Property matrices from unit-impulse response functions of unbounded medium (1994 – 1997)

Antonio Paronesso

Dynamic unbounded medium-structure interaction

In a dynamic unbounded medium-structure-interaction problem based on the substructure method (Fig. 1), the bounded structure and the unbounded structure (semi-infinite or infinite) medium are coupled on the structure-medium interface. In general, the structure will exhibit non-linear behaviour while the unbounded medium is always assumed to remain linear.

a) b)

Fig. 1. Problem definition of dynamic unbounded medium-structure-interaction analysis. a) Coupled system. b) Unbounded medium.

The interaction force $\{R(t)\}$-displacement $\{u(t)\}$ relationship with respect to the degrees of freedom of the nodes on the structure-medium interface of the unbounded medium is global in space and time (Fig. 1b).

$$\{R(t)\} = [K_\infty]\{u(t)\} + [C_\infty]\{\dot{u}(t)\} + \int_0^t [S_r(t-t)]\{u(t)\}dt$$

The first two terms on the right-hand side representing the instantaneous response denote the singular part with $[K_\infty]$ and $[C_\infty]$ defining the high-frequency limit ($w \to \infty$)of the dynamic-stiffness matrix $[S(w)]$. The third term on the right-hand side describing the lingering response is equal to the regular part (subscript r) consisting of the convolution of the corresponding unit-impulse response function $[S_r(t)]$ and the displacement.
The interaction forces at a specific time depend on the time histories of the displacements in all nodes from the start of the excitation onwards. In the rigorous formulation, a large computational effort (proportional to the *square* of the number of time steps) and storage requirement result, which makes it unrealistic to evaluate large practical problems.

Linear system theory

To reduce the computational effort, concepts of linear system theory can be applied. A systematic detailed description of the various procedures starting from either $[S_r(t_j)]$ or $[S_r(w_j)]$ and leading to either a continuous-time or discrete-time formulation is contained in the Reference . All methods are based on a *rational approximation* of the frequency response of the unbounded medium. This corresponds to introducing some internal degrees of freedom in addition to those present on the structure-medium interface. The unbounded medium is thus represented as a model with a finite number of degrees of freedom. The computational effort becomes proportional to the number of time steps and the storage requirement is also greatly reduced.

Using a balancing approximation a system of *second-order differential equations with the symmetric mass, damping and static-stiffness* matrices can be established ($[M]$, $[C]$ and $[K]$).

$$[M]\begin{Bmatrix}\{\ddot{w}(t)\}\\ \{\ddot{u}(t)\}\end{Bmatrix} + [C]\begin{Bmatrix}\{\dot{w}(t)\}\\ \{\dot{u}(t)\}\end{Bmatrix} + [K]\begin{Bmatrix}\{w(t)\}\\ \{u(t)\}\end{Bmatrix} = \begin{Bmatrix}\{0\}\\ \{R(t)\}\end{Bmatrix}$$

This equation for the model of the unbounded medium is in the same form as the equation of motion in finite-element analysis of a structure with a finite number of degrees of freedom. The static-stiffness, damping and mass matrices are called the property matrices.

In-plane motion of semi-infinite layer

The in-plane motion of a semi-infinite layer with a free and fixed boundary extending to infinity of constant depth is addressed (Fig. 2). On the vertical structure-medium interface 4 line finite elements are introduced.

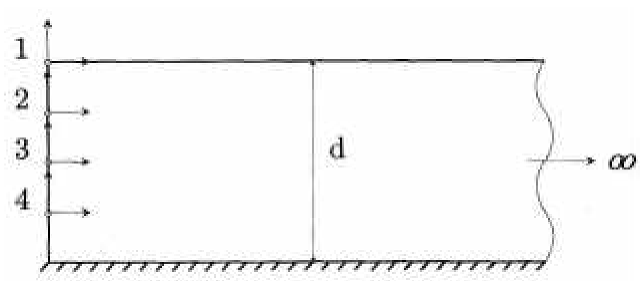

Fig. 2. In-plane motion of a semi-infinite layer fixed at its base with discretization on structure-medium interface.

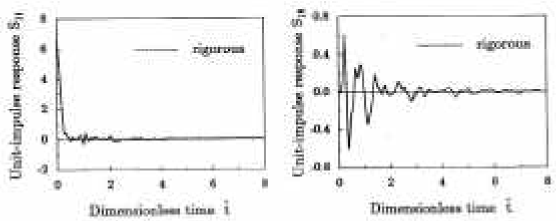

Fig. 3: Unit impulse response coefficients.

The symmetric matrix $[S_r(t_j)]$ of dimensions 8x8 is calculated. The comparison between the rigorous result and the approximation is shown in Fig. 3. Excellent agreement results.

REFERENCE

A. Paronesso, *Rational Approximation and Realization of Generalized Force-Displacement Relationship of an Unbounded Medium*, PhD Thesis, Laboratory of Hydraulic Constructions, Department of Civil Engineering, Swiss Federal Institute of Technology Lausanne, February 1997.