On the shear-current effect: toward understanding why theories and simulations have mutually and separately conflicted

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Large-scale dynamo (LSD): Amplification of magnetic energy beyond the energy-dominant scale of turbulence

Shear-current effect (SC/SCE): Non-helical turbulence + shear flow lead to LSD

Might operate in weakly stratified flows, planetary cores (source of helicity unclear); may coexist with other dynamo drivers ($\Omega \times J$, incoherent α, etc.)

Basic setup in all theories and most simulations:

Cartesian geometry; mean fields defined by xy-average;
forced isotropic turbulence perturbed by a shear flow
Compare with: MRI turbulence

PCUM 2021

Shear-current effect:

Mean electromotive force $\mathscr{E}_i = \langle \mathbf{u} \times \mathbf{b} \rangle_i = \alpha_{ij} B_j - \beta_{ij} J_j$ with negative β_{21}

$+$ Background shear flow $\mathbf{U} = -Sx\hat{\mathbf{y}}$

Amplification of mean magnetic energy: large-scale dynamo

Main question:

What are the respective contributions to β_{21} from turbulent velocity and magnetic fields?

$$\beta_{21} = \frac{\tau}{3}\beta_{21}^{u}\langle u^2 \rangle + \frac{\tau}{3}\beta_{21}^{b}\langle b^2 \rangle$$

$\propto \langle b^2 \rangle$, magnetic SCE

$\propto \langle u^2 \rangle$, kinetic SCE

Theory	Simulation	Kinetic/ Magnetic SCE?	Remarks
†Rogachevskii & Kleeorin (2003)		Y/	
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Rädler & Stepanov (2006)		N/	
Rüdiger & Kitchatinov (2006)		N/	
	Brandenburg et al. (2008)	N/	$\mathrm{Re} = \mathcal{O}(1), \mathrm{Rm} \lesssim \mathcal{O}(100)$
†Pipin (2008)		Y/Y	
Sridhar & Subramanian (2009)		N/	Quasi-linear in shearing frame
Sridhar & Singh (2010)		N/	Quasi-linear in shearing frame
Singh & Sridhar (2011)		N/	Quasi-linear in shearing frame
Squire & Bhattacharjee (2015a)		N/Y	
	Squire & Bhattacharjee (2015c)	N/Y	$\mathrm{Re} = \mathrm{Rm} \simeq 5$
	*Squire & Bhattacharjee (2015b)	N/Y	$\mathrm{Re} = \mathrm{Rm} \lesssim 15$
	Singh & Jingade (2015)	N/	$\min\{\mathrm{Re}, \mathrm{Rm}\} < 1$
	*Squire & Bhattacharjee (2016)	N/Y	$8\mathrm{Re} = \mathrm{Rm} \lesssim 5 \langle u^2 \rangle / \langle B^2 \rangle$
	Käpylä et al. (2020)	N/N	MHD burgulence, $\mathrm{Re} < 1$, $\mathrm{Rm} < 15$

Re & Rm defined by u_{rms} and k_f

†theory = using τ closure (~ applies for Re & Rm $\gg 1$)
otherwise using SOCA or quasi-linear (~ applies for Re & Rm < 1)

*simulation = using the projection method
otherwise using test field methods

	Theory		Simulation	
	τ closure	SOCA/Quasi-linear	Test field	Projection method
Kinetic SC	Yes	No	No	No
Magnetic SC	Yes	Yes	No	Yes

(Wissing+2021: MRI with SPH, no SCE in ustr+nf, ustr+znf, or str+nf; contradicts Shi+2016)

Theories and simulations have mutually and separately conflicted

$$\partial_t \langle u \rangle = \langle uu \rangle + \cdots$$
$$\partial_t \langle uu \rangle = \langle uuu \rangle + \cdots$$
$$\cdots$$

Second-order-correlation-approximation (SOCA):

Drops nonlinear terms

$\rightarrow \partial_t \langle uu \rangle = \cdots + \nu \nabla^2 \langle uu \rangle$

Justified at low Reynolds numbers or low Strouhal number (=correlation time/eddy turnover time)

Spectral-τ or minimal-τ:

Replaces order ≥ 3 terms by a damping term

$\rightarrow \partial_t \langle uu \rangle = \cdots - \langle uu \rangle / \tau(k)$ (in Fourier space)

A closure at high Reynolds numbers (hence small dissipation terms)

Collectively we can write $\partial_t \langle ub \rangle = \text{linear terms} + ck^{\lambda} \langle ub \rangle$

where $ck^{\lambda} = \begin{cases} -\nu k^{2} & \text{for SOCA} \\ -\tau k^{q-1} & \text{for spectral } \tau \end{cases}$

q = spectral index (e.g. 5/3 for Kolmogorov)

ZB21: to the $\mathcal{O}(\mathrm{Sh})$ order, $\beta_{21}^{u} < 0$ only if $\lambda < 1$, thus:

In SOCA, $\beta_{21}^{u} > 0$ always $\rightarrow$ **no kinetic SC**

In spectral-τ, $\beta_{21}^{u} < 0$ if $q < 2$ (Rogachevskii & Kleeorin 2003, 2004)

In both theories magnetic SC always works ($\beta_{21}^{b} < 0$), regardless of the spectral index q

	τ closure	SOCA/Quasi-linear	Test field
Kinetic SC	Yes	No	No
Magnetic SC	Yes	Yes	No

Simulations not finding KSC: too low Re and Rm?

Squire & Bhattacharjee (2015c)	N/Y	$\mathrm{Re}=\mathrm{Rm}\simeq 5$
*Squire & Bhattacharjee (2015b)	N/Y	$\mathrm{Re}=\mathrm{Rm}\lesssim 15$
Singh & Jingade (2015)	N/	$\min\{\mathrm{Re},\mathrm{Rm}\}<1$
*Squire & Bhattacharjee (2016)	N/Y	$8\mathrm{Re}=\mathrm{Rm}\lesssim 5\langle u^2\rangle/\langle B^2\rangle$
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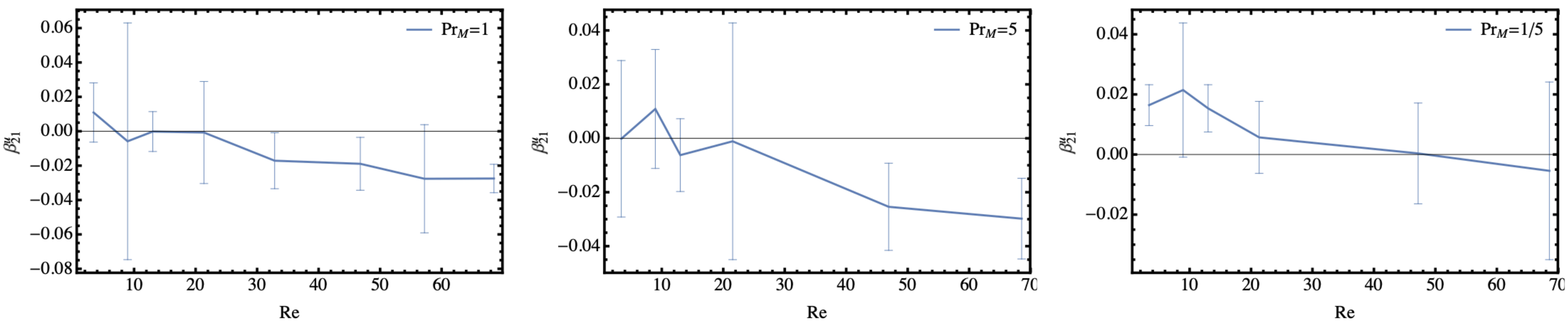
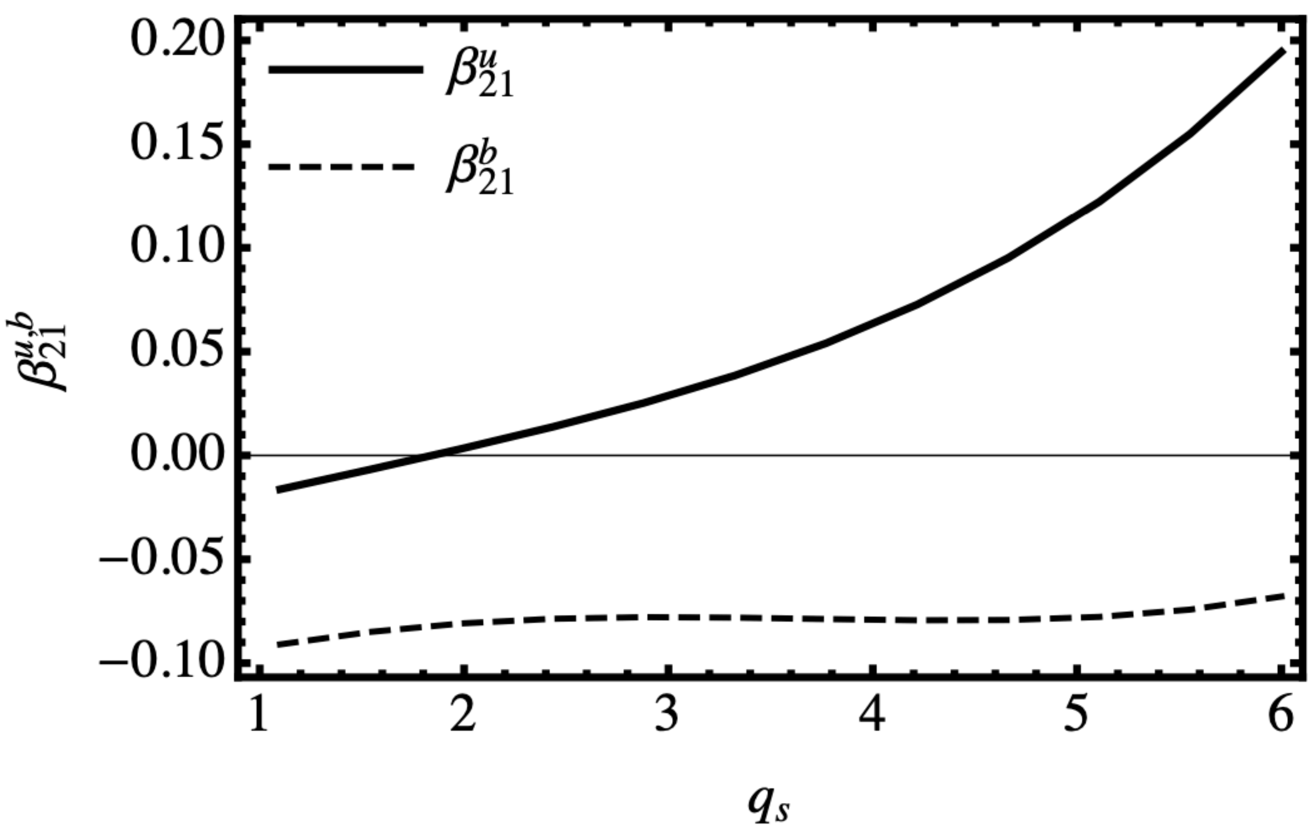
Kinematic test field method (no magnetic background)

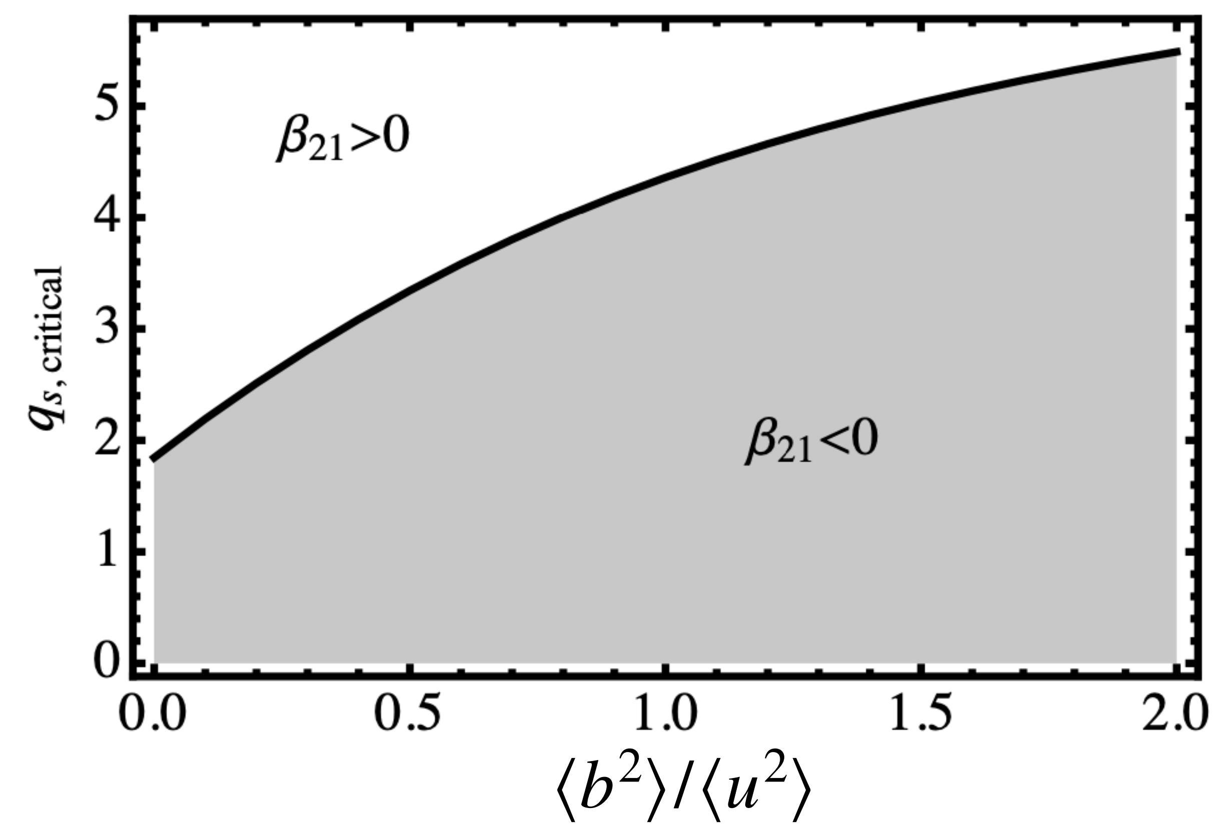


→ evidence of transiting from positive to negative values (but with large error bars)

	τ closure	SOCA/Quasi-linear	Test field
Kinetic SC	Yes	No	No at small Re Yes at large Re?
Magnetic SC	Yes	Yes	No at Re,Rm<15

Why no magnetic SC detected in test field methods even though both theories predicted it?





$$\beta_{21} = \frac{\tau}{3}\beta_{21}^{u}\langle u^2\rangle + \frac{\tau}{3}\beta_{21}^{b}\langle b^2\rangle$$

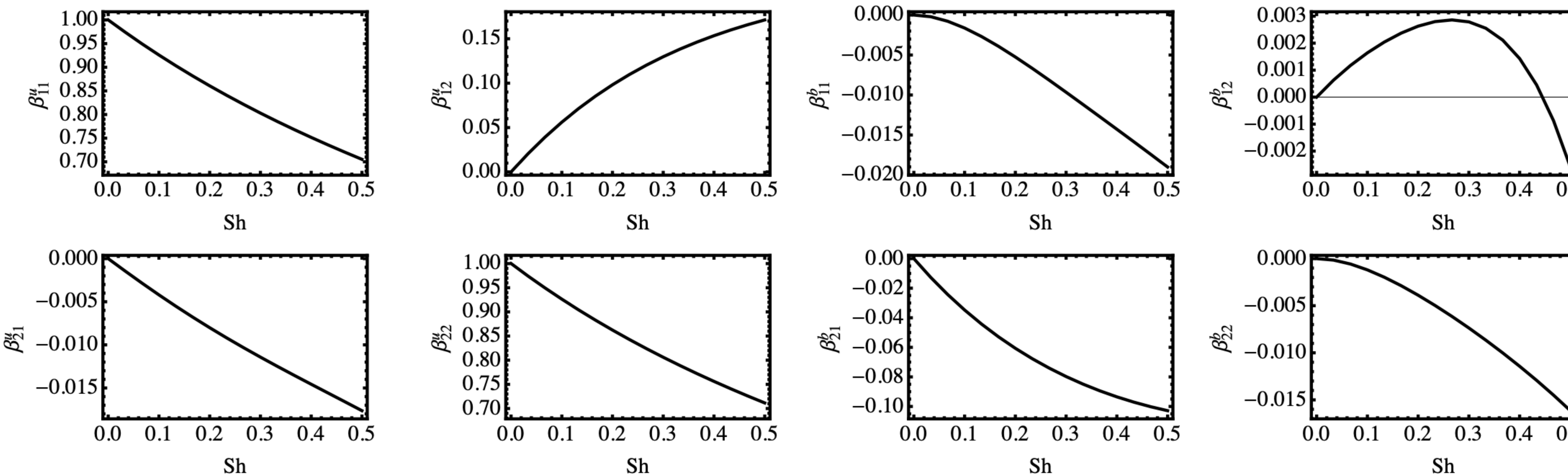
Kinetic SC depends more sensitively on spectral index

$\rightarrow$ for steep spectra (as at low Re & Rm),

$|\beta_{21}^{u}|\langle u^2\rangle > |\beta_{21}^{b}|\langle b^2\rangle$ **even if** $\langle u^2\rangle \lesssim \langle b^2\rangle$

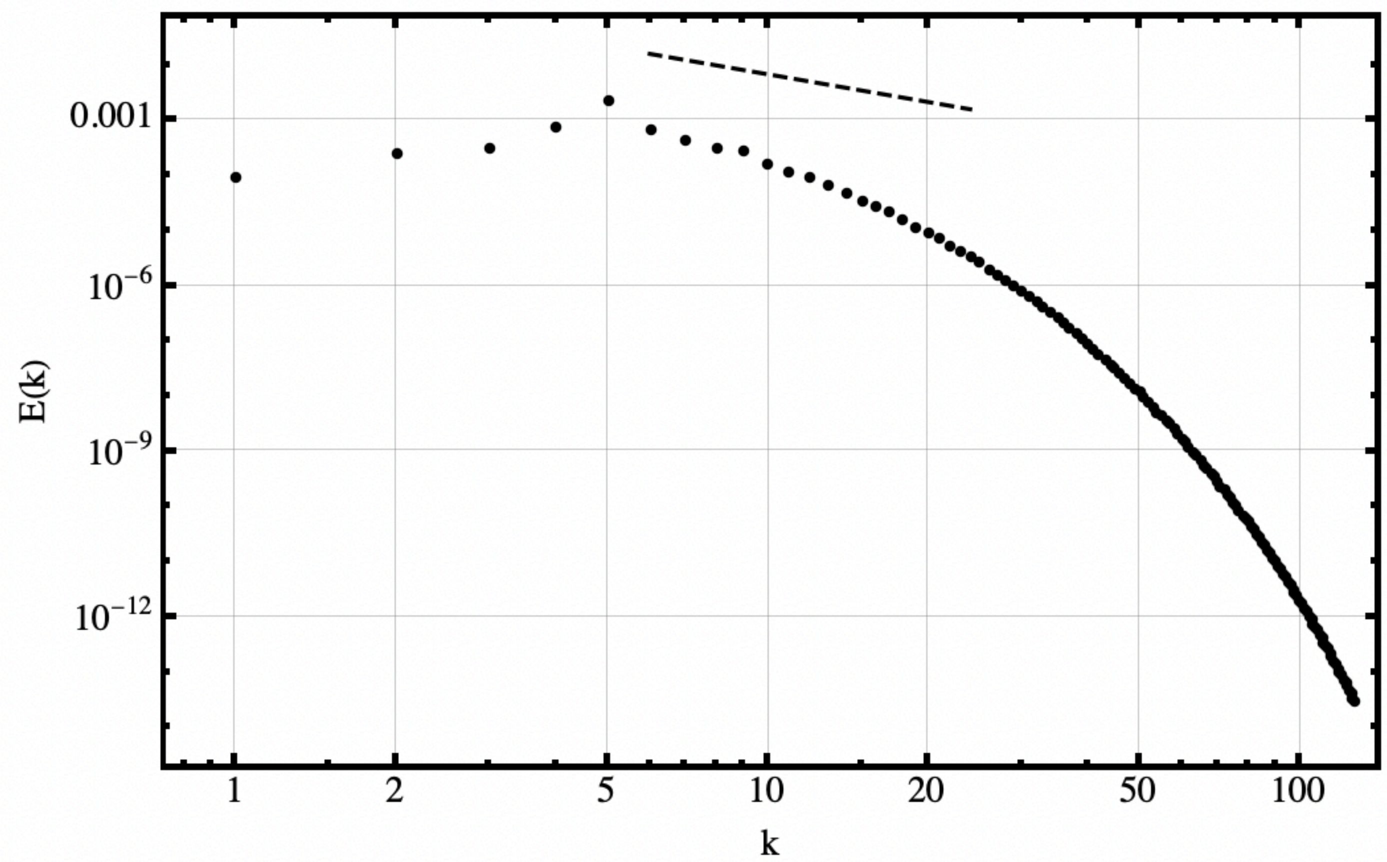
The full solution of β_{ij} for the pure shear case

(Cases of shear + rotation / pure rotation / burgulence in ZB21)



1. Comparable β_{12} and β_{21} → may invalidate setting $\beta_{12} = 0$ in the projection method
2. No dynamo cycle period → SCE at best subdominant in simulations, unless β_{ij} itself is periodic (requires nonlinear theory)

Subtlety 1: The spectral index



The energy-dominant modes are $k = 4{,}5{,}6$, but:

1. $q(5{,}6)$ very large: a sudden steep slope because of forcing

   → destructive to SCE even if $q < 2$ inside the inertial range

2. $q(4{,}5) < 0$: positive slope at $k < k_f$

   → constructive to SCE, but not considered in theories

The combined effect is…?

Subtlety 2: The boundary condition

Theories: normally period boundary conditions

$$f(x+L,y)=f(x,y)$$

Shearing box simulations: shear periodic boundary condition

$$f(x+L,y-StL)=f(x,y)$$

In all theories, $\hat{P}_{ij}=\delta_{ij}-\frac{\partial_i\partial_j}{\partial^2}$ is found crucial to produce SCE, but does using different b.c. change the solution because of ∂^{-2}??

A possible solution: shearing coordinates (Sridhar, Subramanian, Singh …), but

1. no calculation for magnetic SCE yet
2. How to incorporate a closure at large Re & Rm?

Summary:

Assuming using normally or shear periodic b.c. does not qualitatively change the story:

Different powers of k in the closure term; Need large enough Re & Rm, and shallow enough spectrum

Yes(?) at large Re & Rm

	τ closure	SOCA/Quasi-linear	Test field	Projection
Kinetic SC	Yes	No	No at Rm<~15	No at Rm<~15
Magnetic SC	Yes	Yes	No at Rm<~15	Yes(?) at Rm<~15

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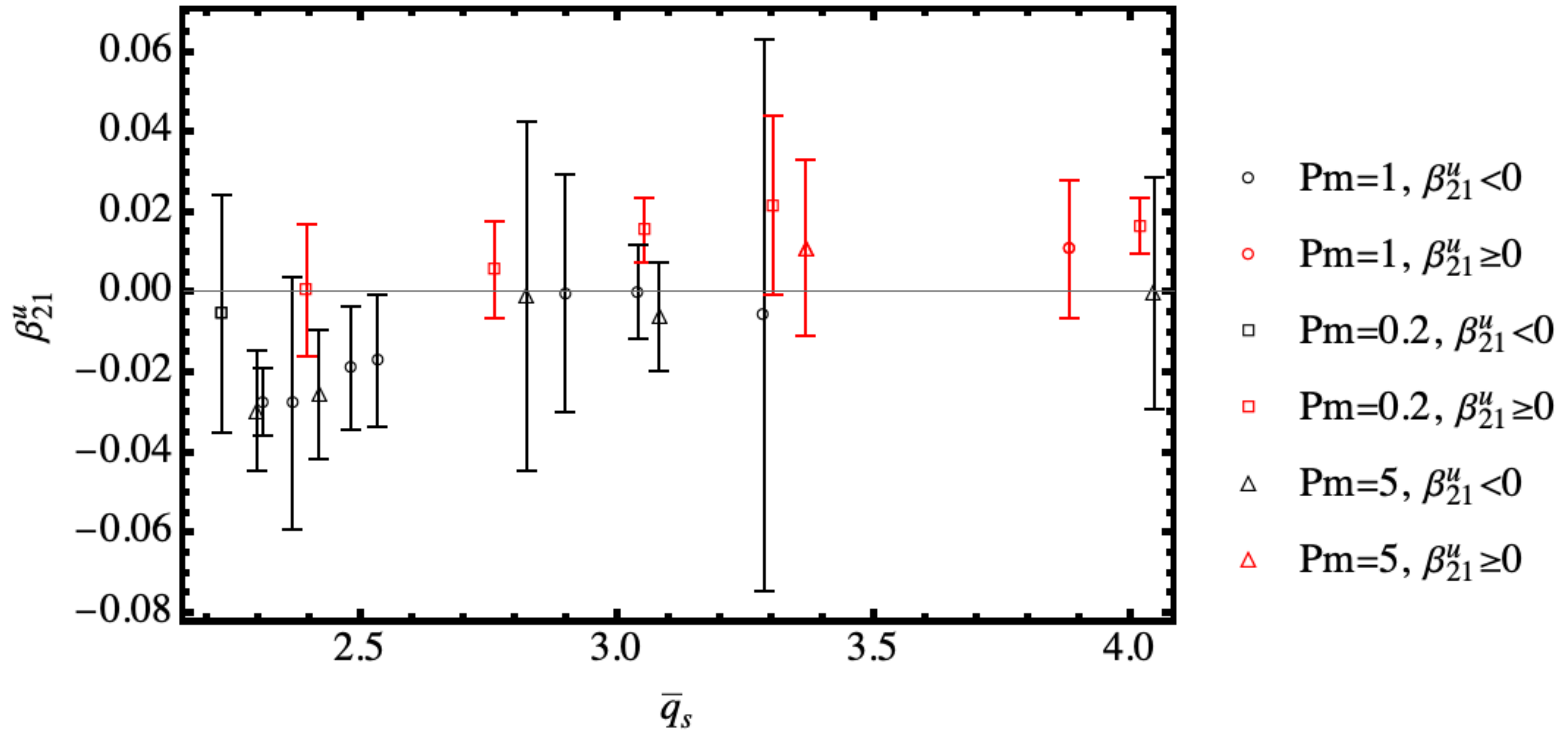
Too steep spectrum of u^2?

Some unresolved issues:

1. More careful justification of the slope dependence
2. Any influence from different boundary conditions?
3. Physical picture compatible the slope dependence



$$q_s(k) = -\frac{k}{E(k)}\frac{E(k+1)-E(k)}{(k+1)-k}$$

$$\bar{q}_s(k) = \frac{\sum_{k\geq k_f} q_s(k)E(k)}{\sum_{k\geq k_f} E(k)}$$

$$(\Delta k = 1)$$

For all Pm, fitted critical exponent is {{x → 3.15362}}

	Statistic	P-Value
Pearson Correlation	0.70625	0.000500877

For Pm=1, fitted critical exponent is {{x → 3.30541}}

	Statistic	P-Value
Pearson Correlation	0.925254	0.000986349

For Pm=0.2, fitted critical exponent is {{x → 2.29412}}

	Statistic	P-Value
Pearson Correlation	0.844132	0.0345487

For Pm=5, fitted critical exponent is {{x → 3.46678}}

	Statistic	P-Value
Pearson Correlation	0.773489	0.07115

Subtlety: Role of the pressure gradient term

$$\partial_t \mathbf{u} = -\nabla p + \text{other terms}$$

If $\mathbf{u}$ is incompressible, then p is not dynamical but instead determined by other fields and and boundary conditions:

$$\partial_t \mathbf{u} = \hat{\mathbf{P}}(\text{other terms}),\ \hat{P}_{ij} = \delta_{ij} - \frac{\partial_i \partial_j}{\partial^2}$$

Theory: $\hat{\mathbf{P}}$ is necessary for both kinetic and magnetic SCEs (ZB21, Squire & Bhattacharjee 2016)

Simulation: not the case? (Käpylä+2020)

Käpylä+2020:

Run	Re_M	$\lambda/(\eta_0 k_f^2)$	η_{xx}/η_0	η_{yy}/η_0	η_{yx}/η_0
FK1a	2.1	−0.0354	0.557 ± 0.006	0.547 ± 0.007	0.048 ± 0.001
FK1b	11.9	0.0140	0.608 ± 0.015	0.598 ± 0.014	0.023 ± 0.001
FK8a	2.1	−0.0008	0.572 ± 0.010	0.563 ± 0.011	0.044 ± 0.002
FK8b	12.7	0.0166	0.641 ± 0.019	0.634 ± 0.017	0.023 ± 0.001
SK1a	2.0	0.0006	0.367 ± 0.001	0.393 ± 0.002	−0.003 ± 0.000
SK1b	12.3	0.0183	0.440 ± 0.004	0.412 ± 0.001	−0.011 ± 0.002
SK4a	2.1	−0.0042	0.367 ± 0.003	0.390 ± 0.003	−0.004 ± 0.000
SK4b	13.3	0.0185	0.334 ± 0.037	0.339 ± 0.044	−0.004 ± 0.005
SK8a	2.1	0.0033	0.367 ± 0.003	0.390 ± 0.004	−0.003 ± 0.000
SK8b	12.8	0.0192	0.401 ± 0.005	0.424 ± 0.005	−0.015 ± 0.000
SKM1a	1.9	⋯	1.794 ± 0.039	1.278 ± 0.045	0.200 ± 0.025
SKM4a	2.1	⋯	2.012 ± 0.179	1.191 ± 0.014	0.221 ± 0.012
SKM8a	1.8	⋯	3.054 ± 0.625	1.481 ± 0.131	0.338 ± 0.064
SKM16a	2.0	⋯	2.238 ± 0.552	1.215 ± 0.010	0.249 ± 0.062
SKM1ad	2.1	0.0103	1.228 ± 0.214	1.326 ± 0.074	0.247 ± 0.043
SKM4ad	1.9	0.0315	1.279 ± 0.150	1.455 ± 0.066	0.222 ± 0.022
SKM8ad	1.5	0.0948	1.688 ± 0.165	2.040 ± 0.150	0.516 ± 0.061
SKM16ad	1.9	0.0344	1.231 ± 0.070	1.589 ± 0.019	0.364 ± 0.116

MHD burgulence:

$$\partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} = \cancel{-\nabla p} + \mathbf{J}\times\mathbf{B} + \nu\nabla^2\mathbf{u}$$

But…

$$\mathbf{J}\times\mathbf{B} = -\nabla\frac{B^2}{2} + \mathbf{B}\cdot\nabla\mathbf{B}$$