Fundamentals in Biophotonics

Photon /wave particle duality

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Blackbody radiation

- Hot objects glow (toaster coils, light bulbs, the sun).

- As the temperature increases the color shifts from Red (700 nm) to Blue (400 nm)

A blackbody is a hypothetical object that is a perfect absorber of electromagnetic radiation at all wavelengths

Stars closely approximate the behavior of blackbodies, as do other hot, dense objects

The intensities of radiation emitted at various wavelengths by a blackbody at a given temperature are shown by a blackbody curve
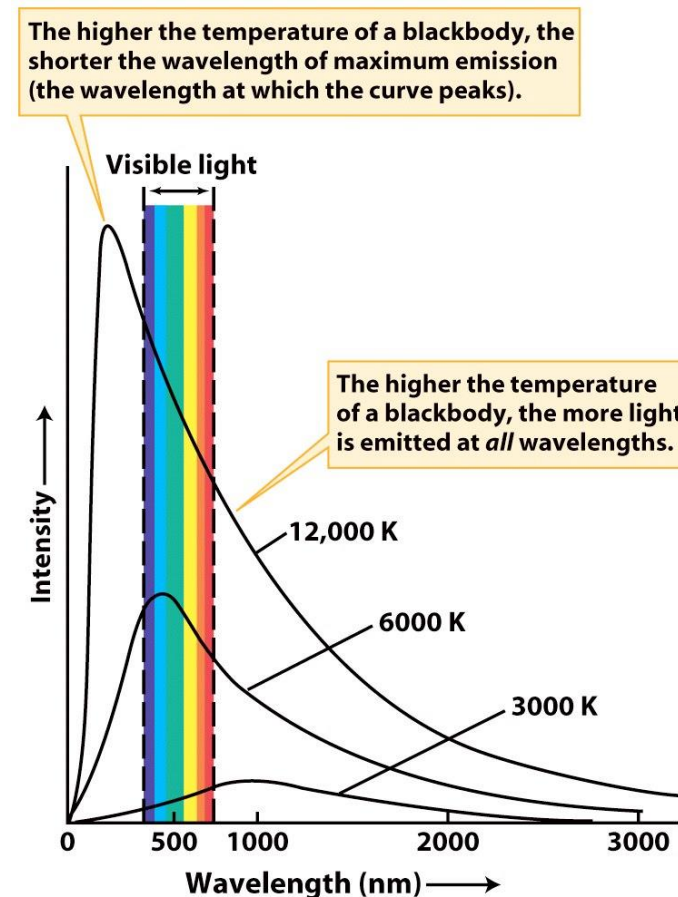
Hot:
glows deep red

(a)

Hotter:
glows reddish-orange

(b)

Even hotter:
glows yellowish-white

(c)
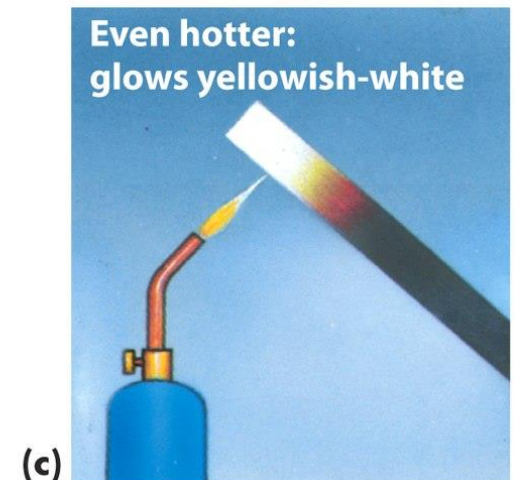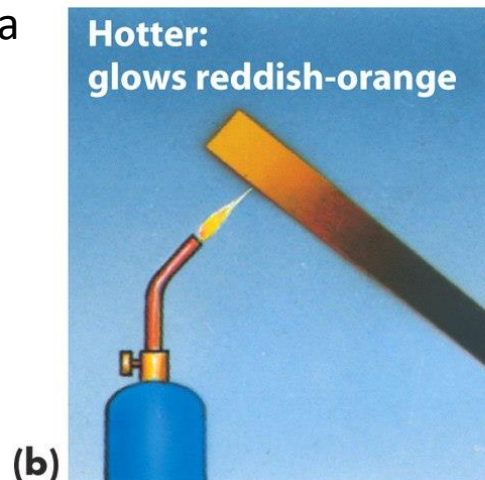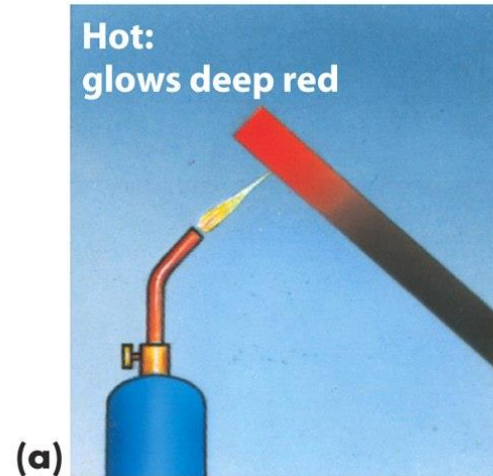
Wien's Law

$$\lambda_{max} = \frac{0.0029 \text{ K m}}{T}$$

λ_{max} = wavelength of maximum emission of the object (in meters)

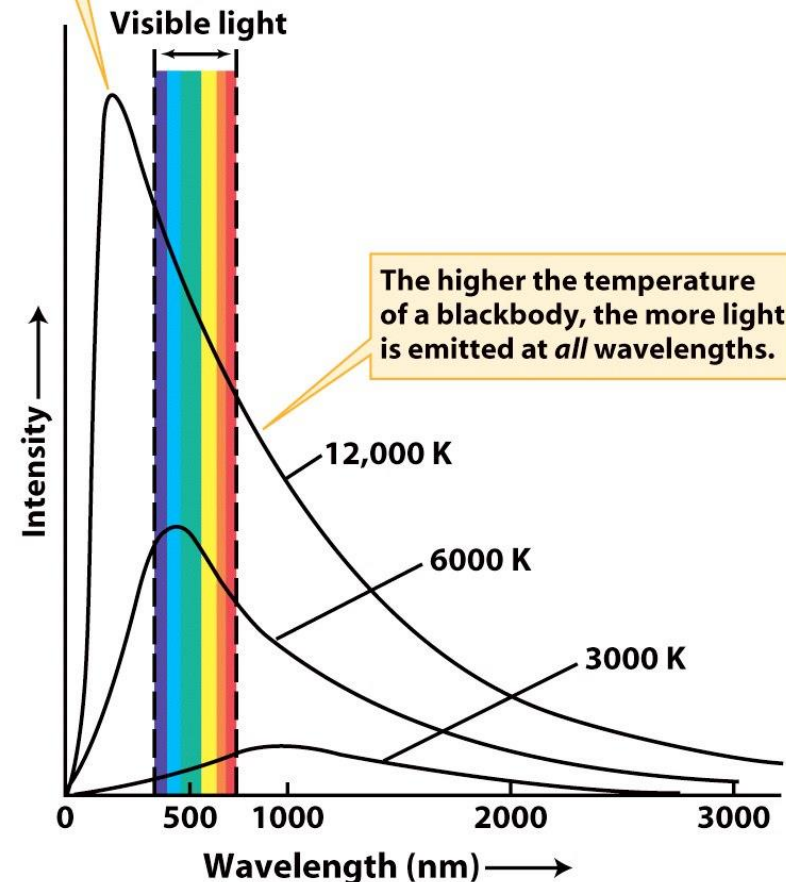T = temperature of the object (in kelvins)

Wien's law states that the dominant wavelength at which a blackbody emits electromagnetic radiation is inversely proportional to the Kelvin temperature of the object

The higher the temperature of a blackbody, the shorter the wavelength of maximum emission (the wavelength at which the curve peaks).

Visible light

The higher the temperature of a blackbody, the more light is emitted at *all* wavelengths.

Intensity

12,000 K

6000 K

3000 K

0   500   1000   2000   3000

Wavelength (nm) ⟶

Black body radiation

- The classical physics prediction was completely wrong! (It said that an infinite amount of energy should be radiated by an object at finite temperature)

Wien's law

Rayleigh–Jeans law

$$I(\lambda, T) = \frac{2\pi ckT}{\lambda^4}$$

The higher the temperature of a blackbody, the shorter the wavelength of maximum emission (the wavelength at which the curve peaks).

Visible light

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12,000 K

6000 K

3000 K

Intensity

0   500   1000   2000   3000

Wavelength (nm) ⟶

Stefan-Boltzman law

$$j = \sigma T^4$$

Max Planck found he could explain these curves if he assumed that electromagnetic energy was radiated in discrete chunks, rather than continuously.

The "quanta" of electromagnetic energy is called the photon.

Energy carried by a single photon is

$$E = h\nu = \hbar\omega,$$

Planck's constant: h = 6.626 x 10^{-34} Joule sec

Electromagnetic-Optics Theory of Light in a Resonator

- The concept of the photon and the rules of photon optics are introduced by considering light inside an optical resonator (cavity). This is a convenient choice because it restricts the space under consideration to a simple geometry. However, the presence of the resonator turns out not to be an important restriction in the argument; the results can be shown to be independent of its presence.

(a)

(b)
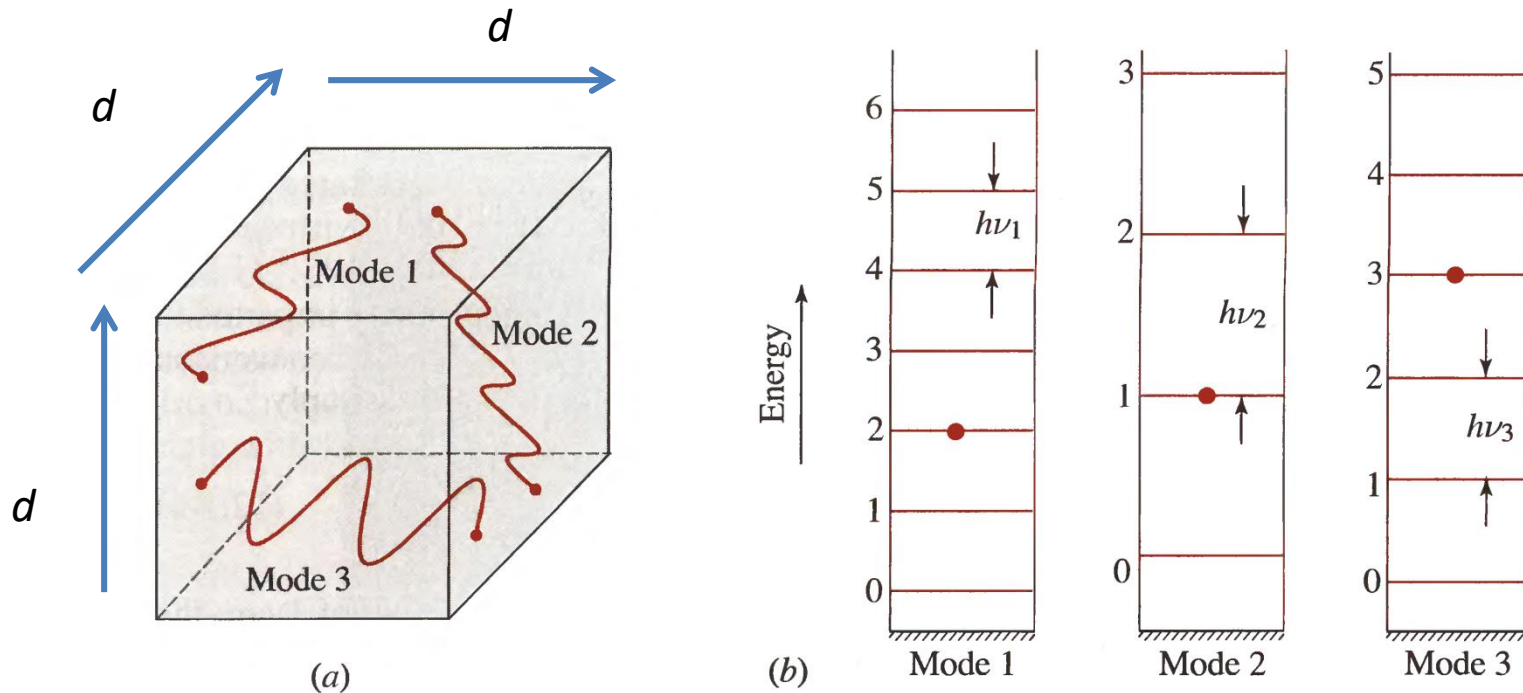
Electromagnetic-Optics Theory of Light in a Resonator

- light inside a lossless resonator of volume V is completely characterized by an electromagnetic field that takes the form of a superposition of discrete orthogonal modes of different spatial distributions, different frequencies, and different polarizations.

$$\mathbf{E}(\mathbf{r},t) = \sum_{\mathbf{q}} A_{\mathbf{q}} U_{\mathbf{q}}(\mathbf{r}) \exp(j2\pi\nu_{\mathbf{q}}t)\, \widehat{\mathbf{e}}_{\mathbf{q}}. \qquad \int_V |U_{\mathbf{q}}(\mathbf{r})|^2\, d\mathbf{r} = 1.$$

In a cubic resonator of dimension d, a convenient choice of the spatial expansion functions is the set of standing waves

$$U_{\mathbf{q}}(\mathbf{r}) = \left(\frac{2}{d}\right)^{3/2} \sin\left(q_x \frac{\pi}{d} x\right) \sin\left(q_y \frac{\pi}{d} y\right) \sin\left(q_z \frac{\pi}{d} z\right),$$

Energy contained in the mode q is given

$$E_{\mathbf{q}} = \tfrac{1}{2}\epsilon \int_V |A_{\mathbf{q}}|^2\, |U_{\mathbf{q}}(\mathbf{r})|^2\, d\mathbf{r} = \tfrac{1}{2}\epsilon |A_{\mathbf{q}}|^2,$$

The energy of a mode is quantized with only integral units of this fixed energy permitted. Each unit of energy is carried by ONE photon and the mode may carry an arbitrary number of photons.

Photon energy

- Photon optics provides that the energy of an electromagnetic mode is quantized to discrete levels separated by the energy of a photon . The energy of a photon in a mode of frequency v is

$$E = h\nu = \hbar\omega,$$

- A mode containing zero photons carries some energy which is called the **zero-point energy.**

- TOTAL energy of n photons

$$E_n = \left(n + \tfrac{1}{2}\right) h\nu, \qquad n = 0, 1, 2, \ldots$$

- The order of magnitude of photon energy- EXAMPLE Infrared photon

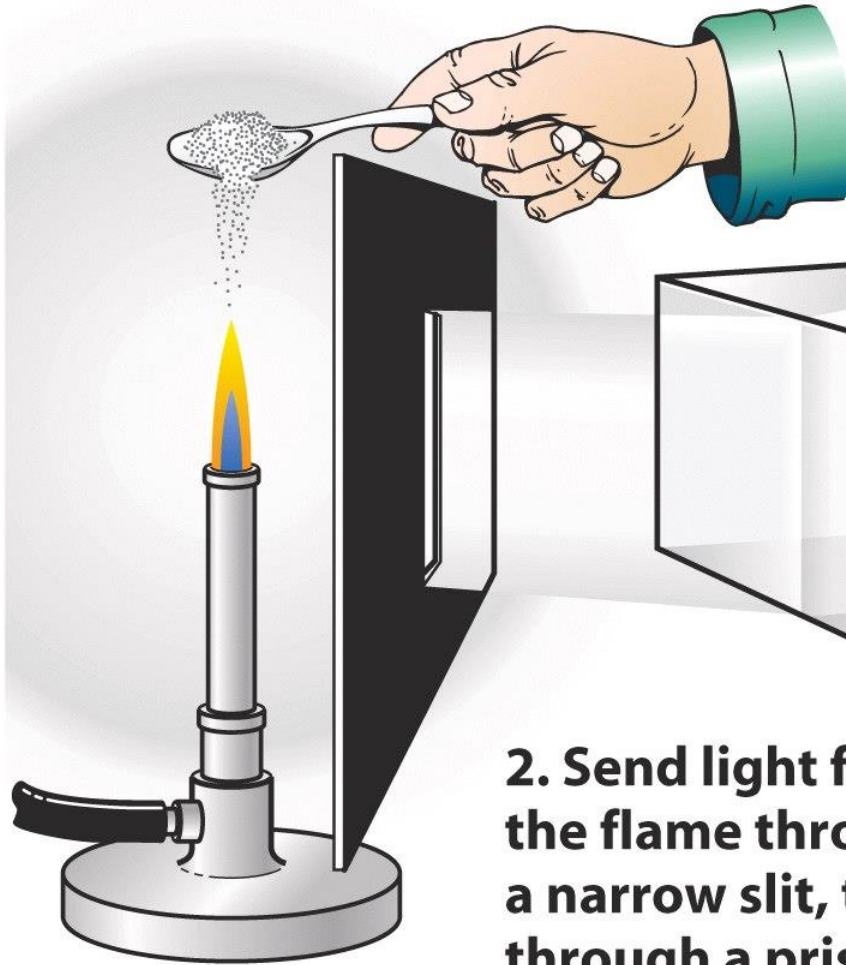$$\lambda = 1\ \mu\text{m} \rightarrow v = \frac{c}{\lambda} = 3 \times 10^{14}\, Hz$$

$$E = h\nu = 1.99 \times 10^{-19}\, J = (1.99 \times 10^{-19} / 1.6 \times 10^{-19})eV$$
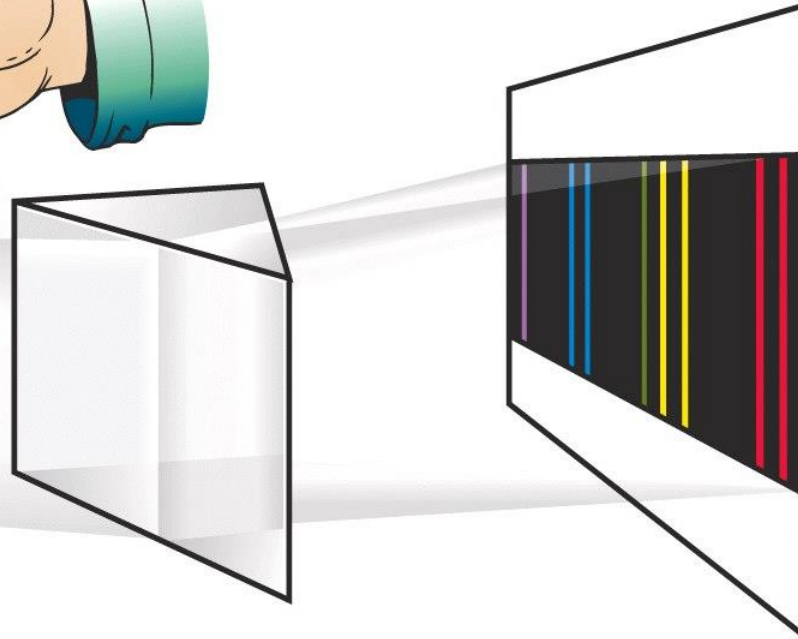
$$E = 1.24 eV$$

$$E\,(\text{eV}) = \frac{1.24}{\lambda_o\,(\mu\text{m})}.$$

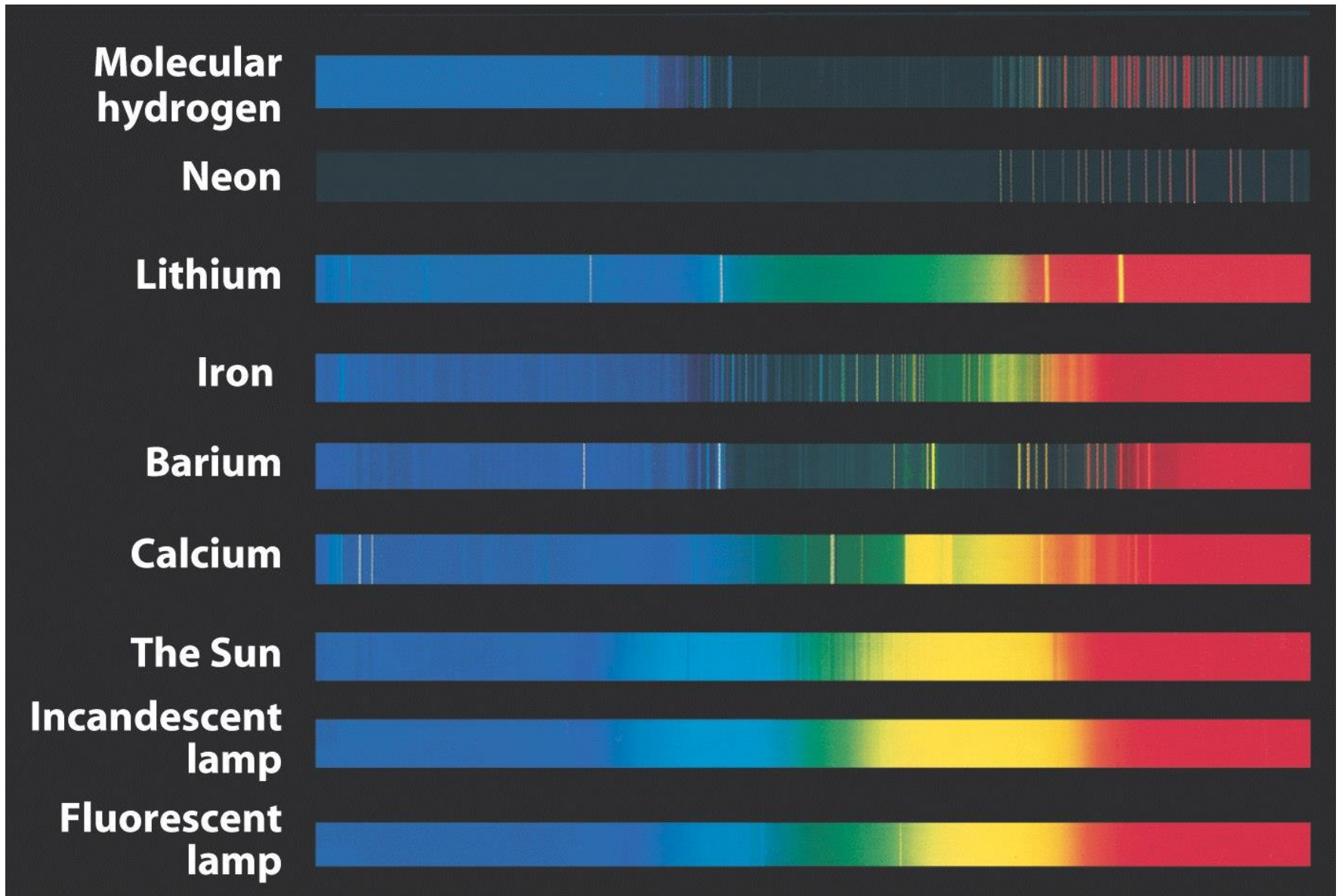1. Add a chemical substance to a flame

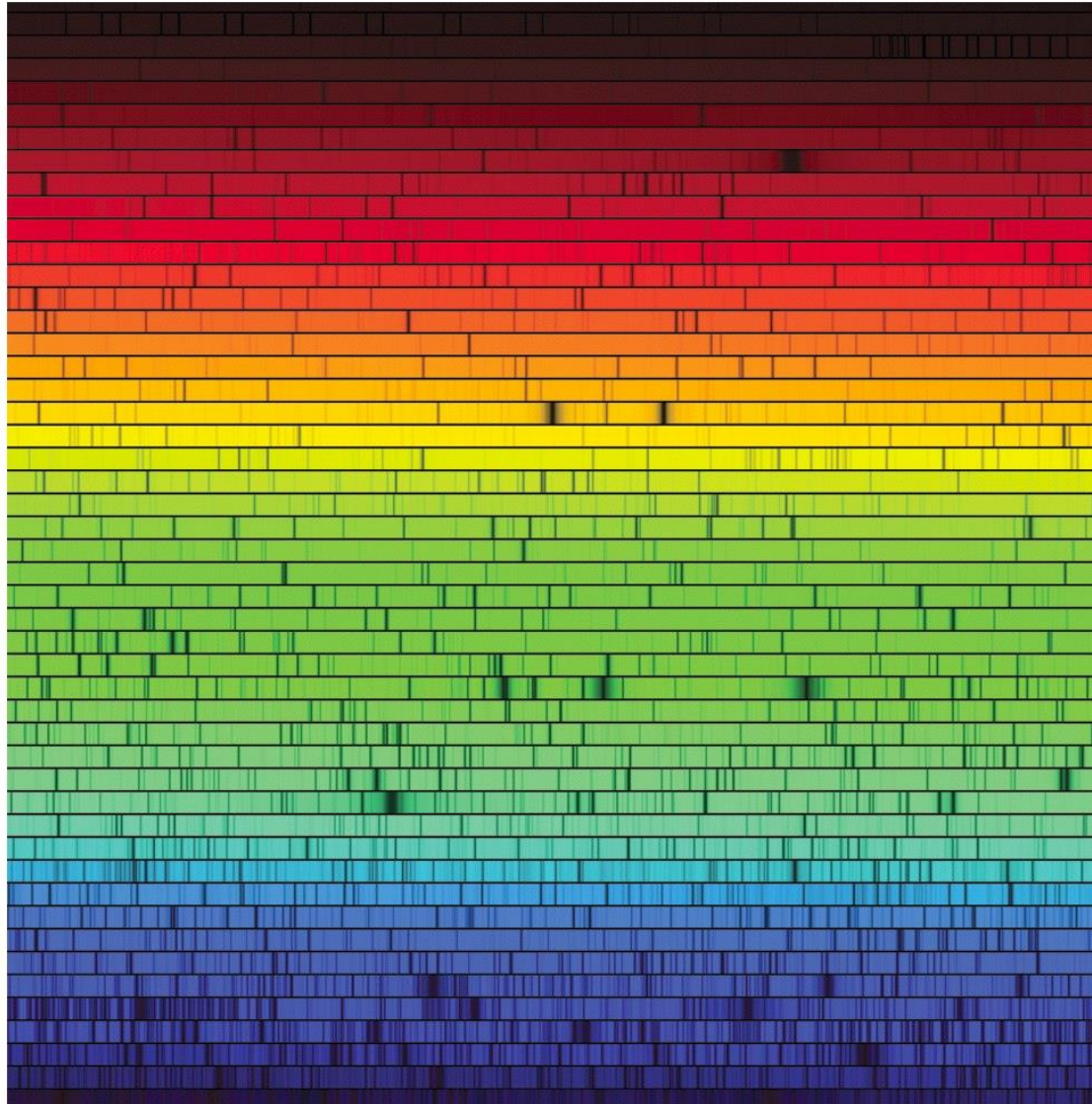2. Send light from the flame through a narrow slit, then through a prism

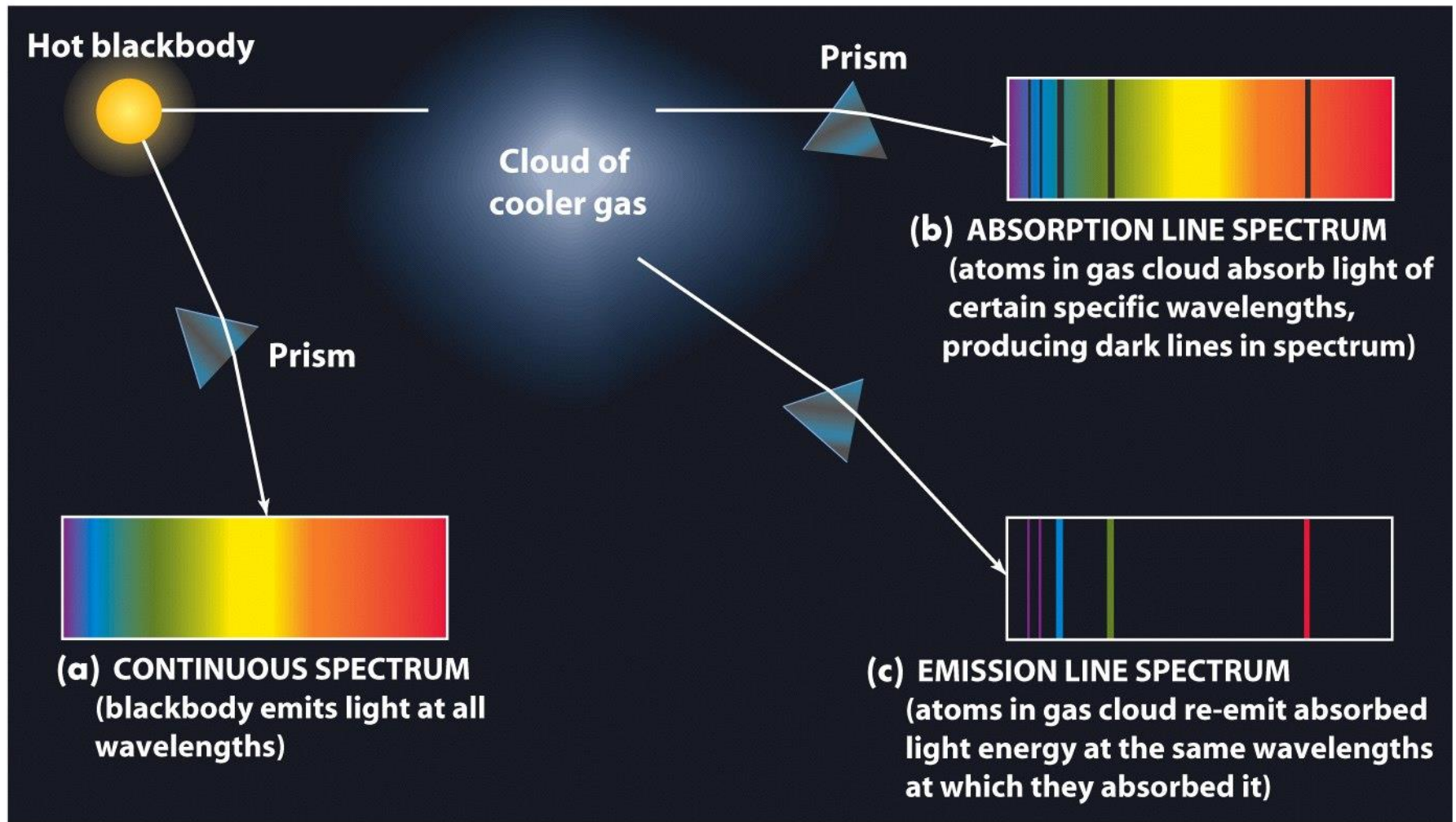3. Bright lines in the spectrum show that the substance emits light at specific wavelengths only

Each chemical element produces its own unique set of spectral lines

Kirchhoff's Laws

Hot blackbody

Prism

Cloud of cooler gas

(b) ABSORPTION LINE SPECTRUM
(atoms in gas cloud absorb light of certain specific wavelengths, producing dark lines in spectrum)

Prism

(a) CONTINUOUS SPECTRUM
(blackbody emits light at all wavelengths)

(c) EMISSION LINE SPECTRUM
(atoms in gas cloud re-emit absorbed light energy at the same wavelengths at which they absorbed it)

- **as a kind of conceptual skeleton**.

Incoming photon,
$\lambda = 656.3$ nm

Emitted photon,
$\lambda = 656.3$ nm

$n = 2$

$n = 3$

(a) Atom absorbs a 656.3-nm photon; absorbed energy causes electron to jump from the $n = 2$ orbit up to the $n = 3$ orbit

(b) Electron falls from the $n = 3$ orbit to the $n = 2$ orbit; energy lost by atom goes into emitting a 656.3-nm photon
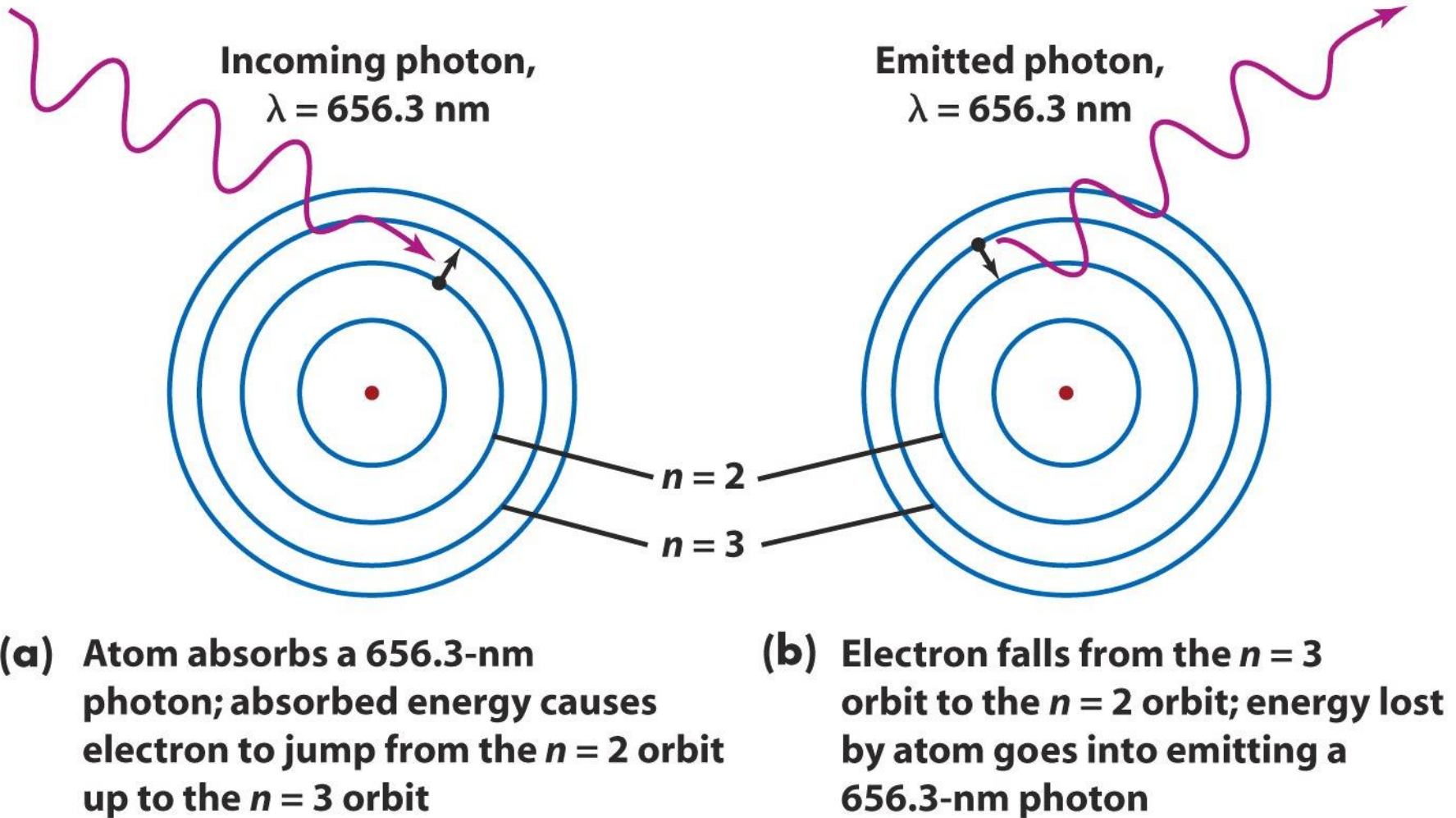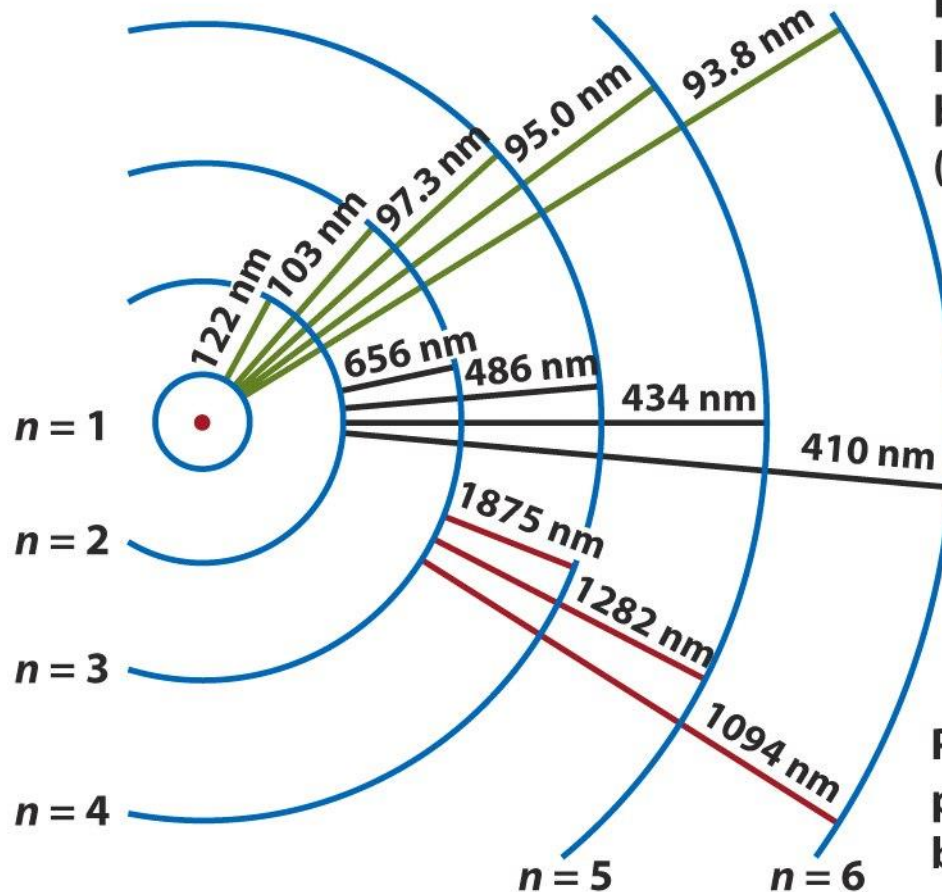
What is known of [photons] comes from observing the results of their being created or annihilated

Eugene Hecht

What is a photon a how light is generated ?


Lyman series (ultraviolet) of spectral lines: produced by electron transitions between the $n = 1$ orbit and higher orbits ($n = 2, 3, 4, …$)

Balmer series (visible and ultraviolet) of spectral lines: produced by electron transitions between the $n = 2$ orbit and higher orbits ($n = 3, 4, 5, …$)

Paschen series (infared) of spectral lines: produced by electron transitions between the $n = 3$ orbit and higher orbits ($n = 4, 5, 6, …$)

What is a photon and how light is generated ?

Bohr's formula for hydrogen wavelengths
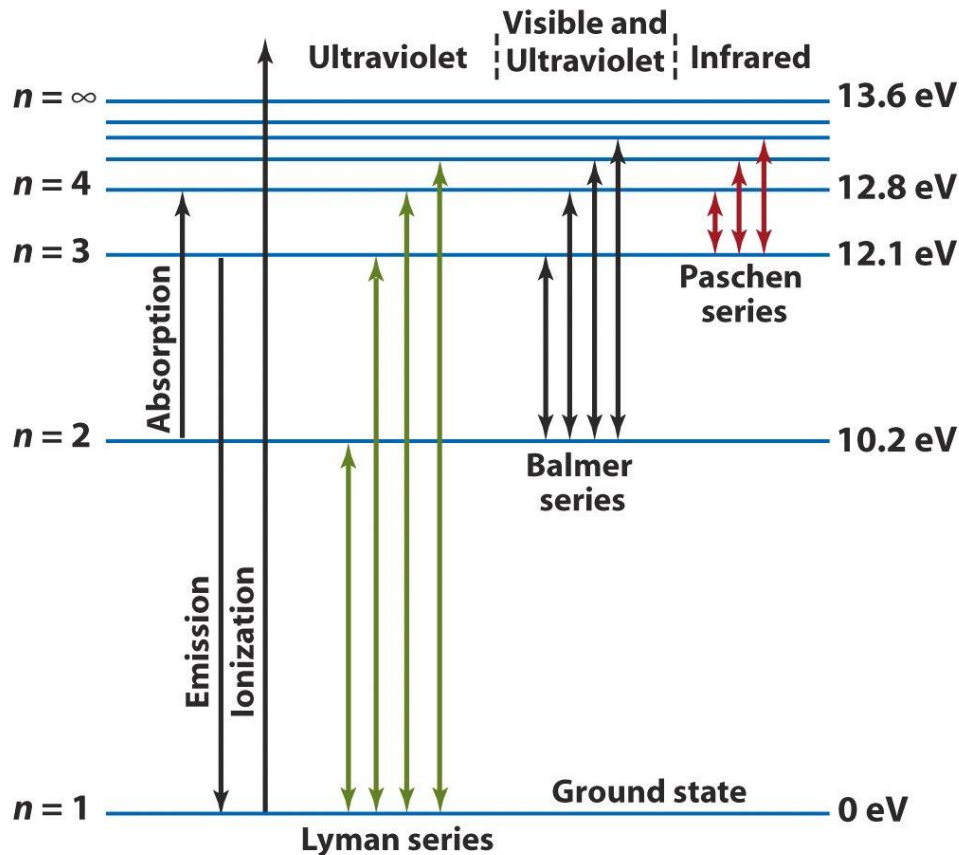
$$\frac{1}{\lambda} = R\left(\frac{1}{N^2} - \frac{1}{n^2}\right)$$

N = number of inner orbit

n = number of outer orbit

R = Rydberg constant (1.097 X 10^7 m^{-1})

λ = wavelength of emitted or absorbed photon

What is a photon a how light is generated ?

- What makes this process special is the **quantized nature of the electron orbits.** As we said, there are only **a special set of allowed orbits for an electron**. Thus, there are only a special allowed sets of energy for an electron. Each orbit has a particular energy associated with it--we say the energy of an orbit is "quantized".

- To move from one orbit outwards to another, the electron *must absorb exactly the right amount of energy--it cannot absorb more or less, it has to be exactly right*

- The electron must absorb a *photon* of light with **exactly the right amount of energy.**
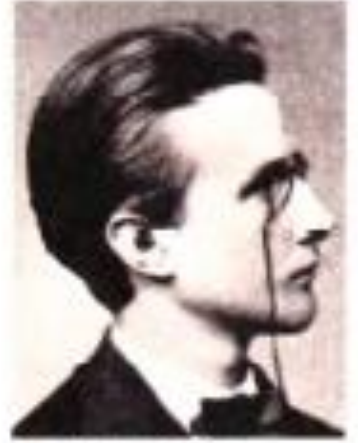
Max Planck (1858-1947)
suggested that the emission and absorption of light by matter takes the form of quanta of energy.
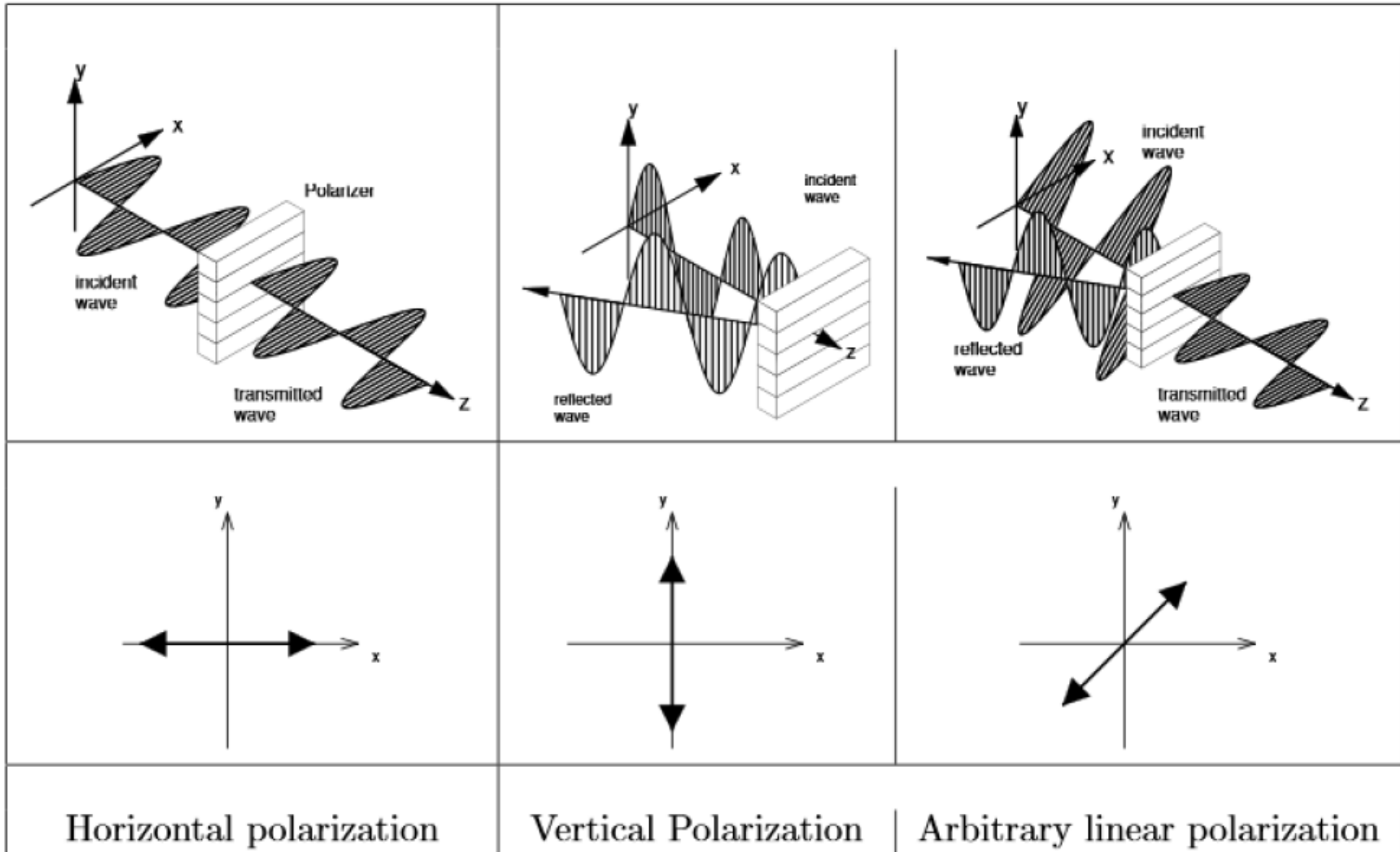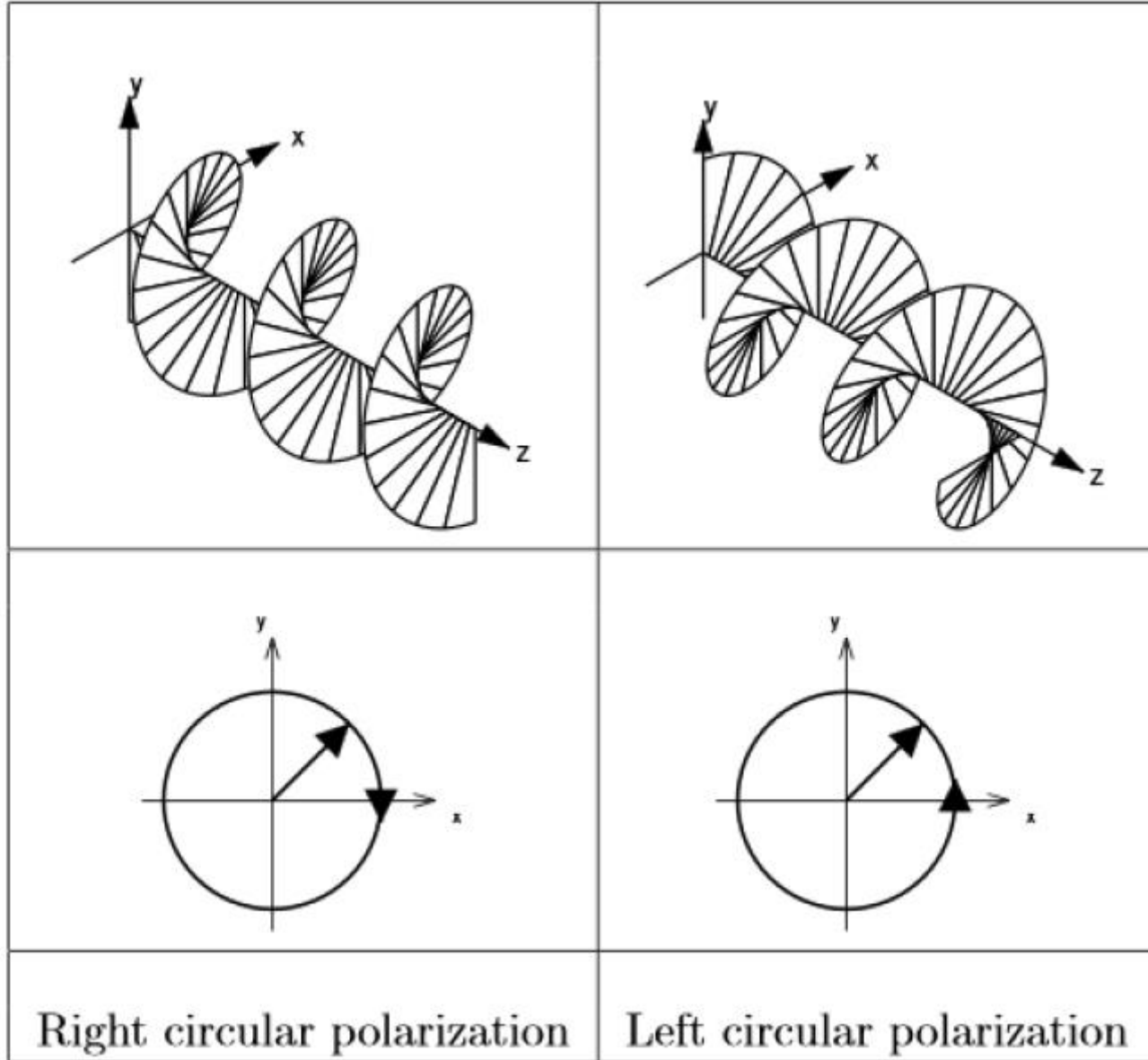

Albert Einstein (1879-1955)
advanced the hypothesis that light itself comprises quanta of energy.

Linearly polarized light incident to a polarizer

- light is travelling in the positive *z*-direction, with angular frequency ω and wavevector **k** = (0,0,*k*), where the wavenumber *k* = ω/*c*.

| Horizontal polarization | Vertical Polarization | Arbitrary linear polarization |

Right circular polarization | Left circular polarization

Photon polarization

- As indicated earlier, light is characterized by a set of modes of **different frequencies, directions, and** **polarizations,** each occupied by a number of photons. For each monochromatic plane wave traveling in some direction, there **are two polarization modes**

$$\mathbf{E}(\mathbf{r},t) = \sum_{\mathbf{q}} A_{\mathbf{q}}\, U_{\mathbf{q}}(\mathbf{r})\, \exp(j2\pi\nu_{\mathbf{q}}t)\, \widehat{\mathbf{e}}_{\mathbf{q}}.$$
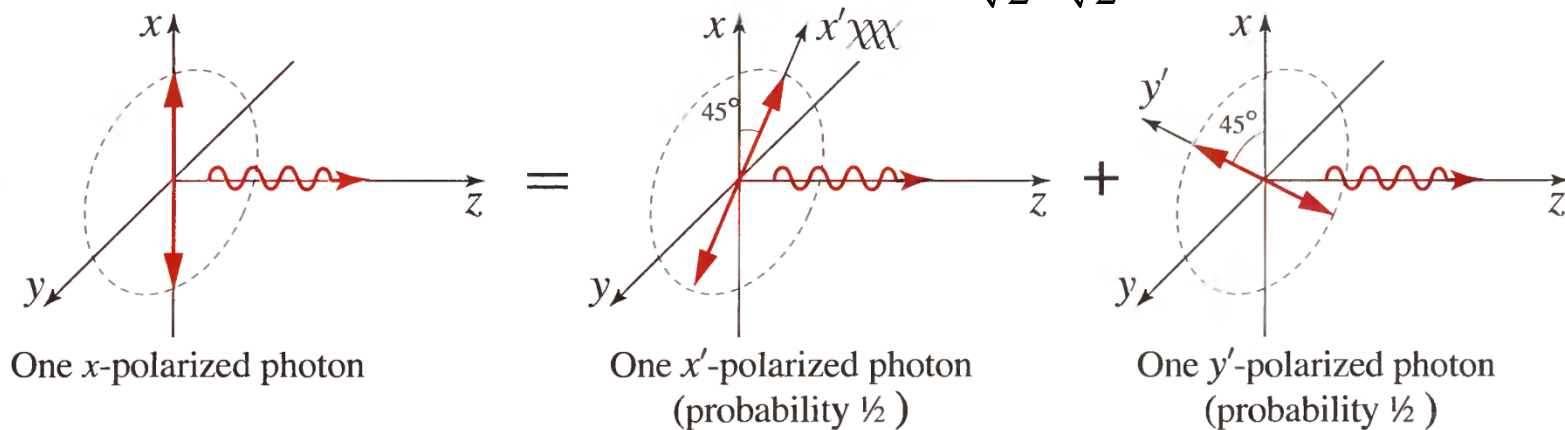
- Since the polarization modes of free space are **degenerate,** they are not unique. One may use modes with linear polarization in the x and y directions, linear polarization in two other orthogonal directions, say x' and y', or right- and left-circular polarizations.

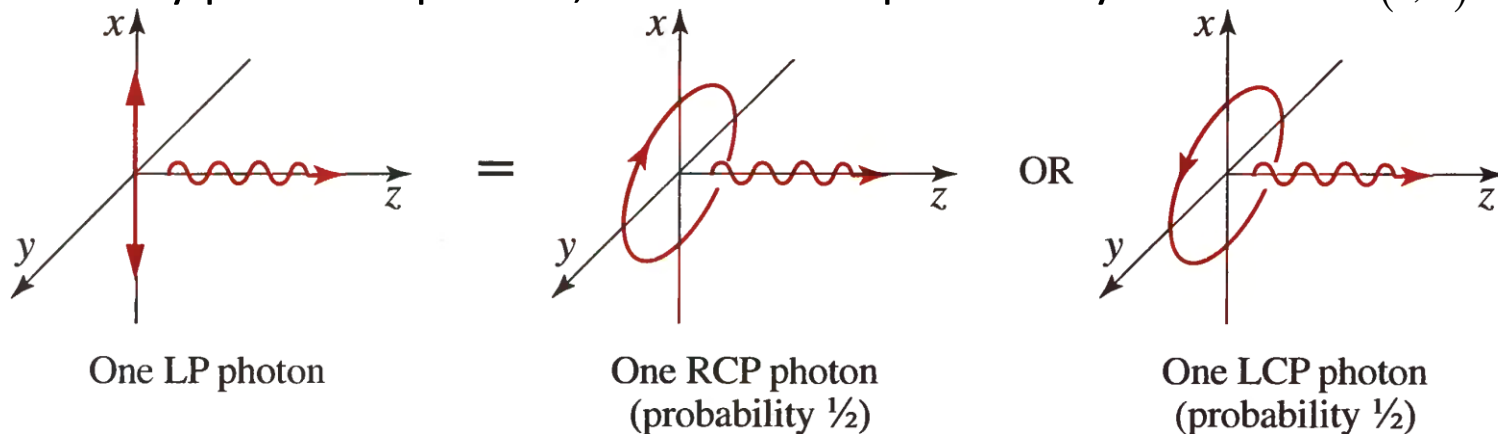$$A_{x'} = \frac{1}{\sqrt{2}}\left(A_x - A_y\right), \qquad A_{y'} = \frac{1}{\sqrt{2}}\left(A_x + A_y\right),$$

The components Ax, Ay are transformed from one coordinate system to another like ordinary Jones vectors, and the new components represent complex probability amplitudes in the new modes. *Thus, a single photon may exist, probabilistically, in more than one mode.*

Photon polarization

- A photon in the x linear polarization mode is the same as a photon in a superposition of the x' linear polarization mode and the y' linear polarization mode with probability ½ each. $Jones\ vector\ (1,0) \rightarrow (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$

One x-polarized photon   =   One x'-polarized photon (probability ½)   +   One y'-polarized photon (probability ½)

A linearly polarized photon is equivalent to the superposition of a right- and a left-circularly polarized photon, ½ each with probability $Jones\ vector\ (1,0) \rightarrow \frac{1}{\sqrt{2}}(1 \pm j)$

.

One LP photon   =   One RCP photon (probability ½)   OR   One LCP photon (probability ½)

6 common examples of normalized Jones vectors.

Polarization	Corresponding Jones vector	
Linear polarized in the x-direction Typically called 'Horizontal'	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	
Linear polarized in the y-direction Typically called 'Vertical'	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$	
Linear polarized at 45° from the x-axis Typically called 'Diagonal' L+45	$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$	
Linear polarized at −45° from the x-axis Typically called 'Anti-Diagonal' L-45	$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$	
Right Hand Circular Polarized Typically called RCP or RHCP	$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ +i \end{pmatrix}$	
Left Hand Circular Polarized Typically called LCP or LHCP	$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}$	

Jones matrices for common optical elements

Optical Element	Jones matrix
horizontal linear polarizer	$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$
vertical linear polarizer	$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$
linear polarizer at θ	$\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$
quarter wave plate (fast axis vertical)	$e^{i\pi/4}\begin{bmatrix} 1 & 0 \\ 0 & -i \end{bmatrix}$
quarter wave plate (fast axis horizontal)	$e^{i\pi/4}\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$

When light crosses an optical element the resulting polarization of the emerging light is found **by taking the product of the Jones matrix of the optical element and the Jones vector of the incident light**. Light which is randomly polarized, partially polarized, or incoherent must be treated using Mueller calculus.

Photon position

- When a photon impinges on a detector of small area **dA** located normal to the direction of propagation at the position r, its indivisibility causes it to be either **wholly detected or not detected at all**

The probability $p(\mathbf{r})dA$ of observing a photon at a point **r** within an incremental area dA, at any time, is proportional to the local optical intensity $I(\mathbf{r}) \propto |U(\mathbf{r})|^2$, so that

$$p(\mathbf{r})\, dA \propto I(\mathbf{r})\, dA.$$
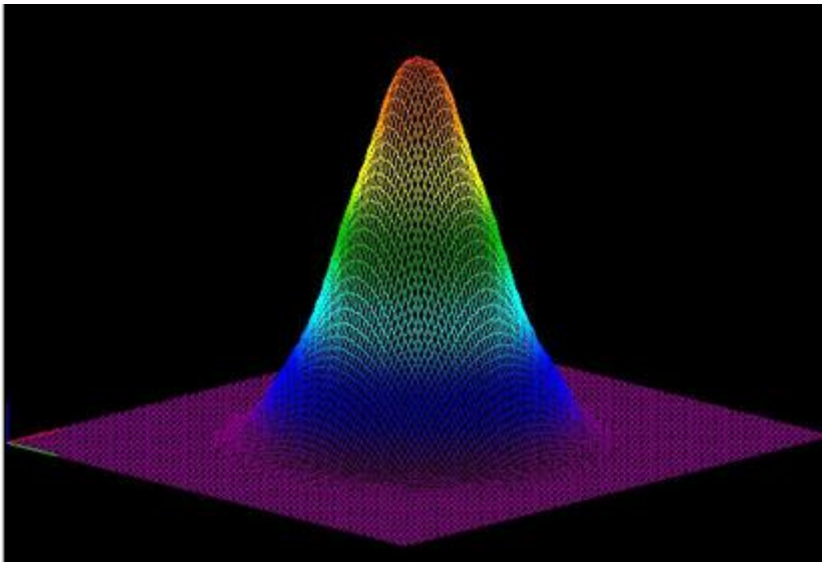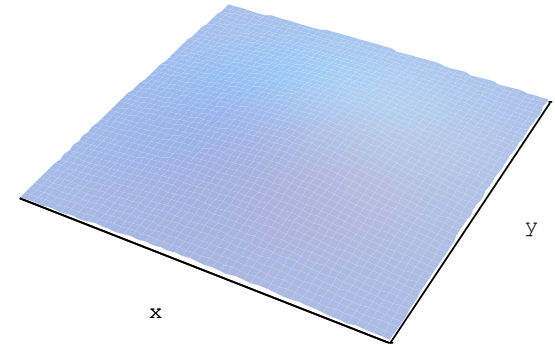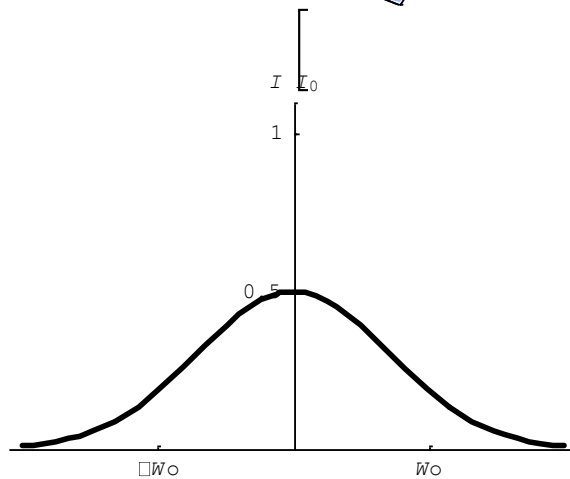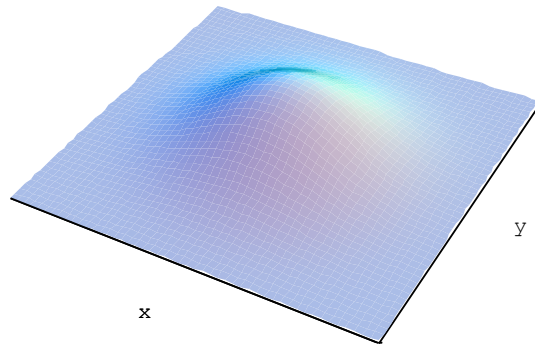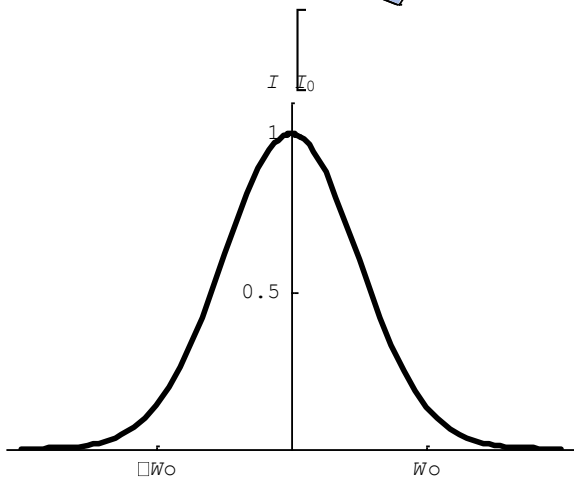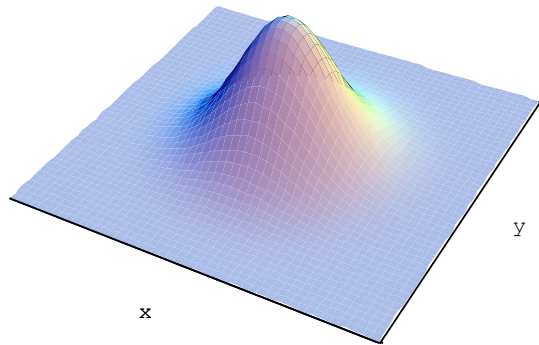
$$U(\mathbf{r})\, \exp(j2\pi\nu t) \text{(}$$

The photon is therefore **more likely to be found at those locations where the intensity is high.**
Optical photons behave as extended and localized entities. This behavior is called **wave particle duality**.
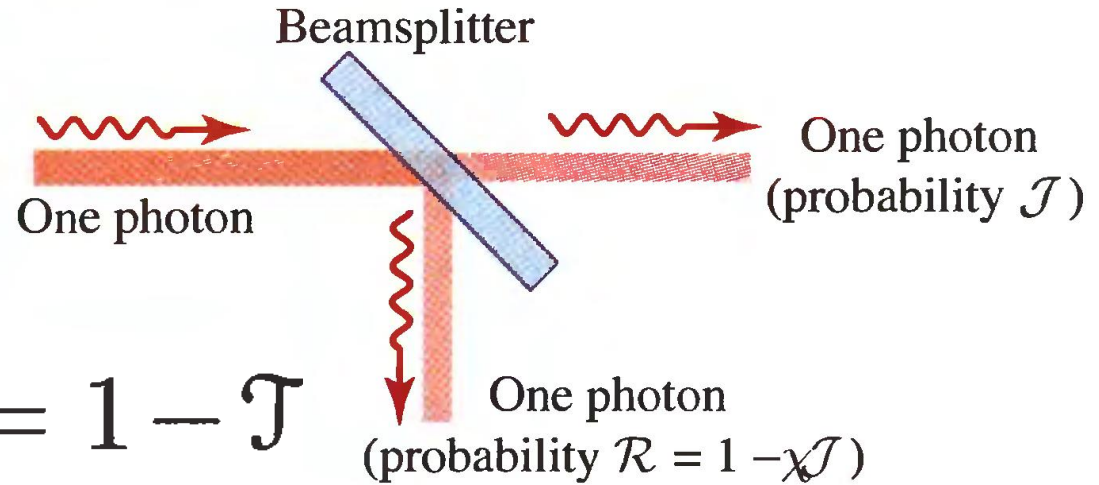The localized nature of photons becomes evident when they are detected.

$$\frac{I(\rho, z)}{I_0} = \left[\frac{W_0}{W(z)}\right]^2 \exp\left[-\frac{2\rho^2}{W^2(z)}\right]$$
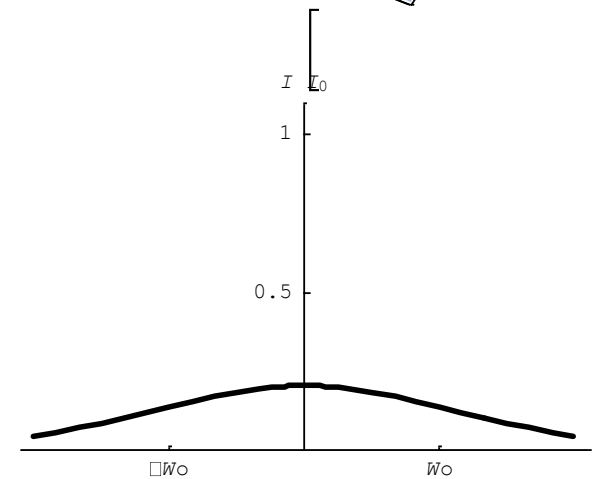
- A photon is indivisible, it must choose between the two possible directions permitted by the beamsplitter.

Beamsplitter

One photon

One photon
(probability $\mathcal{T}$)

One photon
(probability $\mathcal{R} = 1 - \mathcal{T}$)

intensity reflectance $\quad \mathcal{R} = 1 - \mathcal{T}$

intensity transmittance $\quad \mathcal{T}$

The probability that the photon is transmitted is proportional intensity transmittance

$$\mathcal{T} = I_t / I$$

The probability that it is reflected

$$1 - \mathcal{T} = I_r / I.$$

Photon Momentum

In photon optics, the linear momentum of a photon is $\mathbf{p} = (E/c)\hat{\mathbf{k}}$ where $E = \hbar\omega = \hbar c k$ is the photon energy.

- Photons carry momentum

$$\left|\vec{p}\right| = \frac{h}{\lambda}$$

- **Change in momentum corresponds** to the **force** and it can be calculated by the difference is **momentum flux S** between entering and leaving a object

$$\overline{F} = \frac{n}{c}\iint\left(\vec{S}_{in} - \vec{S}_{out}\right)dA$$

- Applying this formula to a 100% reflecting mirror reflecting a 60W lamp gives a pressure of:

$$\vec{F} = 2\frac{n}{c}\iint\left(\vec{S}_{in}\right)dA$$

$$\vec{F} = 2\frac{n}{c}W = 4\times10^{-7}\,N$$

Photon Momentum

- momentum associated with a photon can be transferred to objects of finite mass, giving rise to a force and causing mechanical motion. As an example, light beams can be used to deflect atomic beams traveling perpendicularly to the photons. The term radiation pressure is often used to describe this phenomenon (pressure force/area).

- .
- Sunlight on earth 0.5 nN/cm^2

Gravity pulls on a 1 kg mirror with 9.8 N so the force of the photons is negligible.

- However, if the same light is reflected by a object of 1 μg it can't be ignored!

- Using a laser on a microscopic particle will realize this situation

Physics of optical trapping

- The physics of the trapping mechanism is based on optical gradient and scattering forces arising from the interaction of strongly focused laser light with matter

- Simple models that explain optical trapping behavior can be applied in the **Mie scattering (d >> λ)** and the **Rayleigh scattering (d << λ)** regimes depending on the size of the particle relative to the wavelength of laser light

- A real optical tweezers typically works in the intermediate (d ≈ λ) regime, requiring a rigorous application of complicated approaches such as Generalized Lorentz-Mie Scattering or T-Matrix theory (beyond the scope of this lecture!)

Photon Interference

- Young's double-pinhole interference experiment is generally invoked to demonstrate the wave nature of light
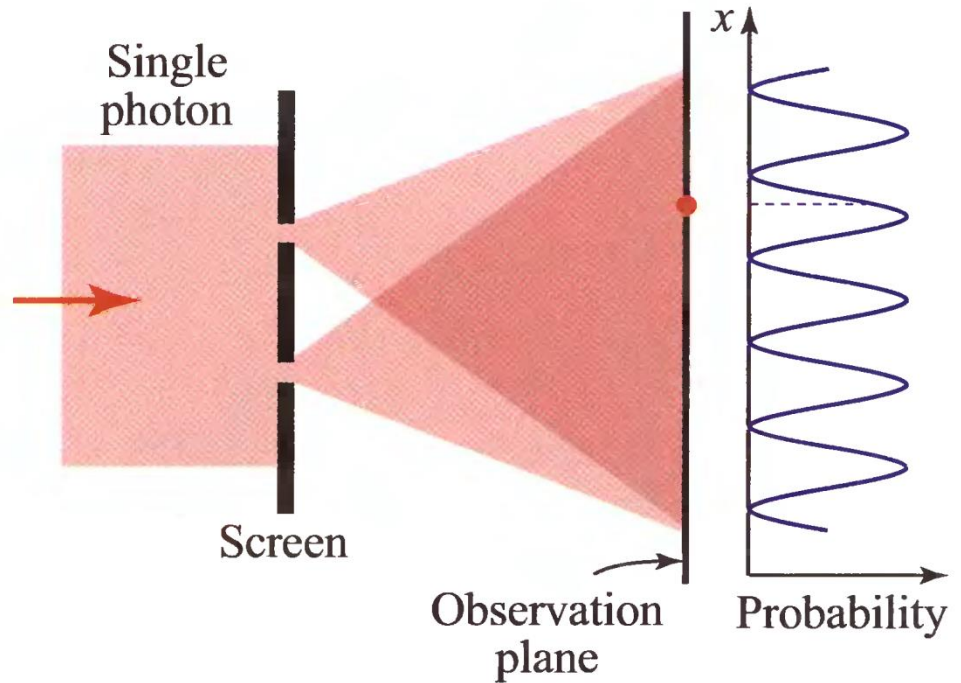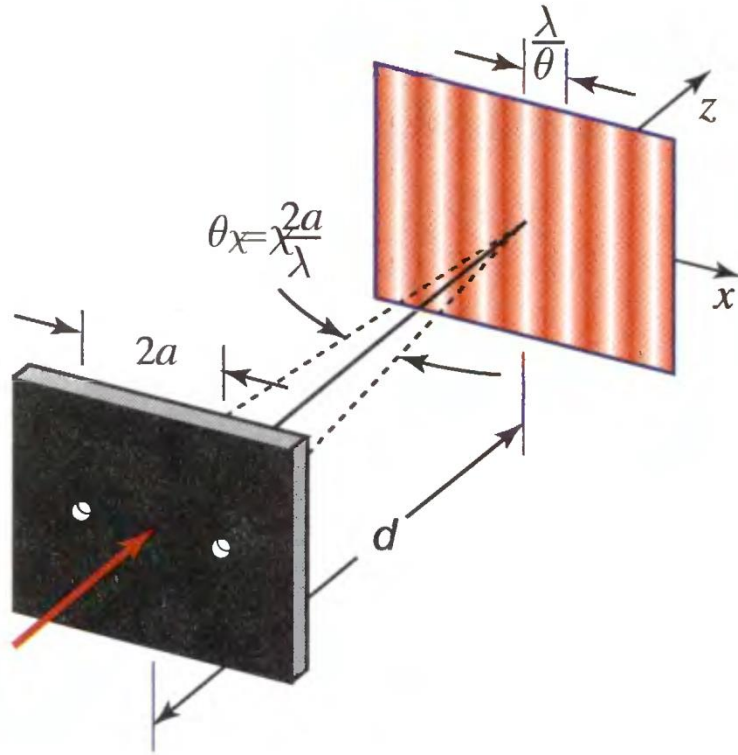

The occurrence of the interference results from the extended nature of the photon, which permits it to pass through both holes of the apparatus. This gives it knowledge of the entire geometry of the experiment when it reaches the observation plane, where it is detected as a single entity.
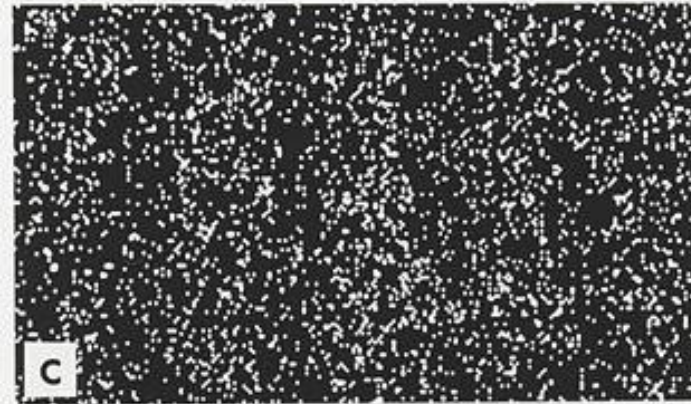
Photon Streams

- The temporal pattern of such photon registrations can be highlighted by examining
- the temporal and spatial behavior separately. Consider the use of a detector with good temporal resolution that integrates light over a finite area A
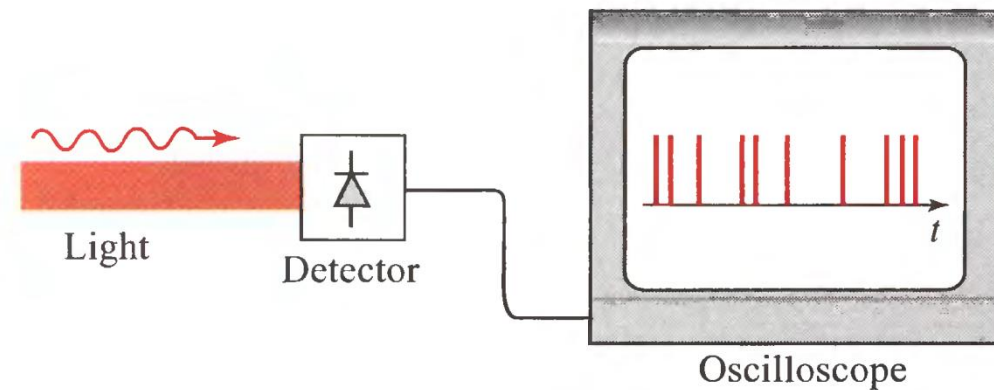
$$P(t) = \int_A I(\mathbf{r}, t) \, d\mathbf{A}.$$

Photon registrations at random localized instants of time for a detector that integrates light over an area A.

Mean Photon-Flux Density

Monochromatic light of frequency v and classical intensity I(r) (Watts/cm ^{2}) carries a

mean photon-flux density

$$\phi(\mathbf{r}) = \frac{I(\mathbf{r})}{h\nu}.$$

- Typical values of q; r for some common sources of light

Source	Mean Photon-Flux Density (photons/s-cm^2)
Starlight	10^6
Moonlight	10^8
Twilight	10^{10}
Indoor light	10^{12}
Sunlight	10^{14}
Laser light[a]	10^{22}

[a]A 10-mW He–Ne laser beam at $\lambda_o = 633$ nm focused to a 20-μm-diameter spot.

The mean photon flux Φ (units of photons /s) is obtained by integrating the mean photon-flux density over a specified area

$$\Phi = \int_A \phi(\mathbf{r})\, d\boldsymbol{A} = \frac{P}{h\overline{\nu}},$$

As an example, 1 nW of optical power, at a wavelength λ 200 nm, delivers to an object an average photon flux $\Phi = 10^9$ photons per second.

Mean Number of photons

- The mean number of photons **n** detected in the area A and in the time interval T is obtained by multiplying the mean photon flux Φ in by the time duration

$$\bar{n} = \Phi T = \frac{E}{h\bar{\nu}},$$

| Classical		Quantum	
Optical intensity	$I(\mathbf{r})$	Photon-flux density	$\phi(\mathbf{r}) = I(\mathbf{r})/h\bar{\nu}$
Optical power	P	Photon flux	$\Phi = P/h\bar{\nu}$
Optical energy	E	Photon number	$\bar{n} = E/h\bar{\nu}$

Time Varying Light

$$\phi(\mathbf{r}, t) = \frac{I(\mathbf{r}, t)}{h\bar{\nu}}.$$

$$\Phi(t) = \int_A \phi(\mathbf{r}, t)\, dA = \frac{P(t)}{h\bar{\nu}}, \quad \bar{n} = \int_0^T \Phi(t)\, dt = \frac{E}{h\bar{\nu}},$$

- For photon streams, the classical intensity **I(r,t)** determines the mean photon- flux density ϕ(r,t). The properties of the light source determine the fluctuations

- the times at which the photons are detected are random, their statistical behavior determined by the source,
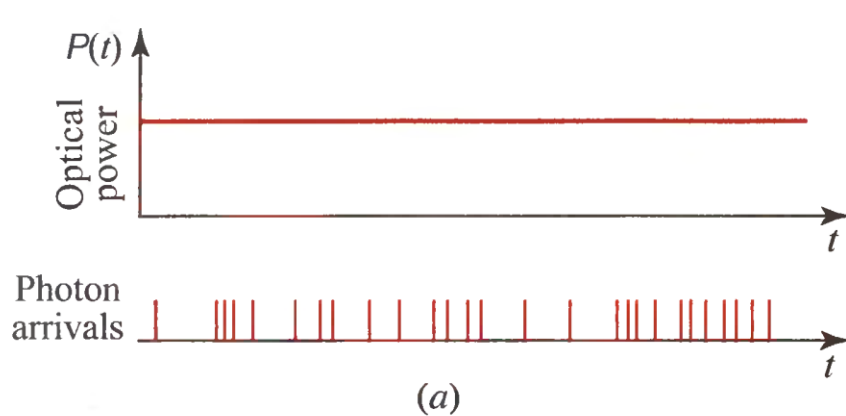
(a)   Constant optical power and the corresponding **random photon arrival times**.
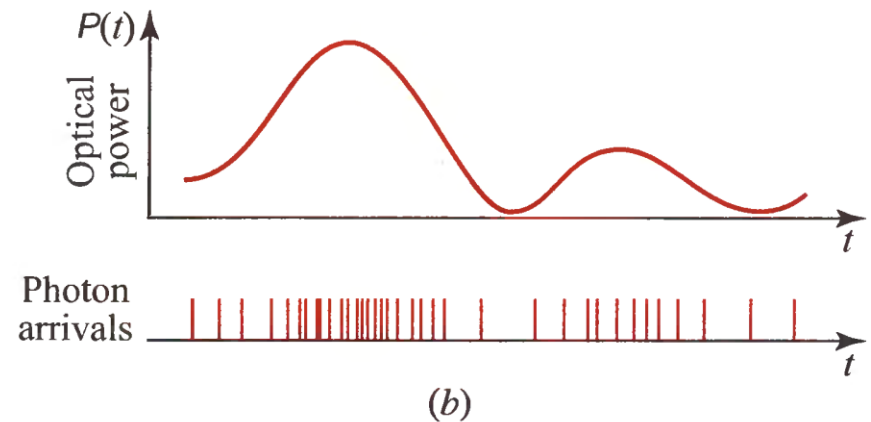
(b) Time-varying optical power and the corresponding random photon arrival times

Randomness of Photon Flow

- An understanding of photon-number statistics is important for applications such as reducing noise in weak images and optimizing optical information transmission.

- **Coherent light** has a constant optical power P. The corresponding mean photon flux Φ=P/hv (photons/s) is also constant, but the actual times of registration of the photons are random. An expression for the probability distribution p(n) can be derived under the assumption that the registrations of photons are statistically independent. The result is the **Poisson distribution**

$$p(n) = \frac{\bar{n}^n \exp(-\bar{n})}{n!}, \qquad n = 0, 1, 2, \ldots$$

- It is not difficult to show in and that the mean of the Poisson distribution is indeed n and its variance is equal to its mean:

$$\sigma_n^2 = \bar{n}.$$

Signal to Noise Ratio

- **The randomness of the number of photons** constitutes a fundamental source of noise that we have to account for when using light to transmit a signal. Representing the mean of the signal as **n,** and its noise by **the root mean square value is** σn, a useful measure of the performance of light as an information-carrying medium is the signal-to-noise ratio (SNR). The SNR of the random number n is defined as
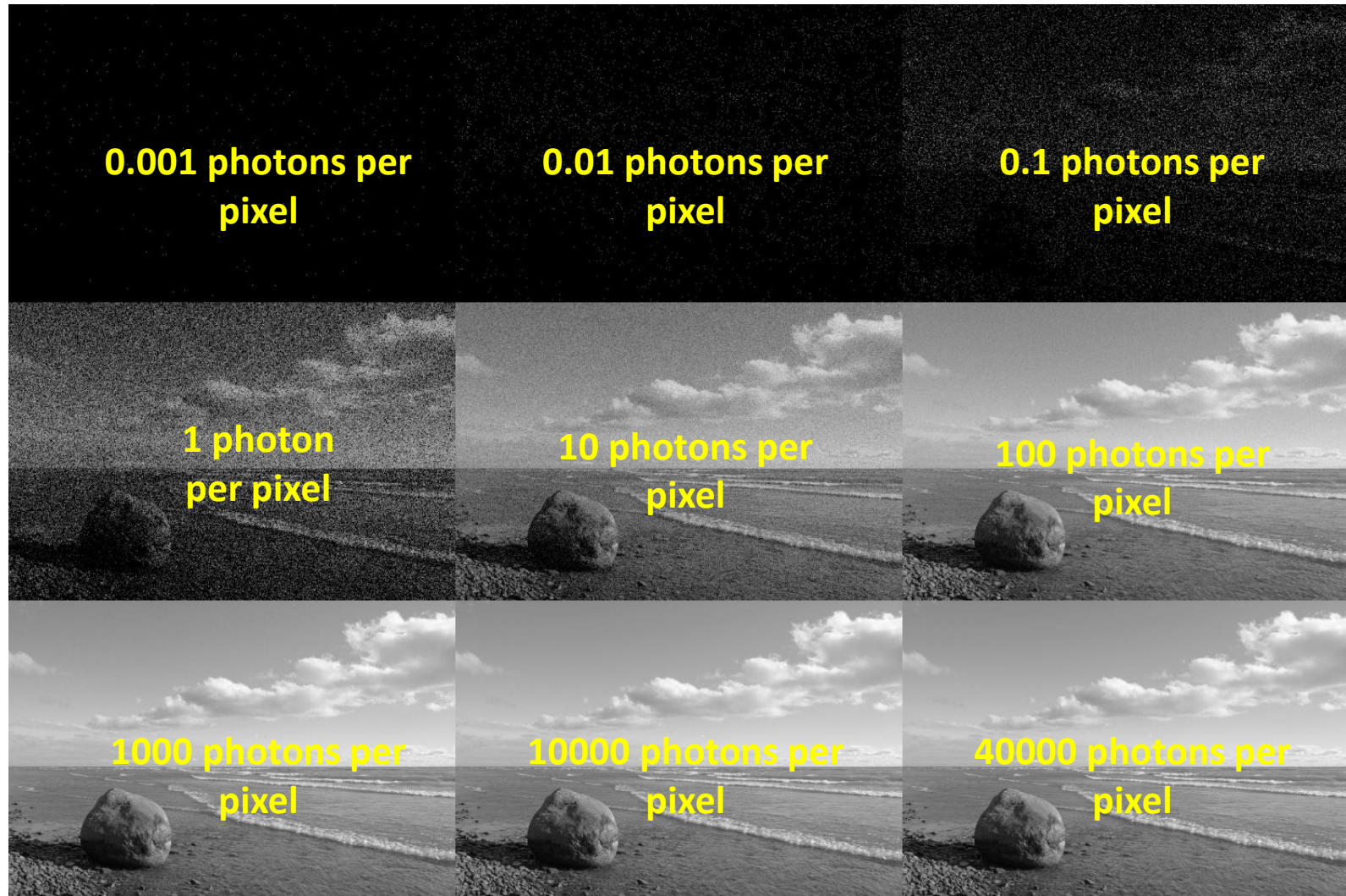
$$SNR = \frac{(\text{mean})^2}{\text{variance}} = \frac{\bar{n}^2}{\sigma_n^2}.$$

For the Poisson distribution

$$SNR = \bar{n},$$

so that the signal-to-noise ratio **increases linearly with the mean number of photon counts.**

A photon noise simulation, using a sample image as a source and a per-pixel Poisson process to model an otherwise perfect camera (quantum efficiency = 1, no read-noise, no thermal noise, etc).

Noise Sources of a Detector

Photon Shot Noise – Counting statistics of the signal photons

      -Originates from the Poisson distribution of signal photons as a function of time

      -Random arrival of photons and electron is governed by Poisson distribution
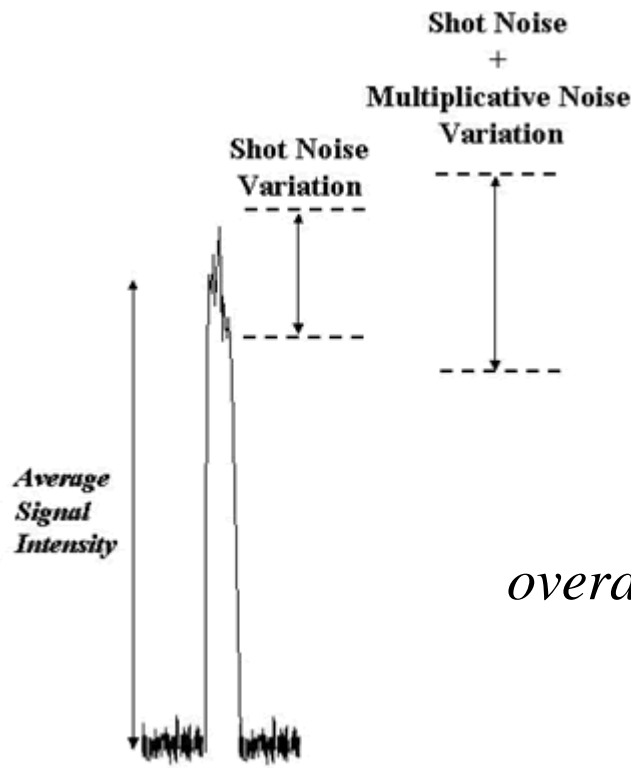
- Dark Current Noise – Counting statistics of spontaneous electron generated in the device
- Johnson Noise – Thermally induced current in the transimpedence amplifier

Shot Noise
+
Multiplicative Noise
Variation

Shot Noise
Variation

Average
Signal
Intensity

current)

$$overal\ noise = \sqrt{(readnoise)^2 + (darknoise)^2 + (shotnoise)^2}$$

Photon Shot Noise

- Shot noise is white noise, just like Johnson noise. Does not exist unless current is driven through the device.

- This is termed **"white noise"** why?

- Because, like in white light, **all frequencies are equally represented**

- Standard deviation (or noise) and the photon noise limited Signal-to-Noise-Ratio (SNR) associated with detecting a mean of 'N' photons are given by

$$Noise\,(photon) \approx \sqrt{N}$$

$$SNR(photon) = \frac{Signal}{Noise} \approx \frac{N}{\sqrt{N}} = \sqrt{N}$$
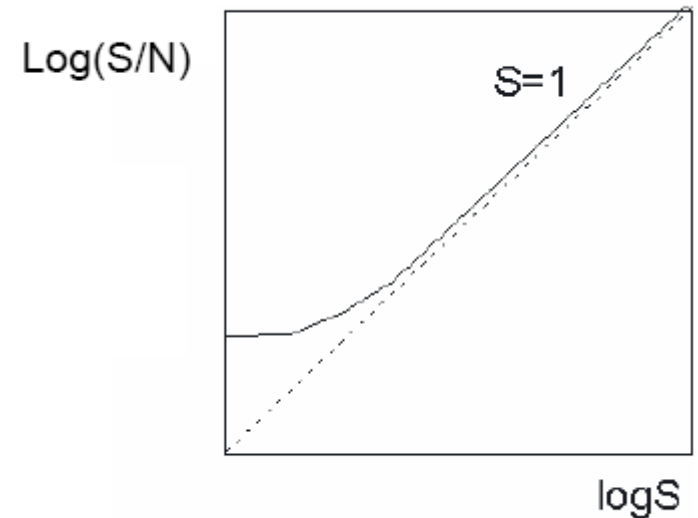
- Means to **enhance** S/N
  - Signal averaging: internally or externally
  - Signal smoothing: boxcar averaging, moving average, polynomial smoothing (**keyword: convolution**)
  - Filtering in the frequency domain: