The relevance of lensing-convergence terms for photo-z surveys

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Swiss Euclid Days, EPFL February 2020
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- **the redshift $z$ is perturbed** by peculiar velocity, the gravitational potential at the source and the integrated Sachs Wolfe effect. The first of these terms is at present routinely taken into account via redshift space distortions (RSD, Kaiser 1987). The other redshift perturbations are relevant mainly on very large scales.
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- On larger scales there are additional light cone effects (e.g. the Shapiro time delay, the Doppler term and more, (Yoo et al., 2009, Bonvin & Durrer, Challinor & Lewis 2011), but these are mainly visible close to the Hubble scale or at very low redshift, $z \leq 0.1$. 

Galaxy number counts

We count the number of galaxies inside a redshift bin $dz$ and a solid angle as seen at the observer $d\Omega_o$, 

$$dN = A(z, n)dzd\Omega_o, \quad \Delta(z, n) = \frac{A(z, n) - \bar{A}(z)}{\bar{A}(z)}$$

$$A(z, n) = \rho(z, n)\frac{dV}{dzd\Omega_o} = \bar{\rho}(\bar{z})(1 + \rho'/\rho\delta z)(1 + \delta(\bar{z}, n))\frac{dV}{dzd\Omega_o}$$

$$dV = r(z, n)^2 drd\Omega_s = \bar{r}(\bar{z})^2 \left(1 + 2\frac{\delta r}{\bar{r}}\right) \frac{dr}{dz} \left| \frac{d\Omega_s}{d\Omega_o} \right| dzd\Omega_o$$

$$\frac{dr}{dz} = \left(\frac{d\bar{r}}{d\bar{z}} + \frac{d\delta r}{d\bar{z}}\right) \left(1 - \frac{d\delta z}{d\bar{z}}\right) \approx H(z)^{-1} \left(1 - v_r - H^{-1} \frac{dv_r}{dr}\right), \quad \frac{\delta z}{1 + z} = v_r.$$ 

This is the term which generates the well known RSD.
Galaxy number counts - lensing

\[
\frac{d\Omega_s}{d\Omega_o} = 1 + \frac{d\delta \theta_s^a}{d\theta_o^a}
\]

\[
\delta \theta_s^a = -2 \nabla^a \int_0^s \frac{dr}{r_s r} (r_s - r) \psi_W \quad \psi_W = \frac{1}{2} (\Phi + \Psi)
\]

\[
\frac{d\delta \theta_s^a}{d\theta_o^a} = -2 \Delta \int_0^s \frac{dr}{r_s r} (r_s - r) \psi_W = -2 \kappa \quad r_s = r(z)
\]

\[
\frac{d\Omega_s}{d\Omega_o} = 1 - 2 \kappa.
\]

In addition to this purely geometrical effect there is the magnification bias $5s(z)\kappa$, where

\[
s(z, m^*_s) = \frac{2}{5} \frac{d\bar{n}(z, L)}{d \log L}.
\]
The galaxy number counts power spectrum

\[ \Delta(z, n) \simeq b \delta_z(z, n) + \frac{1}{H(z)} v_r(z, n) - (2 - 5s(z)) \kappa(z, n) \]

\[ \Delta(z, n) = \sum a_{\ell m}(z) Y_{\ell m}(n), \quad \langle a_{\ell m}(z) a^*_{\ell' m'}(z') \rangle = \delta_{\ell \ell'} \delta_{mm'} C_\ell(z, z') \]

\[ \langle \Delta(z, n) \Delta(z', n') \rangle = \frac{1}{4\pi} \sum_\ell (2\ell + 1) C_\ell(z, z') P_\ell(n \cdot n') \]
The observable angular matter power spectrum: slim redshift bins

\[
\frac{C_l}{(l+1)} = \frac{\pi}{2} \left( \frac{z}{z'} = 1 \right)
\]

\[
\frac{C_l}{(l+1)} = \frac{\pi}{2} \left( \frac{z}{z'} = 2 \right)
\]

\[
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\]

---
- the full result
- density only
- RSD and RSD-density correlation
- terms containing $\kappa$ ($b = 1$ and $s = 0$).

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The observable angular matter power spectrum: wide redshift bins

Contributions to the transverse power spectrum at redshift \( z = 3, \Delta z = 0.3 \) (from Bonvin & RD 2011)

\[
\begin{align*}
C_\ell^{\delta\delta} \text{ (red)}, & \quad C_\ell^{RSD} \text{ (green)}, \quad 2C_\ell^{\delta RSD} \text{ (blue)}, \quad C_\ell^{\text{lensing}} \text{ (magenta)}. \\
\end{align*}
\]
$E_g$ statistics

More than a decade ago Zhang et al. (2007) have proposed the so called $E_g$ statistics which is very sensitive to deviations from Einstein gravity. It consists of measuring the lensing potential, the density-lensing correlation and the redshift space distortion independently.

$$E_g(\ell, z) = \Gamma(z) \frac{C^\kappa g_\ell}{\beta C^{gg}_\ell} \left( = \frac{\Omega_{m0}}{f(z)} \right)$$

The last equal sign holds for GR: $E_g$ is independent of the galaxy bias and of scale. But $C^\kappa g_\ell$ and $C^{gg}_\ell$ are not directly measurable! But $C^{\kappa\Delta}_\ell(z, z')$, $C^{\Delta\Delta}_\ell(z, z')$ and $C^{\kappa\kappa}_\ell(z, z')$ are.

$$C^{\kappa\Delta}_\ell(z, z') \simeq C^{\kappa g}_\ell(z, z') - (2 - 5s(z')) C^{\kappa\kappa}_\ell(z, z')$$

$$C^{\Delta\Delta}_\ell(z, z') \simeq C^{gg}_\ell(z, z') - (2 - 5s(z')) C^{g\kappa}_\ell(z, z') - (2 - 5s(z)) C^{\kappa g}_\ell(z, z')$$

$$+ (2 - 5s(z')) (2 - 5s(z)) C^{\kappa\kappa}_\ell(z, z')$$

The corrections can be considerable!
The corrections for the 1st year DES results setting (the unknown) $s(z) = 0$. 
(From Ghosh & Durrer 2019)
The corrections for the Euclid photometric survey with $s(z)$ taken from Montanari & Durrer (2015).
(From Ghosh & Durrer 2019)
Conclusions

- **Gravitational lensing** does not only generate shear, but it **affects galaxy number counts** by enhancing the transversal volume (reducing the observed density) and enhancing the flux of sources (enhancing the observed density).
- This effect must be taken into account especially for **photometric redshift surveys** with relatively large bin widths, $\Delta z/(z + 1) > 0.01$, or when correlating different redshifts.
- In high precision spectroscopic surveys, lensing is only relevant at high redshifts and for sparse populations.
- When correlating shear and galaxy counts the lensing effect cannot be neglected.
- Lensing seriously affects e.g. **$E_g$ statistics** in future surveys.
- Interestingly for the Euclid specifications at $z \sim 1$, we have $s(z) \sim 0.4$ and the two effects cancel to good approximation.