Lecture 10 – **Populations of Neurons: Fokker-Planck equation**

Course Neural Networks and Biological Modeling

- -Review of Integrate-and-Fire
- -Definition of homogeneous Population
- -Density of membrane potentials
- -Flux
- -Fokker-Planck Equation
- -Threshold and Reset

BOOK: Spiking Neuron Models,
W. Gerstner and W. Kistler
Cambridge University Press, 2002

Chapter 6

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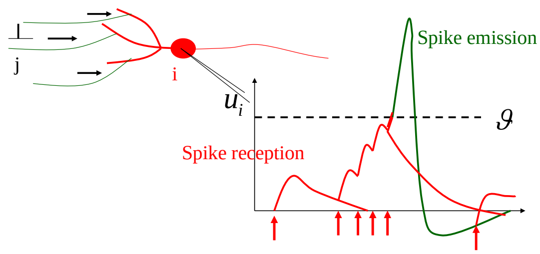
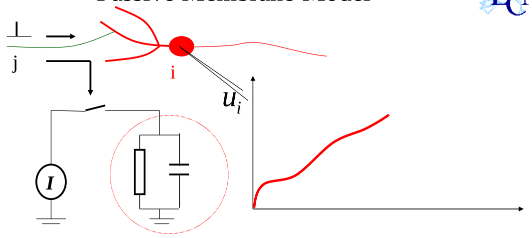
Integrate-and-fire type models



-spikes are events
-threshold
-spike/reset/refractoriness

Passive Membrane Model



$$\tau \cdot \frac{d}{dt} u = -(u - u_{eq}) + RI(t)$$

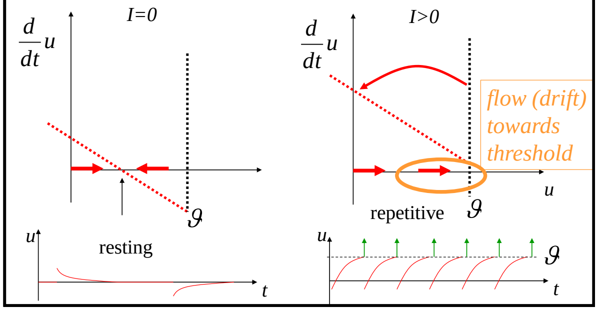
$$\tau \cdot \frac{d}{dt} V = -V + RI(t); \quad V = (u - u_{eq})$$

Integrate-and-fire Models

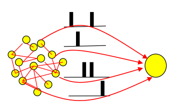
$$\tau \cdot \frac{d}{dt} u = -(u - u_{eq}) + RI(t)$$

LIF

If firing: $u \rightarrow u_{reset}$



Diffusive noise (stochastic spike arrival)



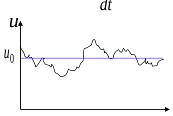
Stochastic spike arrival:
excitation, total rate R_e
inhibition, total rate R_i

Synaptic current pulses

$$\tau \frac{d}{dt} u = -(u - u_{eq}) + R\{ \underbrace{\sum_{k,f} q_e \delta(t - t_k^f)}_{\text{EPSC}} - \underbrace{\sum_{k',f'} q_i \delta(t - t_{k'}^{f'})}_{\text{IPSC}} \}$$

See last week

$$\tau \frac{d}{dt} u = -(u - u_{eq}) \quad + R I^{mean}(t) + \quad \xi(t)$$



Langevin equation,
Ornstein Uhlenbeck process

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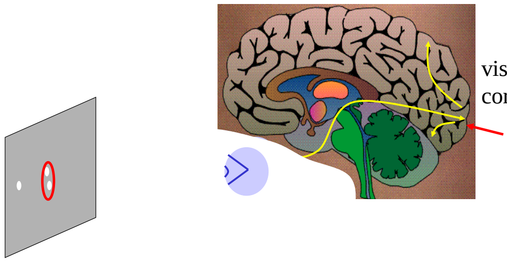
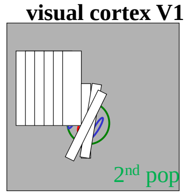
From single Neurons to Populations

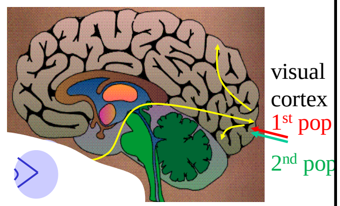


Receptive fields, see lecture 4

From single Neurons to Populations

Receptive fields:
visual cortex V1

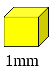




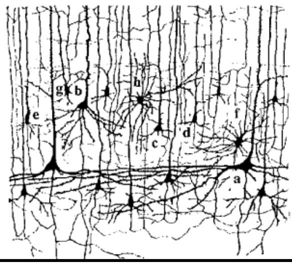
Orientation selective

Neighboring neurons have similar properties

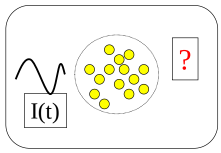
From single neurons to populations



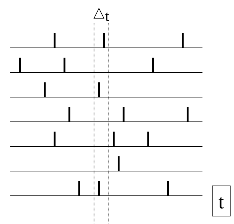
10 000 neurons
3 km wires



Populations of spiking neurons



population dynamics?

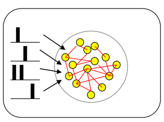


Homogeneous network:
- each neuron receives input from k neurons in network
- each neuron receives the same (mean) external input

population activity

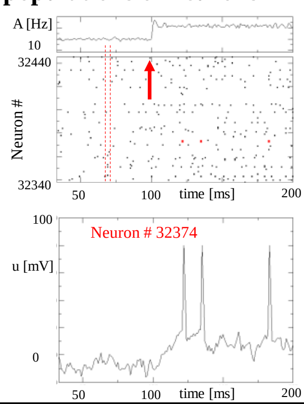
$$A(t) = \frac{n(t;t+\Delta t)}{N\Delta t}$$

Random firing in a populations of neurons

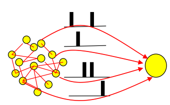


input {-low rate, -high rate}

Population
- 50 000 neurons
- 20 percent inhibitory
- **randomly connected**



Diffusive noise (stochastic spike arrival)

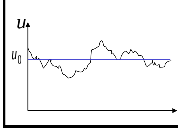


Stochastic spike arrival:
excitation, total rate R_e
inhibition, total rate R_i

For any arbitrary neuron in the population

$$\tau \frac{d}{dt} u = -u + R\{ \underbrace{\sum_{k,f} q_e \delta(t-t_k^f)}_{\text{EPSC}} - \underbrace{\sum_{k',f'} q_i \delta(t-t_{k'}^{f'})}_{\text{IPSC}} \}$$

current pulses



Lecture 10 – **Populations of Neurons: Fokker-Planck equation**

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Chapter 6

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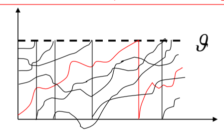
Diffusive noise (stochastic spike arrival)



Blackboard: density of potentials?

For any arbitrary neuron in the population

$$\tau \frac{d}{dt}u = -u + R\Big(\sum_{k,f} q_e \delta(t-t_k^f) - \sum_{k',f'} q_i \delta(t-t_{k'}^{f'})\Big)$$

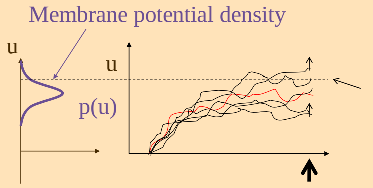
EPSC IPSC

$$\frac{d}{dt}u = -\frac{u}{\tau} + \sum_{k,f}\frac{q_e}{C}\delta(t-t_k^f) - \sum_{k',f'}\frac{q_i}{C}\delta(t-t_{k'}^{f'}) + I^{ext}(t)$$

external input

Exercise 1: flux caused by stochastic spike arrival

Next lecture: 10h15



Reference level u_0

a) Jump at time t

b) spike arrival rate ν

c) spike arrival rate $\sum_k \nu_k$

What is the flux across u_0?

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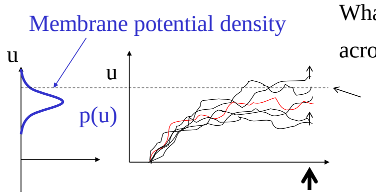
flux – two possibilities

What is the flux across u_0?



Reference level u_0

Jumps caused at spike arrival rate ν

a) flux caused by jumps due to stochastic spike arrival

b) flux caused by systematic drift

Blackboard: Slope and density of potentials

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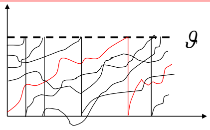
Diffusive noise (stochastic spike arrival)



Blackboard: Derive Fokker-Planck equation

For any arbitrary neuron in the population

$$\frac{d}{dt}u = -\frac{u}{\tau} + \sum_{k,f}\frac{q_e}{C}\delta(t-t_k^f) - \sum_{k',f'}\frac{q_i}{C}\delta(t-t_{k'}^{f'}) + \frac{1}{C}I^{ext}(t)$$

EPSC IPSC external input

Continuity equation:

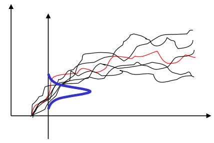
$$\frac{\partial}{\partial t}p(u,t) = -\frac{\partial}{\partial u}J(u,t)$$

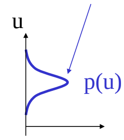
Flux: - jump (spike arrival)
- drift (slope of trajectory)

Diffusive noise (stochastic spike arrival), no threshold

Membrane potential density

blackboard





Fokker-Planck

$$\tau\cdot\frac{\partial}{\partial t}p(u,t) = -\frac{\partial}{\partial u}[\gamma(u)\,p(u,t)] + \sigma^2\frac{\partial^2}{\partial u^2}p(u,t)$$

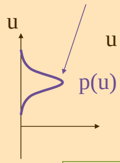
drift $\gamma(u) = -u + \tau\sum_k \nu_k w_k$

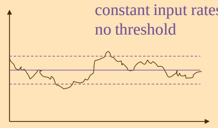
diffusion $\sigma^2 = \frac{1}{2}\tau\sum_k \nu_k w_k^2$

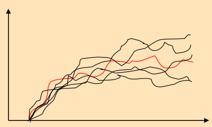
spike arrival rate

Exercise 2: solution of free Fokker-Planck equation

Membrane potential density: Gaussian







Fokker-Planck

$$\tau\cdot\frac{\partial}{\partial t}p(u,t) = -\frac{\partial}{\partial u}[\gamma(u)\,p(u,t)] + \sigma^2\frac{\partial^2}{\partial u^2}p(u,t)$$

drift $\gamma(u) = -u + \tau\sum_k \nu_k w_k + RI(t)$

diffusion $\sigma^2 = \frac{1}{2}\tau\sum_k \nu_k w_k^2$

spike arrival rate

Next lecture: 11h15

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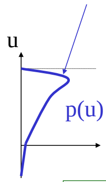
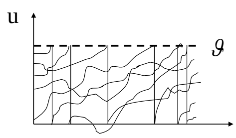
Diffusive noise (stochastic spike arrival) with threshold

Membrane potential density





blackboard

Fokker-Planck

$$\tau\cdot\frac{\partial}{\partial t}p(u,t) = -\frac{\partial}{\partial u}[\gamma(u)\,p(u,t)] + \sigma^2\frac{\partial^2}{\partial u^2}p(u,t) + \tau\,A(t)\,\delta(u-u_{reset})$$

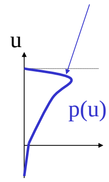
drift $\gamma(u) = -u + \tau\sum_k \nu_k w_k + RI$

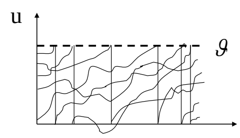
diffusion $\sigma^2 = \frac{1}{2}\tau\sum_k \nu_k w_k^2$

spike arrival rate

Diffusive noise (stochastic spike arrival) with threshold

Membrane potential density





blackboard

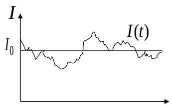
Population Firing rate *A(t):* flux at threshold

Population firing rate $A(t)$ = f =single neuron firing rate

Synaptic current pulses

$$\tau \frac{d}{dt}u = -(u-u_{eq}) + R\{ \sum_{k,f} q_e \delta(t-t_k^f) - \sum_{k',f'} q_i \delta(t-t_{k'}^{f'}) \}$$

EPSC IPSC

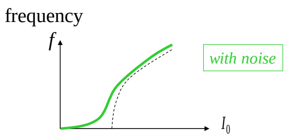


$$\tau \frac{d}{dt}u = -(u-u_{eq}) + R I^{mean}(t) + \xi(t)$$

$$\tau \frac{d}{dt}u = -(u-u_{eq}) + R I(t)$$

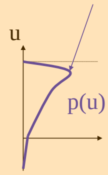
$$I(t) = [I_o + I_{noise}]$$

effective noise current



Exercise 3: Diffusive noise + Threshold

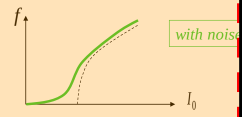
Membrane potential density



u Fokker-Planck

$$\tau \cdot \frac{\partial}{\partial t} p(u,t) = -\frac{\partial}{\partial u}[\gamma(u) p(u,t)] + \tfrac{1}{2}\sigma^2 \frac{\partial^2}{\partial u^2} p(u,t) + source$$

$A=$



Miniproject: 12h00

- Calculate distribution *p(u)*
- Determine population firing rate *A*

THE END