Noisy neurons and coding (again): variance, autocorrelation, stochastic resonance

Lecture 9 *of course*:

Neural Networks and Biological Modeling

- -Noise in the brain
- -Variability=noise?
- -Noise Models

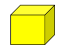
} Review of lecture 8

- -stochastic spike arrival (diffusive noise)
- -Comparison of noise models
- -Coding and stochastic resonance

BOOK: Spiking Neuron Models,
W. Gerstner and W. Kistler
Cambridge University Press, 2002

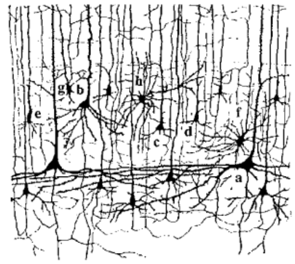
Chapter 5

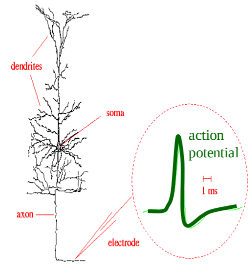
Wulfram Gerstner
EPFL



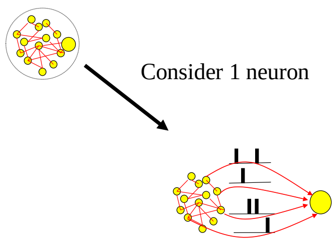
10 000 neurons
3 km wires

Signal:
action potential (spike)



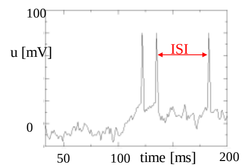


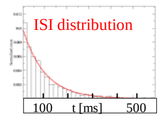
Input to a single neuron



Variability of spike trains =noise?

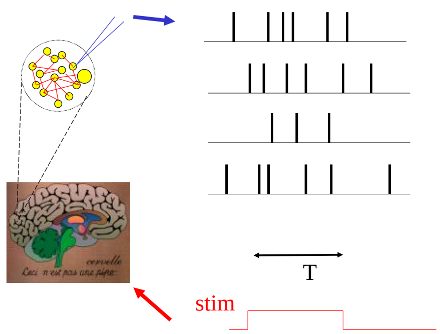
- Variability of interspike intervals





Despite constant external input,
broad interspike interval distribution

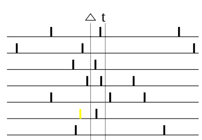
Variability of spike trains across repetitions



Variability of spike trains =noise?

- Variability across repetitions

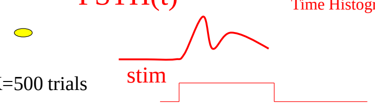
K repetitions



$$PSTH(t) = \frac{n(t; t+\Delta t)}{K\,\Delta t}$$

PSTH(t)

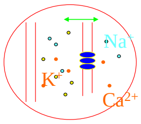
Peri-Stimulus
Time Histogram



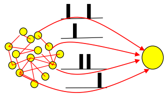
K=500 trials

Variability of spike trains =noise?
sources of noise

- Intrinsic noise (ion channels)

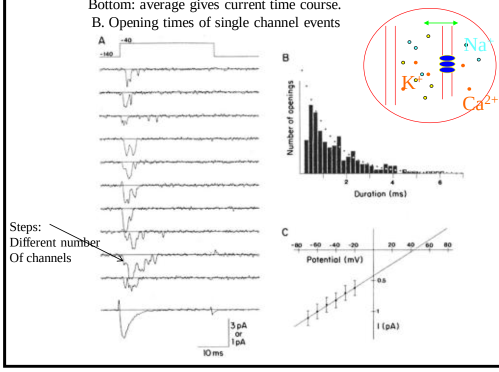


-Network noise (background activity)



Intrinsic noise: Channel noise

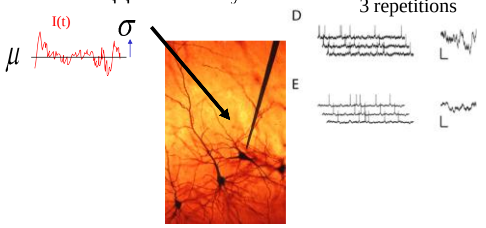
Na+ channel from rat heart.
A 10 sequential traces from a patch containing 4 or 5 channels.
Bottom: average gives current time course.
B. Opening times of single channel events

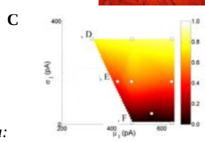


Temporal input shows reliable neurons

Intrinsic reliability of neurons

3 repetitions





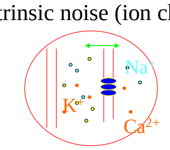
Fluctuating input signal
→ Reliable spike timing

See also: Mainen&Sejnowski

Exp. Data:
A. Rauch, H.Lüscher, U. Berne

Variability of spike trains =noise?
sources of noise

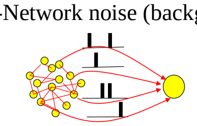
- Intrinsic noise (ion channels)



Less important

-Network noise (background activity) ←

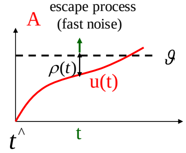
important



Review from lecture 8
-Noise in the brain
-Variability=noise?
-Noise in Integrate-and-fire
→ - mathematical noise models

Noise models

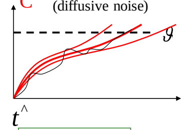
A escape process (fast noise)



escape rate

$$\rho(t) = f(u(t) - \vartheta)$$

C stochastic spike arrival (diffusive noise)



noisy integration

$$\tau \cdot \frac{du_i}{dt} = -u_i + RI + \xi(t)$$

Relation between the two models:
later today

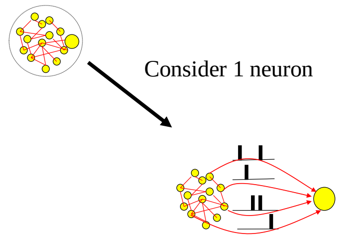
now

Lecture 9 – noisy neurons and coding again: autocorrelation, variance, stochastic resonance

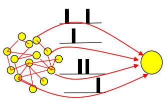
-Noise in the brain
-Variability=noise?
-Noise Models
} Review of lecture 8

→ -stochastic spike arrival (passive membrane)
-Comparison of noise models
-Coding and stochastic resonance

Input to a single neuron: stochastic spike arrival



Input to a single neuron

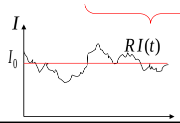


Assumption of Stochastic spike arrival:
in network of exc. neurons,
total spike arrival rate *A(t)*

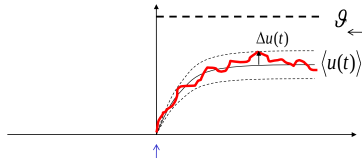
$$\tau \frac{d}{dt}u = -(u-u_{rest}) + RI(t)$$ ← *Passive membrane*

$$RI(t) = R\, I_o + I_{noise}$$

Blackboard



Diffusive noise (stochastic spike arrival)



Assumption:
far away from theshold

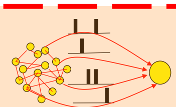
Blackboard

Expected voltage at time *t* $\langle u(t)\rangle = ?$

Variance of voltage at time *t*

$$\langle \Delta u(t)\Delta u(t)\rangle = \langle u(t)u(t)\rangle - \langle u(t)\rangle^2 =$$

Exercise 1 now: Diffusive noise (stochastic spike arrival)



Stochastic spike arrival:
excitation, total rate $\langle S(t)\rangle = \nu$

Synaptic current pulses

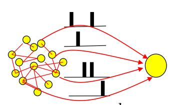
Next lecture: 10h20

$$\tau \frac{d}{dt}u = -(u-u_{rest}) + RS(t)$$

$$S(t) = q_e \sum_f \delta(t-t^f)$$

1. Assume that for t>0 spikes arrive stochastically with rate ν
   - Calculate mean voltage
2. Assume autocorrelation $\langle S(t)S(t')\rangle = \nu\,\delta(t-t') + \nu^2$
   - Calculate $\langle u(t)u(t)\rangle = ?$

Mean current in passive membrane



Assumption of Stochastic spike arrival:
network of exc. neurons,
spike arrival rate ν_k

$$\tau \frac{d}{dt}u = -(u-u_{rest}) + RI(t)$$ ← *Passive membrane*

Synaptic current pulses of shape α

$$RI^{syn}(t) = \sum_k w_k \sum_f \alpha(t-t_k^f) \qquad \int ds\,\alpha(s) = q$$

$$I^{syn}(t) = \frac{1}{R}\sum_k w_k \sum_f \int dt'\,\alpha(t-t')\,\delta(t'-t_k^f)$$

mean $$I_0 = \langle I^{syn}(t)\rangle = \frac{1}{R}\sum_k w_k \int dt'\,\alpha(t-t')\left\langle \sum_f \delta(t'-t_k^f)\right\rangle$$

$$I_0 = \frac{1}{R}\sum_k w_k \int dt'\,\alpha(t-t')\ \nu_k$$ ← Spike arrival rate

$$\int ds\,\alpha(s) = q \qquad RI_0 = \sum_k w_k q \nu_k$$

Indicative evaluation of Bachelor/Master courses given to EPFL students

from Monday, April 18 (TODAY, NOW!!!!)
to Sunday, May 8 at midnight

Summary:

All the courses of all the sections are evaluated.
A single question on the course in general.

Evaluation indicative des cours Bachelor/Master donnés aux étudiants de l'EPFL

dès lundi 18 avril jusqu'au dimanche 8 mai à minuit.

Résumé:

Tous les enseignements de toutes les sections sont évalués
Une seule question sur l'enseignement dans son ensemble
Motiver vos étudiants à répondre, c'est aussi améliorer votre cours
Suivi et consultation des résultats sur:
https://infowww.epfl.ch/imoniteur_ISAP/isacademia.htm
Support technique (en français pour l'instant) sur:
http://wiki.epfl.ch/is-academia.suppot/evalens-bama-enseignant

Stochastic spike arrival – diffusive noise

Part 2 of *Lecture 9 of course*:
Noisy neurons and coding (again):
autocorrelation, variance, stochastic resonance
Neural Networks and Biological Modeling

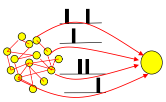
- -review of lecture 8
- -stochastic spike arrival (diffusive noise)
- -Comparison of noise models
- -Coding and stochastic resonance

BOOK: Spiking Neuron Models,
W. Gerstner and W. Kistler
Cambridge University Press, 2002

Chapter 5

Wulfram Gerstner
EPFL

Diffusive noise (stochastic spike arrival)



Stochastic spike arrival:
excitation, total rate R_e
inhibition, total rate R_i

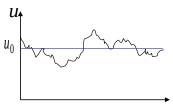
Reuse Blackboard

Passive membrane (no threshold)

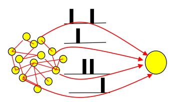
$$\tau \frac{d}{dt} u = -(u - u_{rest}) - \sum_{k,f} \frac{q_e}{C} \delta(t - t_k^f)$$

EPSC

current pulses



Diffusive noise (stochastic spike arrival)



Stochastic spike arrival:
excitation, total rate R_e
inhibition, total rate R_i

Synaptic current pulses

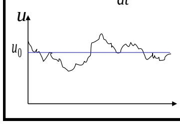
$$\tau \frac{d}{dt} u = -(u - u_{rest}) + \sum_{k,f} \frac{q_e}{C} \delta(t - t_k^f) - \sum_{k',f'} \frac{q_i}{C} \delta(t - t_{k'}^{f'})$$

EPSC IPSC

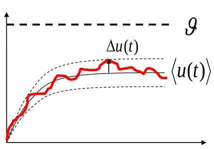
$$\tau \frac{d}{dt} u = -(u - u_{rest}) \quad + R\,I(t) + \quad \xi(t)$$

Blackboard

Langevin equation,
Ornstein Uhlenbeck process

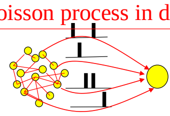


Diffusive noise (stochastic spike arrival)



$$\langle \Delta u(t) \Delta u(t) \rangle = \langle u(t) u(t) \rangle - \langle u(t) \rangle^2 =$$

Justify autocorrelation of spike input: Poisson process in discrete time



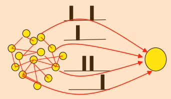
Stochastic spike arrival:

Blackboard

In each small time step Δt
Prob. Of firing $p = \nu \Delta t$

Firing independent between one time step and the next

Exercise: Poisson process in discrete time



Stochastic spike arrival:
excitation, total rate

Next lecture: 11h15

In each small time step Δt
Prob. Of firing $p = \nu \Delta t$

Firing independent between one time step and the next

Show that autocorrelation for $\Delta t \to 0$ $\quad \langle S(t) S(t') \rangle = \nu\, \delta(t - t') + \nu^2$

Show that in an a long interval of duration T, the expected number of spikes is $\langle N(T) \rangle = \nu T$

Comparison of Noise Models and Stochastic Resonance

Part 3 of *Lecture 9 of course*:
Noisy neurons and coding (again):
autocorrelation, variance, stochastic resonance
Neural Networks and Biological Modeling
-stochastic spike arrival (passive membrane)
→ -Comparison of noise models
-Coding and stochastic resonance

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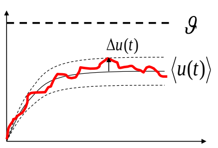
Chapter 5

Wulfram Gerstner
EPFL

Diffusive noise (stochastic spike arrival)

$$\tau \frac{d}{dt} u = -(u - u_{rest}) \quad + R\, I(t) + \quad \xi(t)$$

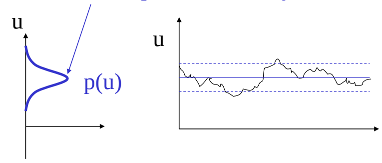


$$\langle \Delta u(t) \Delta u(t) \rangle = \langle u(t) u(t) \rangle - \langle u(t) \rangle^2 =$$

Blackboard

Diffusive noise (stochastic spike arrival)

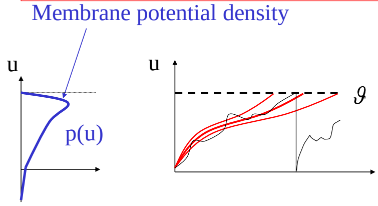
Membrane potential density: Gaussian



constant input rates
no threshold

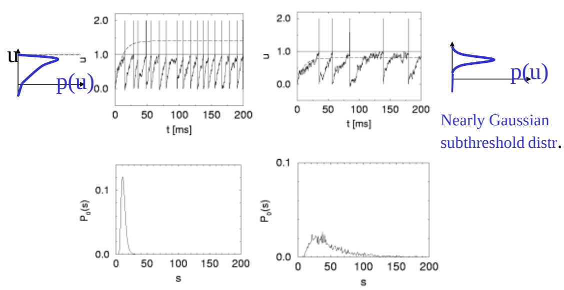
Diffusive noise (stochastic spike arrival)

Membrane potential density



Diffusive noise (stochastic spike arrival)

Superthreshold vs. Subthreshold regime

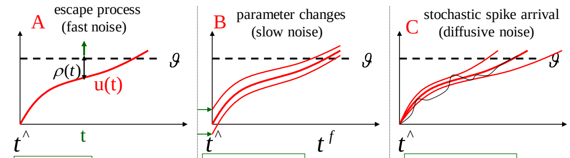


Nearly Gaussian subthreshold distr.

Lecture 9 – noisy neurons and coding again: autocorrelation, variance, stochastic resonance

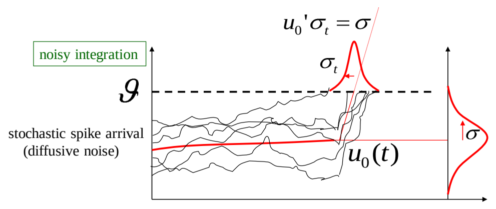
- -review of lecture 8
- -stochastic spike arrival (diffusive noise)
- -Comparison of noise models
- -Coding and stochastic resonance

Noise models



escape rate	stochastic reset	noisy integration
$\rho(t)=f(u(t)-\vartheta)$		$\tau\cdot\frac{du_i}{dt}=-u_i+RI+\xi(t)$
Interval distribution	Interval distribution	noise: white (fast noise), synapse (slow noise)
$P_I(t\mid\hat{t})=\rho(t)\cdot\exp(-\int_{\hat{t}}^{t}\rho(t')dt')$	$P_I(t\mid\hat{t})=G_\sigma(t-t^f)$	$P_I(t\mid\hat{t})$ first passage time problem
escape rate; Survivor function	Gaussian about t^f	(Brunel et al., 2001)

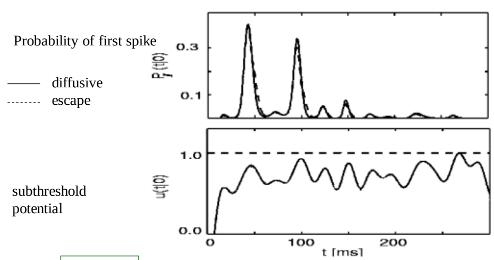
Noise models: from diffusive noise to escape rates



escape rate

$$\rho(t)=f(u_0(t),u'_0(t))\propto\frac{\exp(-\frac{(u_0(t)-\vartheta)^2}{2\sigma^2})}{erf((u_0(t)-\vartheta)/\sigma)}[1+u'_0(t)]$$

Comparison: diffusive noise vs. escape rates



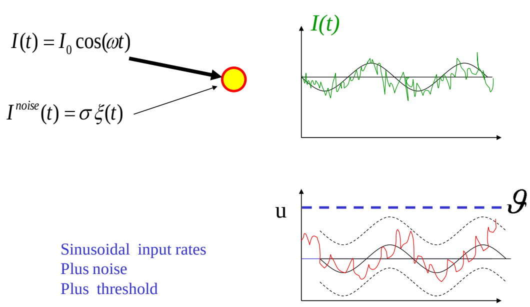
escape rate

$$\rho(t)=f(u_0(t),u'_0(t))\propto\exp(-\frac{(u_0(t)-\vartheta)^2}{2\sigma^2})\;[1+u'_0(t)]$$

Lecture 9 – noisy neurons and coding again: autocorrelation, variance, stochastic resonance

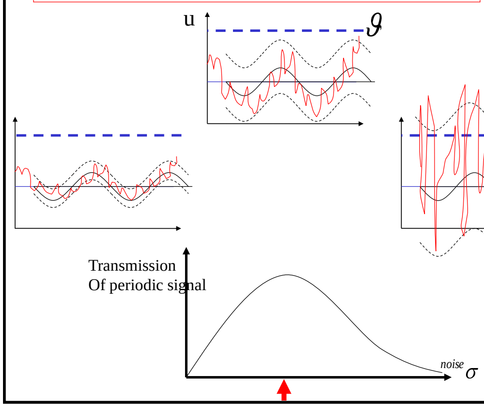
- -Review of lecture 8
- -stochastic spike arrival (diffusive noise)
- -Comparison of noise models
- -Coding and stochastic resonance

Stochastic Resonance: changing the noise level



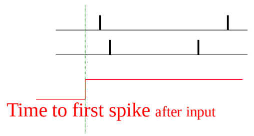
Blackboard

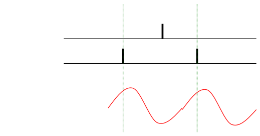
Stochastic Resonance: changing the noise level



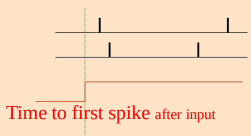
The problem of neural coding:

temporal codes





Exercise 3 now: Coding by time to first spike



Next lecture:
Question session
miniproject
12h00

With deterministic model
With Poisson model
With noisy IF (escape noise)

The end