Name: July 26, 2007

Neural Networks and Biological Modeling

Exam

- The exam lasts 160 minutes.
- Except for one double-sided A4 sheet of handwritten notes, no documentation is allowed.
- *All* answers must be written on the exam sheets.
- The total number of points is 40.

Ex. 1	/8
Ex. 2	/12
Ex. 3	/6
Ex. 4	/8
Ex. 5	/6
Total	/40

Question 1: Model of an ion channel (8 points)

Consider the following model for an ion channel: the electrical current I_{ion} flowing through the channel is given by

$$I_{ion} = g_{ion} p^{n_1} q^{n_2} (u - u_{ion}) \qquad (\blacktriangle)$$

where u is the neuronal membrane potential, g_{ion} and u_{ion} are two constants, and $n_1 = 1$, $n_2 = 2$. The variables p and q obey the dynamics

$$\frac{dp}{dt} = -\frac{p - p_0(u)}{\tau_p(u)}$$

$$\frac{dq}{dt} = -\frac{q - q_0(u)}{\tau_q(u)}$$

with p_0, q_0, τ_p et τ_q as shown on Fig 1.

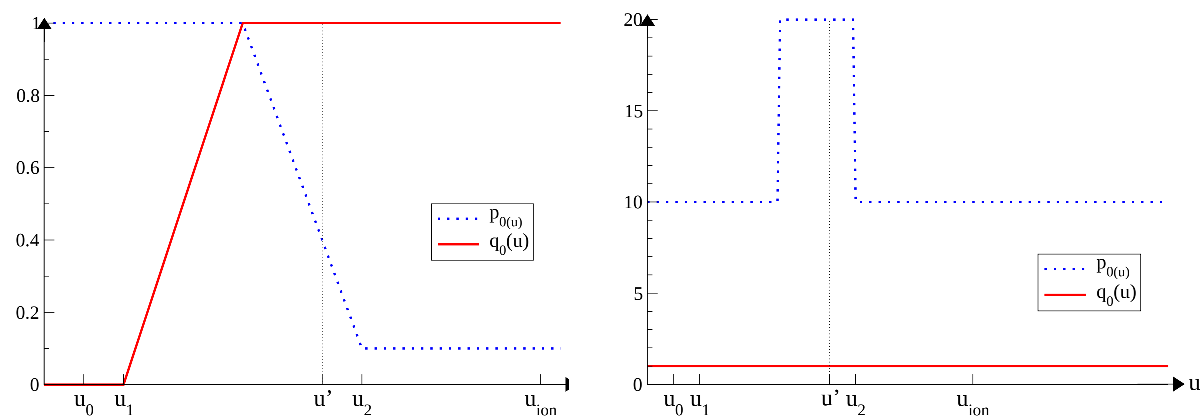


Figure 1: Graphical representation of p_0, q_0, τ_p and τ_q.

1.1 The resting potential of the cell is at u_0 as indicated on the figure. Give the *biological* interpretation of the following parameters and variables:

p : ....

q : ....

g_{ion} : ....

u_{ion} : ....

/2 points

1.2 *Qualitatively*, how does the channel react (in terms of partial or complete opening/closing) to a voltage step? Suppose that for $t < 0$, the neuron is at rest ($u = u_0$). At $t = 0$ the voltage instantaneously jumps to a new value $u' = u_2(1 - \varepsilon)$ with $\varepsilon \ll 1$ (see Fig. 1 for the values of u_0, u', u_2 and u_{ion}), at which it is held constant for all $t \geq 0$.

- For $t < 0$, the channel is ....

  because ....

  ....

- At $t = 1$ ms, it is ....

  because ....

  ....

- At $t = 3$ ms, it is ....

  because ....

  ....

- At $t = 10$ ms, it is ....

  because ....

  ....

- At $t = 20$ ms, it is ....

  because ....

  ....

/2 points

1.3 The neuronal voltage for a passive membrane is given by

$$C\frac{du}{dt} = -g_L(u - u_0) + I_{ext}$$

where $C = 1$ is the membrane capacitance, u is the membrane voltage, and I_{ext} is an externally applied current. Calculate the equilibrium voltage when the neuron is subjected to a very large constant input $I_{ext} = 50$, assuming the following parameter values: $u_0 = -70$, $g_L = 1$.

/1 point

1.4 Repeat this calculation in the case where the active ion channel is also present:

$$C\frac{du}{dt} = -g_L(u - u_0) - I_{ion} + I_{ext}$$

with I_{ion} as defined in Eq.(▲). For this calculation, assume $u_1 = -60$, $u_2 = -20$, $g_{ion} = 10$, $u_{ion} = +20$, and $I_{ext} = 50$.

/2 points

1.5 Is the equilibrium voltage in 1.3 above or below that of 1.4? Why?

....

....

....

/1 point

QUESTION 2: PHASE SPACE ANALYSIS (12 points)

We consider the model consisting of the following equations

$$\frac{d}{dt}x_1(t) = -x_1(t) + h_1^{ext} + (w-\alpha)g(x_1(t)) - \alpha g(x_2(t)) \quad (1)$$

$$\frac{d}{dt}x_2(t) = -x_2(t) + h_2^{ext} + (w-\alpha)g(x_2(t)) - \alpha g(x_1(t)) \quad (2)$$

where the function g(x) is shown in fig.2-1.

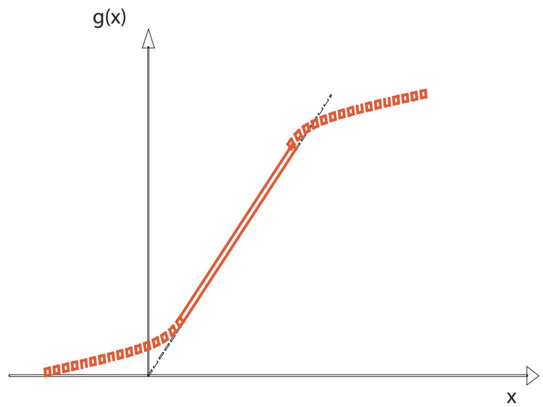


Figure 2

2.1

- Set $\alpha = 0$. What is the interpretation of x_1, x_2, w?

  x_1 : ....

  x_2 : ....

  w : ....

- Set $w = 0$ and $\alpha > 0$. What is the interpretation of α?

  ....

- What does the system of equations (1) and (2) describe?

  ....

  ....

  ....

/3 points

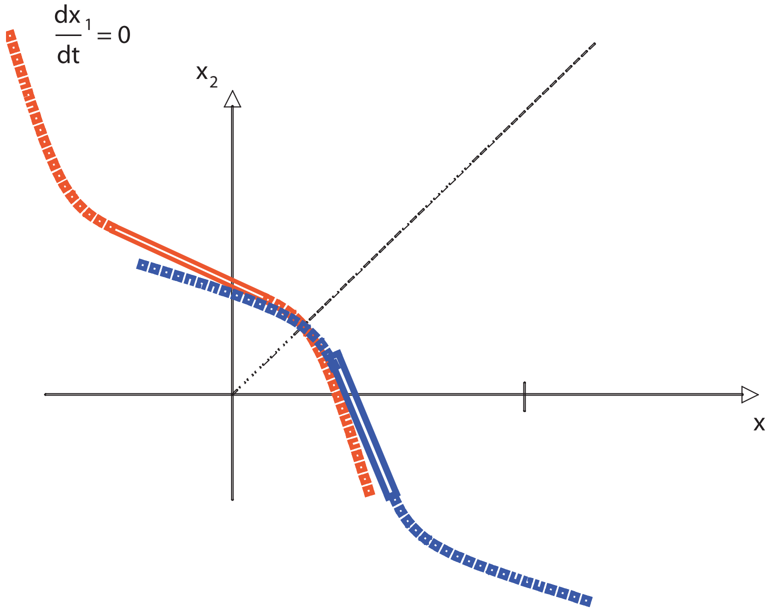


Figure 3

2.2 The nullclines of equations (1) and (2) for $h_1^{ext} = h_2^{ext} = 0.3$ is shown below in fig.3.

- Draw on Fig.3 the flux arrows at the point $(0, 0)$

  ....

- Draw the flux arrows on the nullclines.

  ....

- Mark the fixed-point or fixed-points.

  ....

- What can you say about the stability of the fixed point(s)? Write on the graph "s" for stable and "u" for unstable next to the fixed point(s).

/3 points

2.3 Suppose we increase h_2^{ext}.

- How do the nullclines change?

 ....

- Redraw them on Fig.4

/2 points

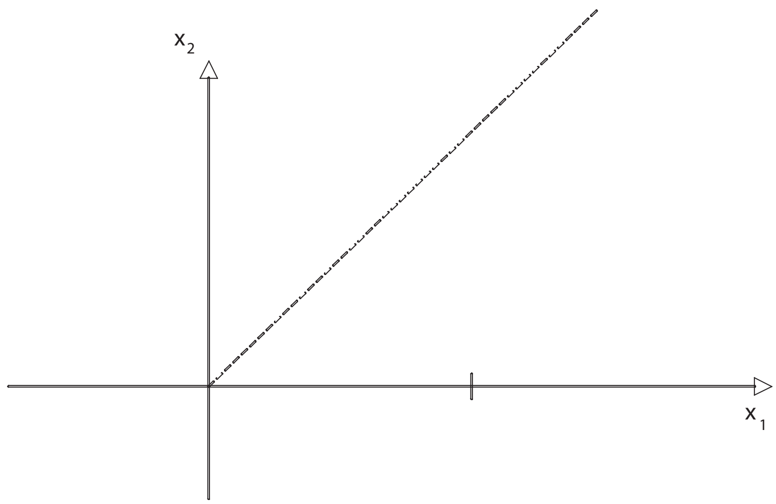


Figure 4

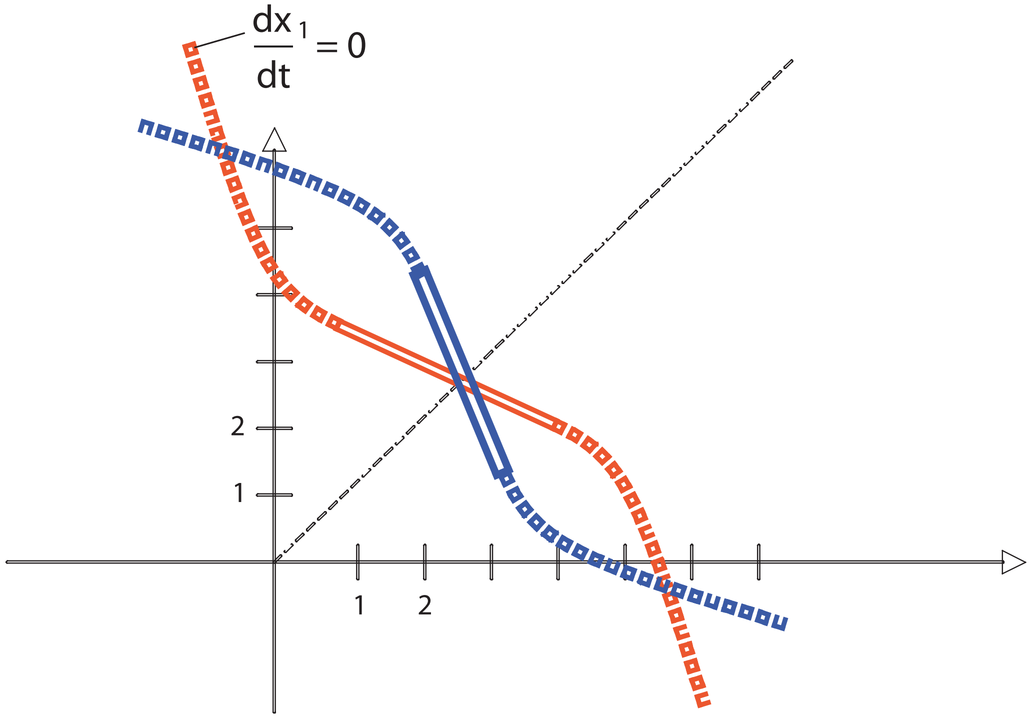


Figure 5

2.4 Now consider the situation where the nullclines are as shown in fig.5

- Mark the fixed point or fixed points.

/1 point

- Indicate stability (s = stable, u = unstable) on the graph.

/1 point

- For the three different initial conditions, $(1, 2)$ $(2, 1)$ and $(6, 5)$, draw the trajectories of the vector (x_1, x_2).

/2 points

Question 3: Reinforcement learning (6 points)

Consider a rat navigating on a platform with 5 boxes (A, B,..., E) as shown in Fig. 6. The rat is initially placed in the start box A, with two food rewards $r = 0.1$ and $R = 2$ placed on the transition between boxes according to Fig. 6. The possible actions are described by the arrows. When the goal is reached, i.e. box E, the rat is placed back in the initial box A and the exploration starts again.

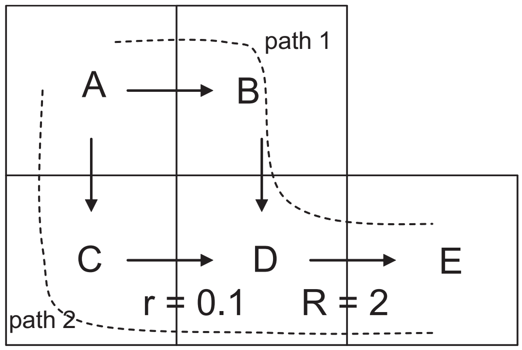


Figure 6

3.1 Fill in the state-and-action graph Fig. 7 in order to sketch the problem the rat faces (circles are states and squares are actions), by placing labels on squares and circles and linking them with appropriate lines (see example). The initial state is indicated by A. /1 point

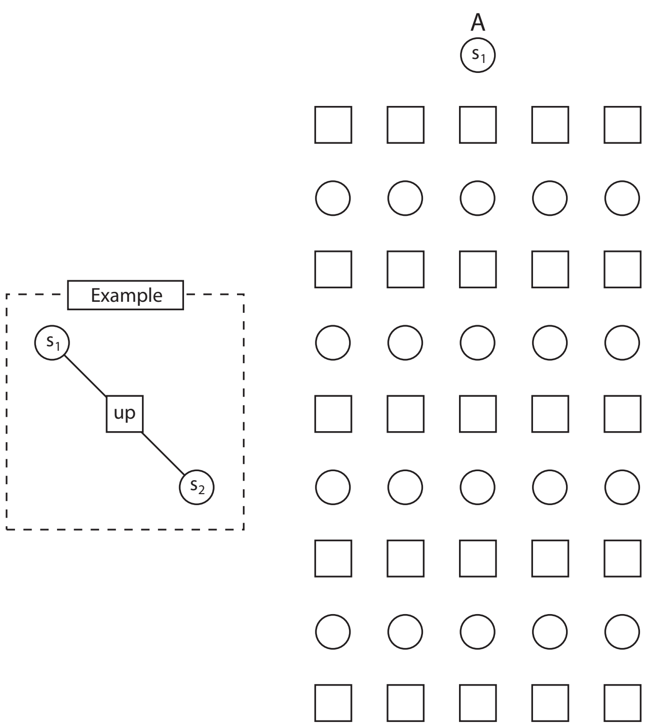


Figure 7

3.2 For this problem we use the following algorithm

$$\Delta Q(s,a) = \eta \left[r - \left(Q(s,a) - Q(s',a^*) \right) \right]$$

with $a^* = \arg\max_a Q_a(s,a)$. Explain all the terms.

a : ....

a^* : ....

s : ....

s' : ....

Q : ....

ΔQ : ....

η : ....

r : ....

/2 points

3.3 The rat can take either path 1 or path 2 (dashed lines). In the first trial the rat takes path 1. All the Q-values are initialized to zero. Fill in the empty table in order to give the Q-values after the first trial (left table) and after the second trial (right table).

	a_1	a_2	.....	.....
s_1				
s_2				
.....				
.....				
.....				
.....				
.....				

	a_1	a_2	.....	.....
s_1				
s_2				
.....				
.....				
.....				
.....				
.....				

/2 points

3.4 Without doing any further calculation, do you think the algorithm will in the end converge to the optimal path? Justify your answer.

....

....

/1 point

Question 4: Associative memory (8 points)

Assume a Hopfield network with the following bit dynamics

$$S_i(t+1) = \text{sgn}[\sum_j w_{ij} S_j(t)].$$

The weight dynamics follows

$$\Delta w_{ij} = (\nu_i^{post} - 0.5)(\nu_j^{pre} - 0.5).$$

4.1 Is the weight dynamics a Hebbian rule? Justify your answer.

....

....

/1 point

4.2 What is the relation between this weight dynamics and that of a Hopfield network?

....

....

....

/2 points

4.3 During the learning session, the pattern "L" and "T" are learnt (Fig 8). "L" is presented for one time step and "T" is presented also for one time step. Weight dynamics is artificially stopped afterwards.

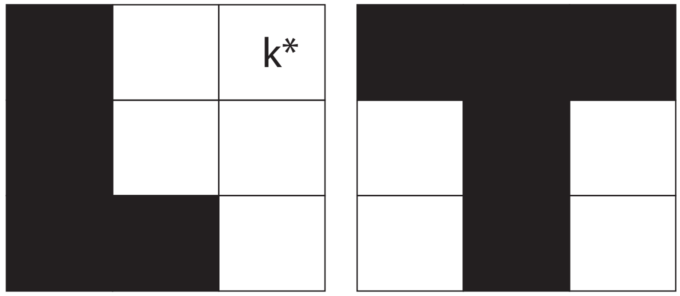


Figure 8

Suppose that, after learning, the network is initialized in the "L" state but the bit of the upper right corner is flipped (the flipped pixel is denoted k^* in Fig 8), what will happen? Calculate the evolution of S_{k^*} for 1 time step. What will happen in the next time step? Comment your result.

/4 points

4.4 In order to store many patterns in a Hopfield network, what should be the pattern properties?

....

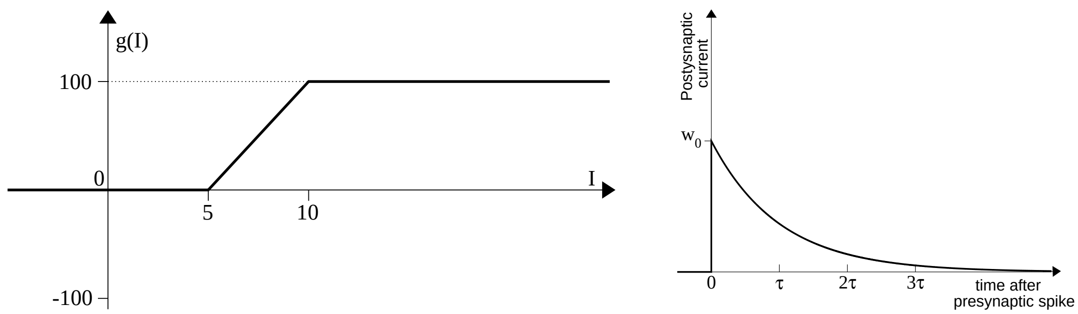
/1 point

Question 5: Population activity (6 points)

Consider a population of $N = 1000$ identical neurons, characterized by a gain function

$$\nu = g(I)$$

where ν is the mean instantaneous firing frequency, and $I = I_{syn} + I_{ext}$ is the total input received by the neuron. We take for g the piecewise linear form shown in Fig 9. Each neuron is connected to every other neuron (including itself) with a synaptic efficacy w.



(a) Gain function $g(I)$ (b) Excitatory postsynaptic current

Figure 9

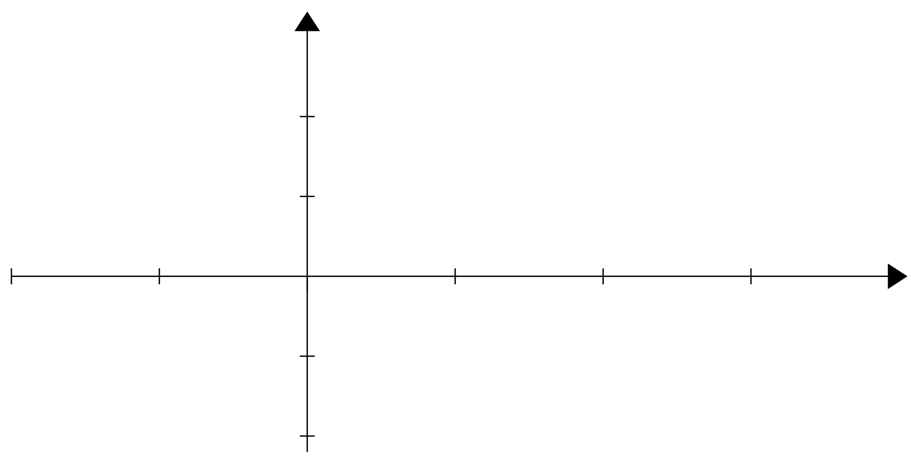
5.1 Suppose that all the neurons are firing asynchronously with a frequency ν. Write down an equation for the total synaptic current I_{syn} received by a single neuron, supposing that each presynaptic spike causes an exponential current pulse of amplitude w_0 and time constant τ (see Fig 9).

/1 point

5.2 Calculate the average current $\langle I \rangle$ received by each neuron.

/2 points

5.3 Using the result of the preceding point, determine graphically in the graph below the existence of fixed points of the population activity when $I_{ext} = 0$. Consider two different values of w_0.



/2 points

5.4 How does the graphical solution change as I_{ext} is varied?

/1 point