Detailed neuron models:
Hodgkin-Huxeley model

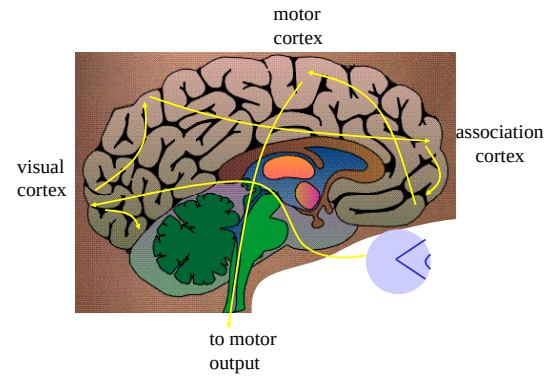
1: Introduction to Hodgkin-Huxley models

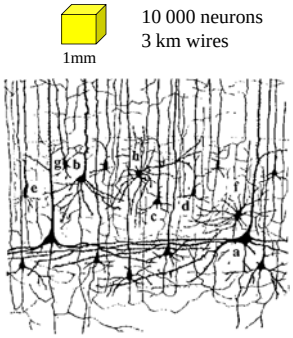
2: threshold in the Hodgkin-Huxeley model

3. Synaptic input (conductance input)

Wulfram Gerstner
http://lcn.epfl.ch

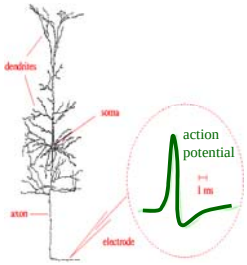
moodle.epfl.ch



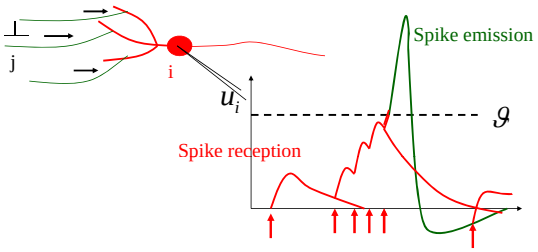


10 000 neurons
3 km wires

Signal:
action potential (spike)

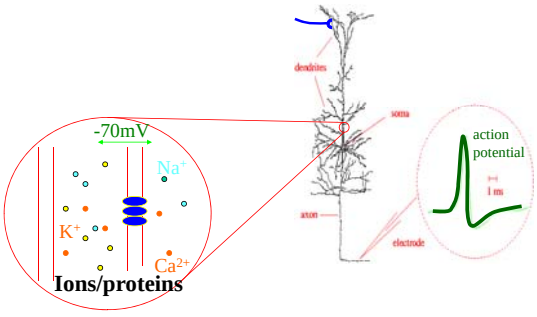


Integrate-and-fire type models



-spikes are events
-threshold
-spike/reset/refractoriness

Hodgkin-Huxley type models



1: Introduction to **Hodgkin-Huxley** models

BOOK: Spiking Neuron Models,
W. Gerstner and W. Kistler
Cambridge University Press, 2002

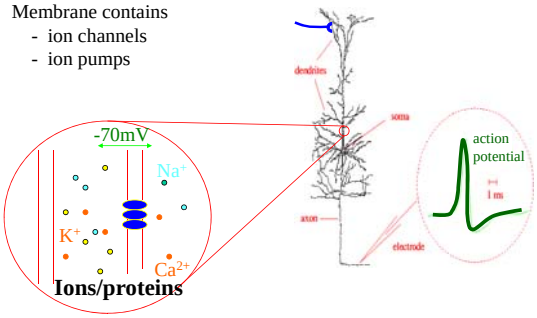
Chapter 2

Chapter 2: Detailed neuron models
Hodgkin-Huxley model

Wulfram Gerstner
http://lcn.epfl.ch

Biophysics of neurons

Cell surrounded by membrane
Membrane contains
- ion channels
- ion pumps

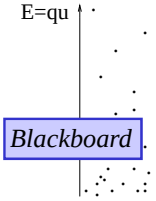


Hodgkin-Huxley Model

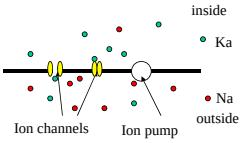
Ion density

$$n \propto e^{-\frac{E}{kT}}$$

E=qu



Blackboard



Ion pump ⇔ Concentration difference

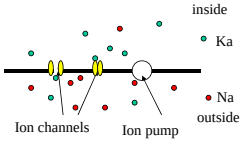
Reversal potential

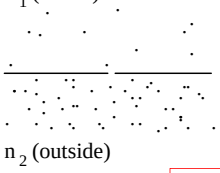
Concentration difference ⇔ voltage difference

Hodgkin-Huxley Model

Ion density

$$n \propto e^{-\frac{E}{kT}}$$



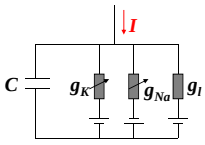


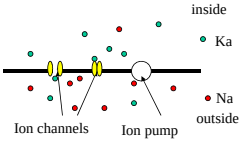
$$\Delta u = u_1 - u_2 = \frac{kT}{q} \ln \frac{n(u_1)}{n(u_2)}$$

Reversal potential

Concentration difference ⇔ voltage difference

Hodgkin-Huxley Model



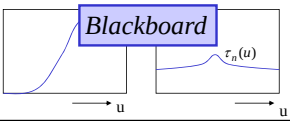


stimulus

$$C\frac{du}{dt} = \underbrace{-g_{Na}m^3h(u-E_{Na})}_{I_{Na}} \underbrace{-g_K n^4(u-E_K)}_{I_K} \underbrace{-g_l(u-E_l)}_{I_{leak}} + I(t)$$

$$\frac{dm}{dt} = \frac{m_0(u)-m}{\tau_m(u)} \qquad \frac{dn}{dt} = \frac{n_0(u)-n}{\tau_n(u)}$$

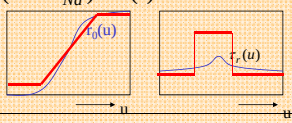
Blackboard

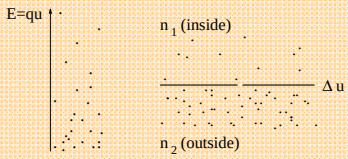


Exercise 1+2 now: dynamics of ion channels

$$C\frac{du}{dt} = -g_{ion} r^{n_1} s^{n_2} (u - E_{Na}) + I(t)$$

$$\frac{dr}{dt} = -\frac{r - r_0(u)}{\tau_r(u)}$$

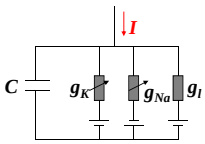


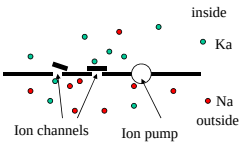


Reversal potential

Next lecture: 10H15

Hodgkin-Huxley Model



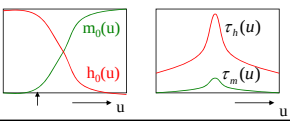


stimulus

$$C\frac{du}{dt} = \underbrace{-g_{Na}m^3h(u-E_{Na})}_{I_{Na}} \underbrace{-g_K n^4(u-E_K)}_{I_K} \underbrace{-g_l(u-E_l)}_{I_{leak}} + I(t)$$

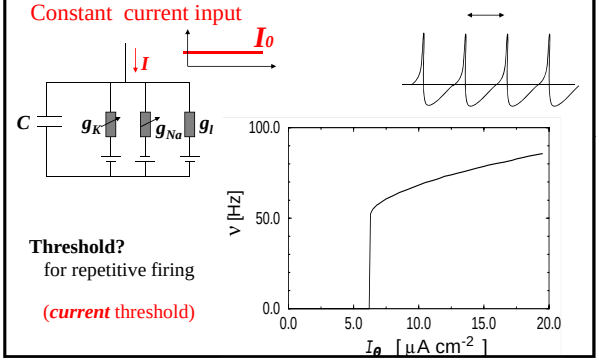
$$\frac{dm}{dt} = -\frac{m - m_0(u)}{\tau_m(u)}$$

$$\frac{dh}{dt} = -\frac{h - h_0(u)}{\tau_h(u)}$$



Hodgkin-Huxley Model

Constant current input

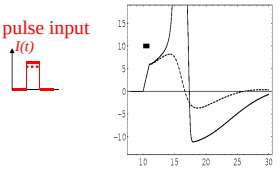


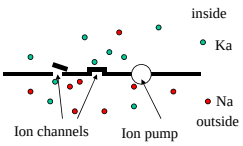
Threshold?
for repetitive firing

(***current*** threshold)

Hodgkin-Huxley Model

pulse input





Threshold?

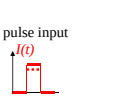
- AP if amplitude 7.0 units
- No AP if amplitude 6.9 units

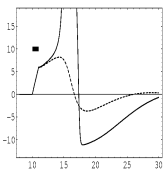
(pulse with 1ms duration)

(and pulse with 0.5 ms duration?)

Hodgkin-Huxley Model

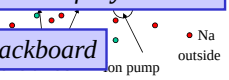
pulse input





*Where is the threshold? question for **you!***

Blackboard

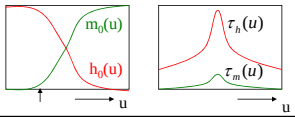


stimulus

$$C\frac{du}{dt} = \underbrace{-g_{Na}m^3h(u-E_{Na})}_{I_{Na}} \underbrace{-g_K n^4(u-E_K)}_{I_K} \underbrace{-g_l(u-E_l)}_{I_{leak}} + I(t)$$

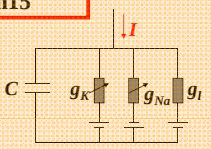
$$\frac{dm}{dt} = -\frac{m-m_0(u)}{\tau_m(u)}$$

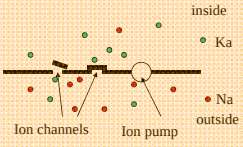
$$\frac{dh}{dt} = -\frac{h-h_0(u)}{\tau_h(u)}$$



Hodgkin-Huxley model – **EXERCISE 3 now**

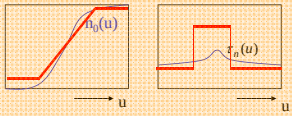
Next lecture 11h15





stimulus

$$C\frac{du}{dt} = -g_K n^4(u-E_K) + I(t)$$

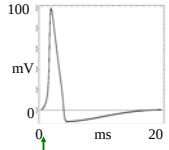


Exercises 3 now!

Hodgkin-Huxley Model

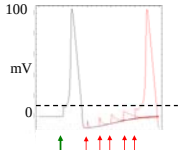
Where is the firing threshold?

Action potential



Strong stimulus

refractoriness

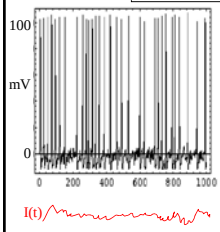


strong stimuli

Strong stimulus

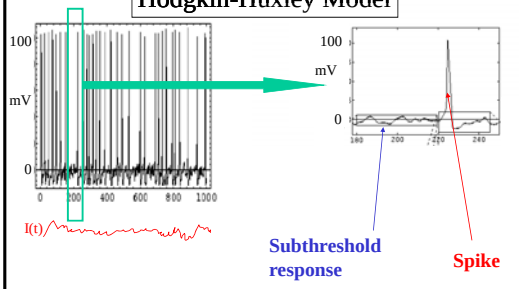
Refractoriness! Harder to elicit a second spike

Hodgkin-Huxley Model



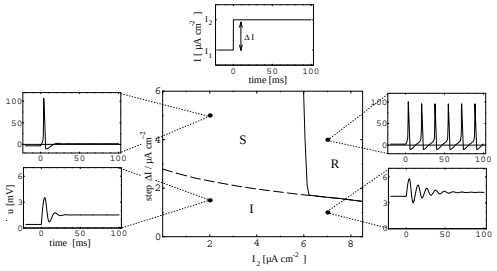
Stimulation with time-dependent input current

Hodgkin-Huxley Model



Hodgkin-Huxley Model

Step current input I_2, $\triangle I$



Hodgkin-Huxley Model

Where is the firing threshold?

pulse input $I(t)$

step input $\triangle I$, I_2

ramp input

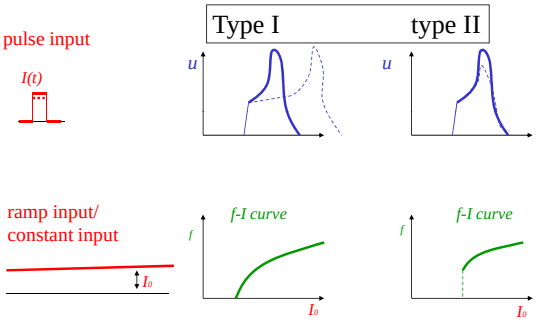
There is no threshold
- no current threshold
- no voltage threshold

'effective' threshold
- depends on typical input

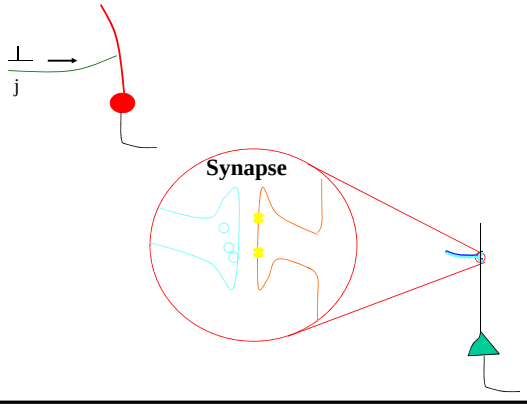
$$C\frac{du}{dt} = -g_{Na}m^3h(u-E_{Na}) - \ldots$$

Type I and type II models

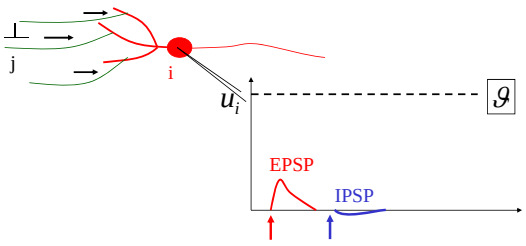
Response at firing threshold?



Model of Synaptic input: conductance change



Model of Synaptic input: conductance change



$$\tau\cdot\frac{d}{dt}u = -(u-u_{rest}) - \sum_{k,f} g_e(t-t_k^f)[u-E_e] - \sum_{k',f'} g_i(t-t_{k'}^{f'})[u-E_i]$$

EPSC IPSC

Detailed neuron models:
Hodgkin-Huxley model

1: Introduction to Hodgkin-Huxley models

2: threshold in the Hodgkin-Huxley model

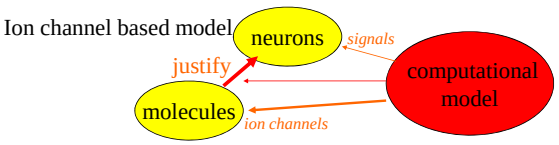
√ 3. Synaptic input (conductance input)

→ 4. Variants of the Hodgkin-Huxley model

Wulfram Gerstner
http://lcn.epfl.ch/

moodle.epfl.ch

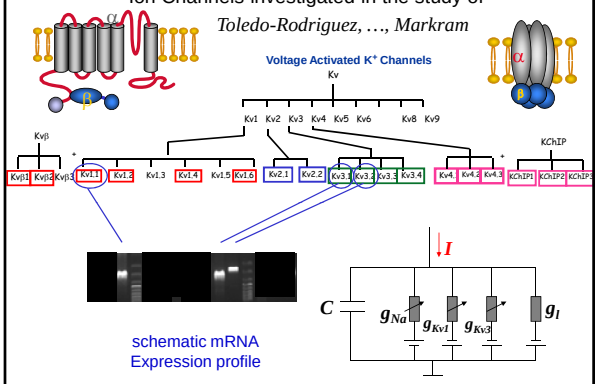
Detour: from molecules to neuron models



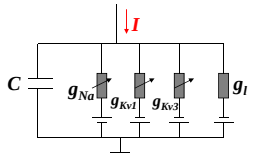
Swiss Federal Institute of Technology Lausanne, EPFL
Laboratory of Computational Neuroscience, LCN, CH 1015 Lausanne

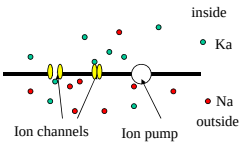
Ion Channels investigated in the study of

Toledo-Rodriguez, …, Markram



Model of fast spiking interneuron





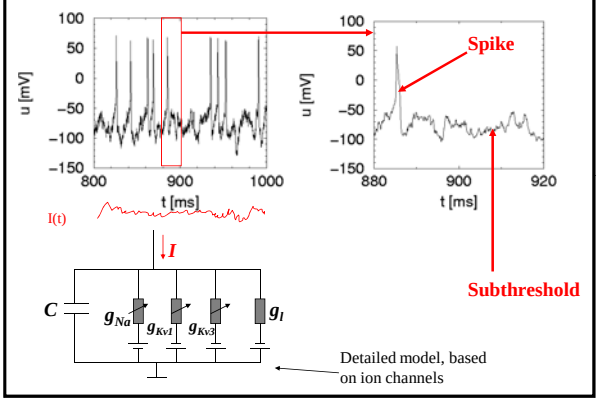
$$C\frac{du}{dt} = \underbrace{-g_{Na} m^3 h (u - E_{Na})}_{I_{Na}} \underbrace{- g_{Kv1} n_{Kv1}^4 (u - E_K) - g_{Kv3} n_{Kv3}^2 (u - E_K)}_{I_K} \underbrace{- g_l (u - E_l)}_{I_{leak}} + I(t)$$

stimulus

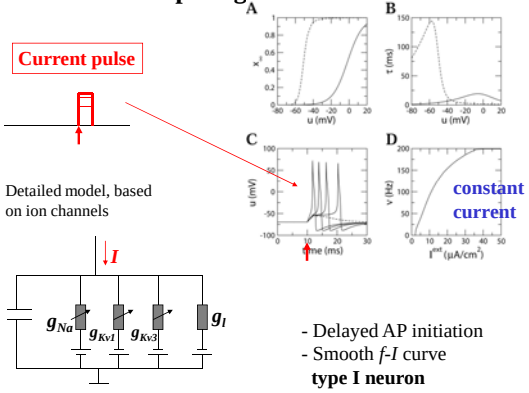
$$\frac{dm}{dt} = \frac{m_0(u) - m}{\tau_m(u)}$$

Erisir et al, 1999
Hodgkin and Huxley, 1952

Model of fast spiking interneuron



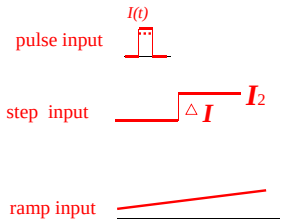
Model of fast spiking interneuron



- Delayed AP initiation
- Smooth *f-I* curve
  type I neuron

Hodgkin-Huxley type models

Where is the firing threshold?



There is no threshold
- no current threshold
- no voltage threshold

'effective' threshold
- depends on typical input

threshold process not immediate
- delayed AP possible

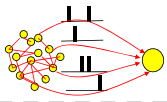
Hodgkin-Huxley Model

Where is the firing threshold?

There is no threshold
- no current threshold
- no voltage threshold

'effective' threshold
- depends on typical input

BUT:
- threshold model is a good approximation
- (voltage threshold)



Exercise 4 **now**:
-Synaptic input

Membrane potential

Synaptic input

$$\tau \frac{d}{dt} u = -(u - u_{rest}) - g_e(t - t_0)[u - E_e]$$

$$g_e(t - t_0) = g_0 \frac{t}{\tau} e^{-\frac{t}{\tau}}$$

Next lecture: 11:57

Computer exercise:
-play with Hodgkin-Huxley model

The end