective variable w

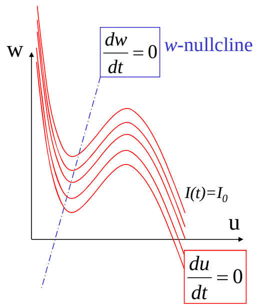
- Phase plane analysis

Reduction of Hodgkin-Huxley Model: 2 dimensional Neuron Models

stimulus

$$\tau \frac{du}{dt} = F(u,w) + I(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$



Phase plane analysis-Exercise 2 NOW

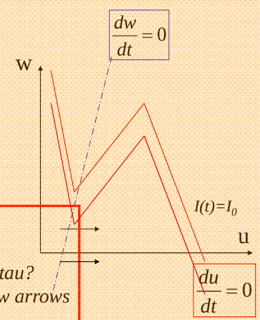
Next lecture:
11h15

stimulus

$$\tau \frac{du}{dt} = au - w + I_0$$

$$\tau_w \frac{dw}{dt} = cu - w$$

Exercise:
2.1. Calculate stability for parameters a,b,influence of tau?
2.2 (i) sketch nullclines and flow arrows
(ii) homework

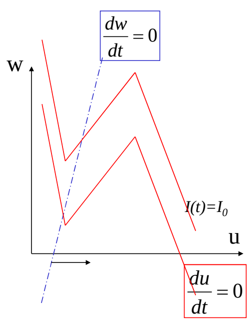


Phase plane analysis- solution of Exercise

stimulus

$$\tau \frac{du}{dt} = au - w + I_0$$

$$\tau_w \frac{dw}{dt} = cu - w$$



Flow and phase plane analysis-solution

stimulus

$$\tau \frac{du}{dt} = au - w + I_0$$

$$\tau_w \frac{dw}{dt} = cu - w$$

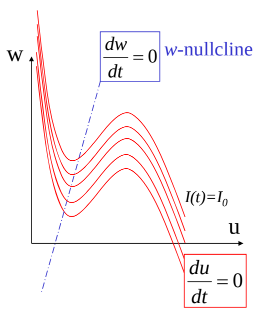
Reduction of Hodgkin-Huxley Model: 2 dimensional Neuron Models

stimulus

$$\tau \frac{du}{dt} = F(u,w) + I(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$

apply constant stimulus I_0



FitzHugh Nagumo Model – limit cycle

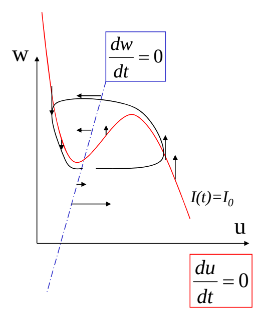
stimulus

$$\tau \frac{du}{dt} = F(u,w) + I(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$

-unstable fixed point
-closed boundary
with arrows pointing inside

⟶ limit cycle



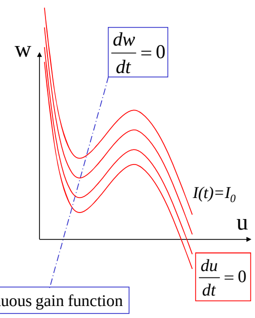
limit cycle

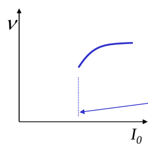
type II Model – constant input

stimulus

$$\tau \frac{du}{dt} = F(u,w) + I(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$





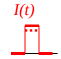
type II Model - pulse input

stimulus

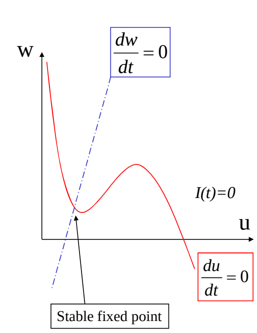
$$\tau \frac{du}{dt} = F(u,w) + I(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$

pulse input



Blackboard

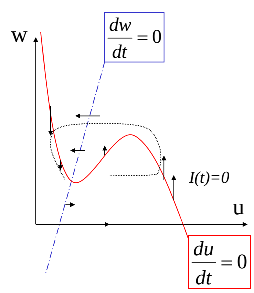


FitzHugh-Nagumo Model
2-dimensional Neuron Models

stimulus

$$\tau \frac{du}{dt} = F(u,w) + I(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$



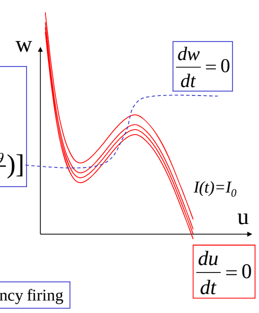
type I Model – constant input

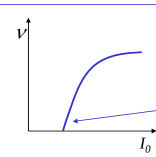
stimulus

$$\tau \frac{du}{dt} = F(u,w) + I(t)$$

$$\frac{dw}{dt} = -\frac{w - w_0(u)}{\tau_w(u)}$$

$$w_0(u) = 0.5[1 + \tanh(\tfrac{u-\theta}{d})]$$



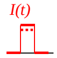


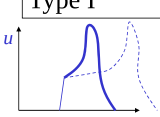
Type I and type II models

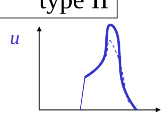
Response at firing threshold?

Type I	type II

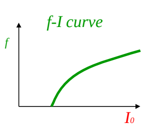
pulse input

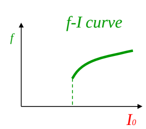






ramp input/
constant input





now 12:00 Computer Show

Computer Exercise – type I/type II model

stimulus

$$\tau \frac{du}{dt} = F(u,w) + I(t)$$

$$\tau_w \frac{dw}{dt} = G(u,w)$$

ramp input/
constant input