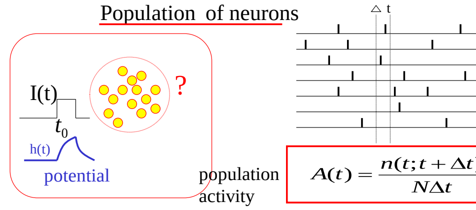
Lecture 12
Course: Neural Networks and Biological Modeling

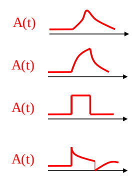
Connected Populations: oscillations, competition and spatial continuum (field equations)

Population of neurons



population activity

$$A(t) = \frac{n(t;t+\Delta t)}{N \Delta t}$$



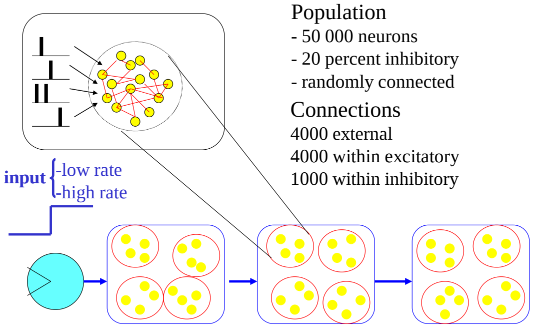
$$\tau \frac{d}{dt} A(t) = -A(t) + g(h(t))$$

$$A(t) = g(h(t)) = g(\int \kappa(s) I(t-s) ds)$$

$$A(t) = g(I(t))$$

$$A(t) = g(I(t), I'(t))$$

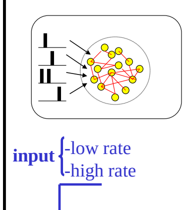
Signal transmission in populations of neurons

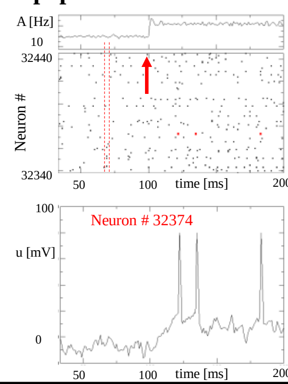


Population
- 50 000 neurons
- 20 percent inhibitory
- randomly connected

Connections
4000 external
4000 within excitatory
1000 within inhibitory

Signal transmission in populations of neurons

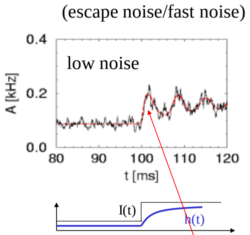




Population
- 50 000 neurons
- 20 percent inhibitory
- randomly connected

Theory of transients

noise model A
(escape noise/fast noise)

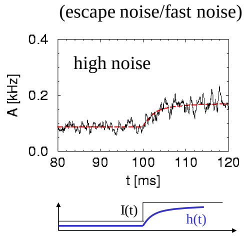


fast transient

$A(t) \approx g(I(t))$

But transient oscillations

noise model A
(escape noise/fast noise)



slow transient

$A(t) = g(h(t))$

High-noise activity equation

blackboard

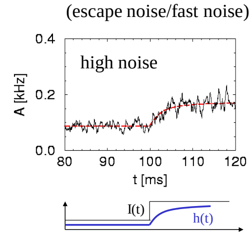
Population activity

$$A(t) = g(h(t))$$

Membrane potential caused by input

$$\tau \frac{d}{dt} h(t) = -h(t) + R\ I(t)$$

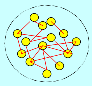
noise model A
(escape noise/fast noise)



slow transient

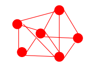
$A(t) = g(h(t))$

Full connectivity



Fully connected network

blackboard



$$\boxed{\tau \frac{d}{dt} h(t) = -h(t) + R\ I(t)}$$

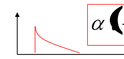
$$I(t) = I^{ext}(t) + I^{net}(t)$$

All spikes, all neurons

$$I^{net}(t) = \sum_j \sum_f w_{ij}\ \alpha(t - t_j^f)$$

Synaptic coupling

$$w_{ij} = \frac{J_0}{N}$$



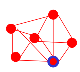
$$\alpha(t - t_j^f)$$

$$\boxed{\tau \frac{d}{dt} h(t) = -h(t) + R\ I(t)}$$

All neurons receive the same total input current ('mean field')

$$I_i(t) = J_0 \int \alpha(s) A(t-s) ds + I^{ext}(t)$$

Index i disappears



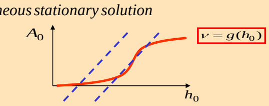
$$w_{ij} = \frac{J_0}{N}$$

All spikes, all neurons

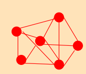
$$I_i(t) = \sum_j \sum_f w_{ij}\ \alpha(t - t_j^f) + I^{ext}(t)$$

Exercises 1, now

Exercise 1: homogeneous stationary solution



Next lecture 10:15



Homogeneous network
All neurons are identical,
Single neuron rate = population rate

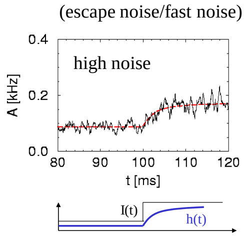
$$\boxed{\nu = A}$$

High-noise activity equation

What is this function *g*?

$$A_0 = g(h_0)$$

noise model A
(escape noise/fast noise)



Population activity

$$A(t) = g(h(t))$$

Membrane potential caused by input

$$\tau \frac{d}{dt} h(t) = -h(t) + R\ I(t)$$

slow transient

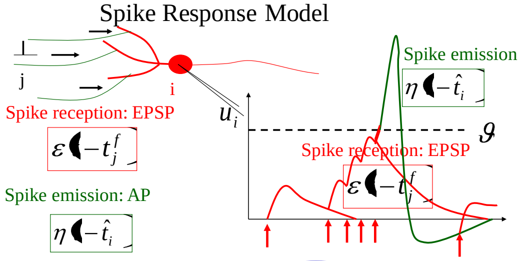
$$A(t) = g(h(t))$$

Note: total membrane potential

$$u(t) = h(t) + \eta(t - \hat{t})$$

Effect of last spike

Spike Response Model



Last spike of i

All spikes, all neurons

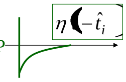
$$u_i(t) = \eta(t - \hat{t}_i) + \sum_j \sum_f w_{ij}\ \varepsilon(t - t_j^f) \quad h_i(t)$$

$$u_i(t) = \vartheta \Rightarrow \text{Firing: } \hat{t}_i = t$$

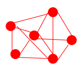
$$\boxed{\tau \frac{d}{dt} h_i(t) = -h_i(t) + R\ I_i(t)}$$

Fully connected network

Spike emission: AP $\eta(t-\hat{t}_i)$



$$u_i(t) = \eta(t-\hat{t}_i) + h_i(t)$$



fully connected N >> 1

$$\tau \tfrac{d}{dt} h(t) = -h(t) + R\, I(t)$$

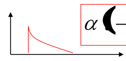
$$I(t) = I^{ext}(t) + I^{netw}(t)$$

All spikes, all neurons

$$I^{netw}(t) = \sum_j \sum_f w_{ij}\, \alpha(t-t_j^f)$$

Synaptic coupling

$$w_{ij} = \frac{J_0}{N}$$

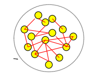


$\alpha(t-t_j^f)$

$$I^{netw}(t) = J_0 \int \alpha(s) A(t-s)\, ds$$

Stationary State/Asynchronous State

A(t)=const



input potential

$$u(t|\hat{t}) = \eta(t-\hat{t}) + h_0$$

$$h_0 = RI_0 = R(J_0 q A_0 + I_0^{ext})$$

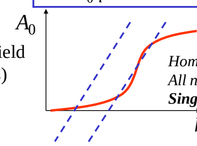
fully connected coupling J/N

$$A_0 = \frac{1}{J_0 q R}(h_0 - R I_0^{ext})$$

$$\int \alpha(s)\, ds = q$$

$\nu = g(h_0)$

typical mean field (Curie Weiss)



Homogeneous network
All neurons are identical,
Single neuron rate = population rate

frequency (single neuron) $\nu = \langle s \rangle^{-1} = \left[\int_0^\infty s\, P_I(\hat{t}+s|\hat{t})\, ds\right]^{-1} = g(h_0)$

blackboard

High-noise activity equation

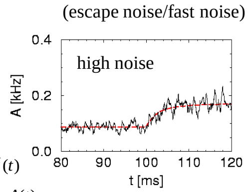
Population activity

$$A(t) = g(h(t))$$

Membrane potential caused by input

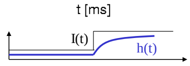
$$\tau \tfrac{d}{dt} h(t) = -h(t) + R\, I(t)$$

noise model A
(escape noise/fast noise)



$$I(t) = I^{ext}(t) + I^{net}(t)$$

$$I(t) = I^{ext}(t) + J_0\, q\, A(t)$$



$$I(t) = I^{ext}(t) + J_0\, q\, g(h(t))$$

slow transient

$$\boxed{A(t) = g(h(t))}$$

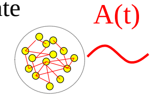
$$\boxed{\tau \tfrac{d}{dt} h(t) = -h(t) + R\, I^{ext}(t) + \gamma\, g(h(t))}$$

Oscillations

Stability of Asynchronous State

A(t)

$$\boxed{A(t) = \int_{-\infty}^{t} P_I(t|\hat{t}) A(\hat{t})\, d\hat{t}}$$



fully connected coupling J/N

linearize

$$A(t) = A_0 + \Delta A(t)$$

$$h(t) = h_0 + \Delta h(t)$$

h: input potential

$$\Delta h(t) = J \int \varepsilon(s) \Delta A(t-s)\, ds$$

$$\Delta A(t) = \int_{-\infty}^{t} P_0(t|\hat{t})\, \Delta A(\hat{t})\, d\hat{t} + A_0 \frac{d}{dt} \int_0^\infty L(x)\, \Delta h(t-x)\, dx$$

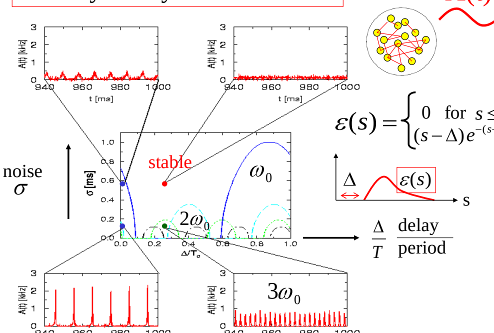
$$\Delta A(t) = A_1 e^{i\omega t + \lambda t}$$

Search for bifurcation points $\lambda = 0$

Stability of Asynchronous State

A(t)

$$\varepsilon(s) = \begin{cases} 0 & \text{for } s \le \Delta \\ (s-\Delta)\, e^{-(s-\Delta)} \end{cases}$$



$\frac{\Delta}{T}$ delay / period

(Gerstner 2000)

High-noise activity equation

Population activity

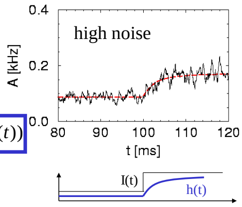
$$A(t) = g(h(t))$$

Membrane potential caused by input

$$\tau \tfrac{d}{dt} h(t) = -h(t) + R\ I(t)$$

$$\boxed{\tau \tfrac{d}{dt} h(t) = -h(t) + R\,I^{ext}(t) + \gamma\, g(h(t))}$$

noise model A
(escape noise/fast noise)



slow transient

$$\boxed{A(t) = g(h(t))}$$

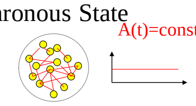
Attention:
- *valid for high noise only, else transients might be wrong*
- *valid for high noise only, else spontaneous oscillations may arise*

Random connectivity

Random Connectivity/Asynchronous State

C inputs per neuron

$$w_{ij} = \begin{cases} \frac{J_0}{C} \\ 0 \end{cases}$$



A(t)=const

randomly connected

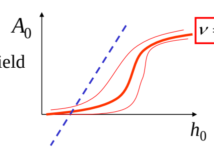
mean drive

$$I_0 = J_0 q A_0 + I_0^{ext}$$

variance

$$\sigma^2 = \frac{J_0}{C} A_0$$

improved mean field



$$\boxed{\nu = g(h_0, \sigma)}$$

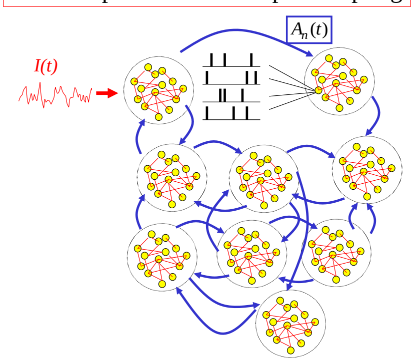
Analogous for column of 1 exc. + 1 inhib. Pop.

frequency (single neuron) $\nu = \langle s \rangle^{-1} = \left[\int_0^\infty P_{I,\sigma}(t+s\,|\,\hat{t})\, ds\right]^{-1} = g(h_0, \sigma)$

(Amit&Brunel 1997, Brunel 2000)

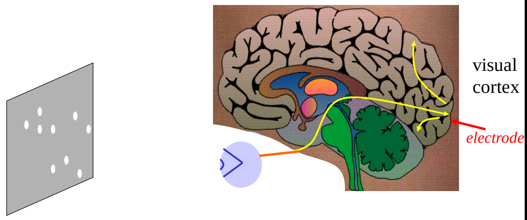
Spatial coupling

Microscopic vs. Macroscopic Coupling

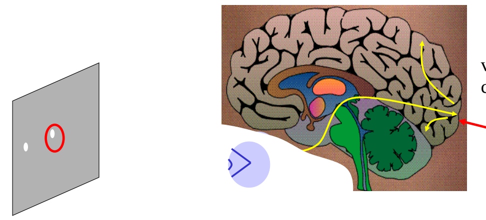


Cortical columns: Orientation tuning

Detour: Receptive fields (see also lecture 4)

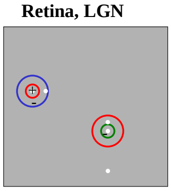


Detour: Receptive field development



Detour: Receptive field development

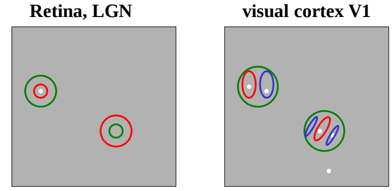
Receptive fields:
Retina, LGN



Detour: Receptive field development

Receptive fields:
Retina, LGN

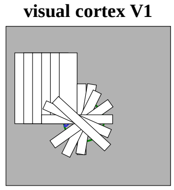
Receptive fields:
visual cortex V1



Orientation selective

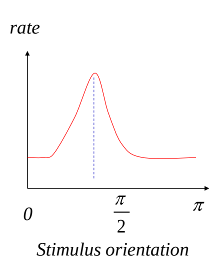
Detour: Receptive field development

Receptive fields:
visual cortex V1

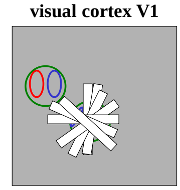


Orientation selective

Detour: orientation selective receptive fields

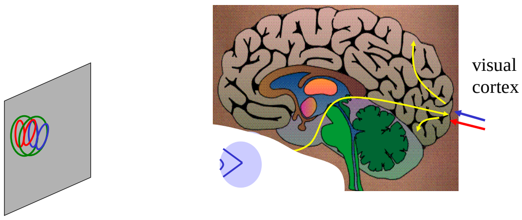


Receptive fields:
visual cortex V1



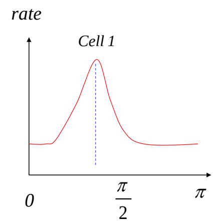
Orientation selective

Detour: Receptive fields

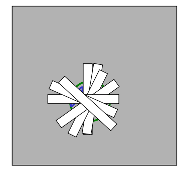


Neighboring cells in visual cortex
Have similar preferred orientation:
cortical orientation map

Detour: orientation selective receptive fields

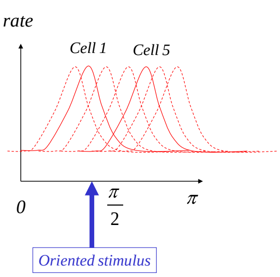


Receptive fields:
visual cortex V1



Orientation selective

Detour: orientation selective receptive fields



Course coding

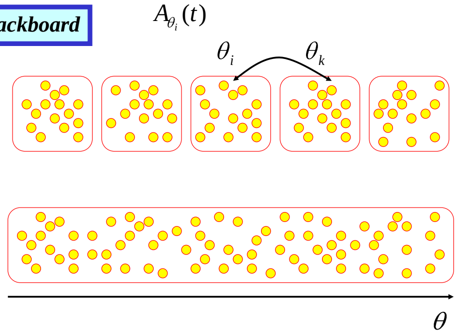
Many cells respond to a single stimulus with different rate

Continuous Networks

Several populations → Continuum

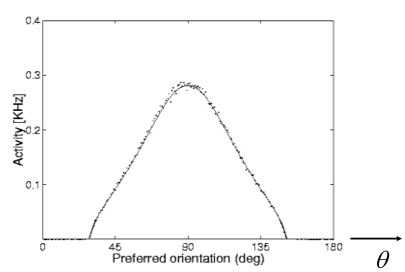
Blackboard

$A_{\theta_i}(t)$

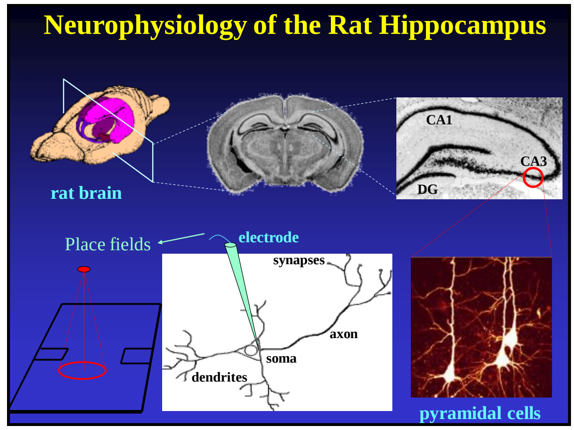


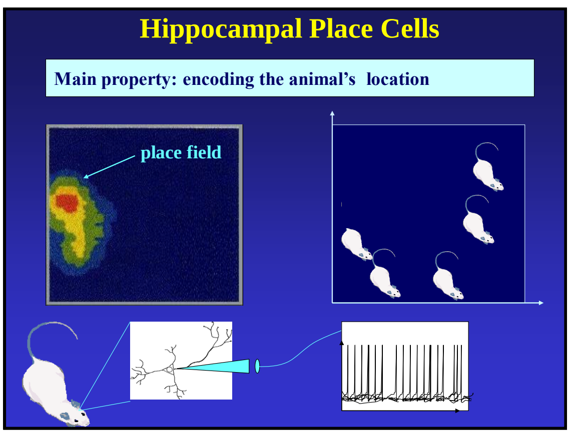
Continuum: stationary profile

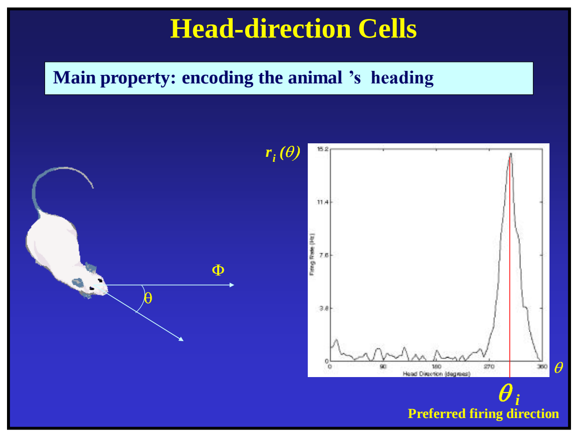
$$A(\theta,t) = A(\theta)$$

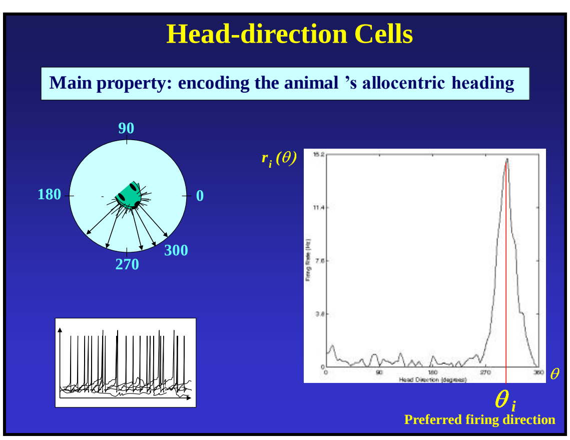


Head direction cells
(and line attractor)





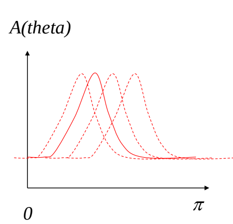




Basic phenomenology

I: Bump formation

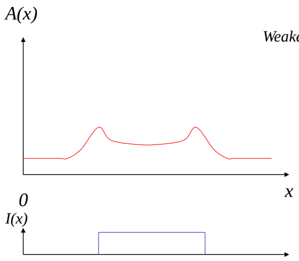
strong lateral connectivity



Basic phenomenology

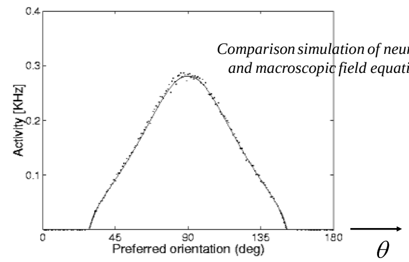
II. Edge enhancement

Weaker lateral connectivity



Continuum: stationary profile

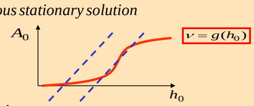
$$A(\theta,t)=A(\theta)$$



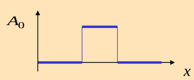
Comparison simulation of neurons and macroscopic field equation

Exercises 2,3,4 from 11:15-12:45

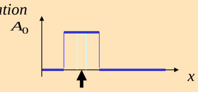
Exercise 1: homogeneous stationary solution



Exercise 2: bump solution



Exercise 3: bump formation



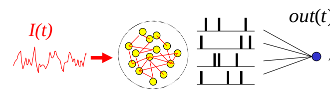
Blob forms where the input is

The end

Liquid state/
Information buffering

Ultra-short-term information buffering ('echo')

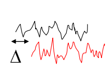
(Maass et al., 2002, H.Jäger 2004)



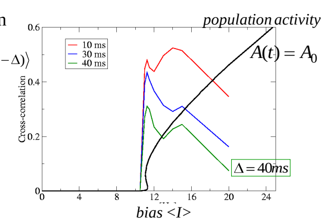
$out(t)=\sum_j \sum_f a_j \varepsilon(t-t_j^f)$

N parameters

How much information

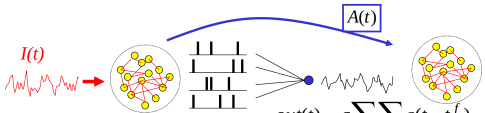


$\langle out(t) I(t-\Delta)\rangle$

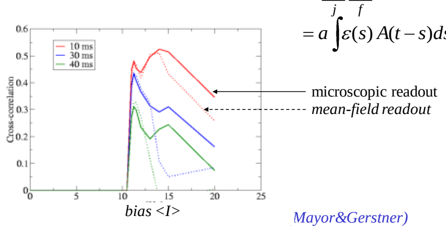


640 excitatory n.
160 inhibitory n.
20ms time constant
50 connections/n.

Ultra-short-term information buffering ('echo')



$$out(t) = a \sum_j \sum_f \varepsilon(t - t_j^f)$$
$$= a \int \varepsilon(s)\, A(t-s)\, ds$$



Mayor&Gerstner)