Learning for Adaptive and Reactive Robot Control
Instructions for exercises of lecture 9

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Introduction

INTRO

This part of the course follows *exercises* 10.1 *to* 10.8 and *programming exercises* 10.1 *to* 10.3 of the book "*Learning for Adaptive and Reactive Robot Control: A Dynamical Systems Approach. MIT Press, 2022*".

1 Theoretical exercises [1h]

1.1

Book correspondence: Ex10.1, p272

Consider a second-order, one-dimensional (1D) DS, which can be used to hit a target at $x = x^*$, with desired velocity $\dot{x} = \dot{x}^*$ at $t = t^*$. Consider first the undamped system for the DS using equation:

$$m\ddot{x} + k(x - x^*) = 0, \quad x(0) = x_0$$

and then consider a DS following equation:

$$m\ddot{x} + d(\dot{x} - \dot{x}^*) + k(x - x^*) = 0, \quad x(0) = x_0$$

Is the damping term required?
What if the DS needs to pass through two points in space with two different velocities (i.e., $x = x_1^*$ with $\dot{x} = \dot{x}_1^*$ at $t = t_1^*$ and $x = x_2^*$ with $\dot{x} = \dot{x}_2^*$ at $t = t_2^*$). Can this scenario be accomplished? Support your answer with analytical and numerical results.
Solution:

The analytical solution of a spring mass system is

$$x(t) = c_1 \sin\left(\sqrt{\frac{k}{m}}t + \Delta\phi\right) \tag{1}$$

$$\dot{x}(t) = \sqrt{\frac{k}{m}}c_1 \cos\left(\sqrt{\frac{k}{m}}t + \Delta\phi\right) \tag{2}$$

Even though there are two unknown parameters in the equation, m, k, their ratio defines the behaviour of the system, Hence, in general the damping term is needed to hit the object at the desired time, desired location with the desired velocity.

Same applies to the two points scenario. In general, it is not possible to pass two points at the specific time with specific velocity.

1.2

Book correspondence: Ex10.2, p273

Consider the DS, designed in exercise 10.1, to hit a nail using a point mass robotic system. If $m = 2$, $k = 25$, and the impact is elastic, what would be the force applied to the nail at each cycle? What is the cycle rate?

Solution:

The analytical solution of the mentioned dynamical system is

$$y(t) = 1.3 \cos\left(\sqrt{25/2}\, t + 0\right)$$

For the velocity we have:

$$v = 1.3\sqrt{25/2} \text{ m/s}$$

the impact is happening at $t_e = \pi/2\sqrt{2/25}$s.

The cycle time is $t_c = 2\pi\sqrt{2/25}$s, it gets cut in half by the elastic impact to $t_e = \pi\sqrt{2/25}$s.

1.3

Book correspondence: Ex10.7, p284

Consider the task of reaching and contacting a large fixed object presented in section 8.4, with motion dynamics for the robot as defined by:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau + J(q)^T F_c$$

and driven by low level controls (task dependent (one of the following)):

$$F = V_x(q,\dot{q})\dot{x} + G_x(q) + M_x(q)\ddot{x} - M_x(q)\Lambda^{-1}(\mathbf{D}(\tilde{x})\dot{\tilde{x}} + \mathbf{K}(\tilde{x})\tilde{x}) + (M_x(q)\Lambda^{-1} - I)F_c$$
$$F = V_x(q,\dot{q})\dot{x} + G_x(q) + M_x(q)\ddot{x} - (\mathbf{D}(\tilde{x})\dot{\tilde{x}} + \mathbf{K}(\tilde{x})\tilde{x})$$
$$F = G_x(q) - (\mathbf{D}(\tilde{x})\dot{\tilde{x}} + \mathbf{K}(\tilde{x})\tilde{x})$$

Design the impedance profile such that the robot is stiff if it is far from the surface and compliant if it is close to the surface.
What would happen if one designed the stiffness profile and kept the damping matrix constant? What role has the inertial matrix?

Solution:

Let us define the distance to the surface $\tilde{x} = x - x_s$. Assuming $\mathbf{K}_1 \ll \mathbf{K}_2$ and one can define

$$\mathbf{K}(\tilde{x}) = \gamma(\tilde{x})\mathbf{K}_1 + (1-\gamma(\tilde{x}))\mathbf{K}_2 \qquad \mathbf{D}(\tilde{x}) = \gamma(\tilde{x})\mathbf{D}_1 + (1-\gamma(\tilde{x}))\mathbf{D}_2 \tag{3}$$

where $\gamma(\tilde{x}) = e^{-l\,\|\tilde{x}\|}$.

One needs to tune l w.r.t. the workspace of the robot. The inertial matrix does not need to be changed as the ratio between $\mathbf{K}(\tilde{x})$, $\mathbf{D}(\tilde{x})$ and Λ is important. Moreover, keeping the damping matrix constant would not deteriorate the stability of the system, However, it might cause overshoot or undershoot behaviour if the ratio between $\mathbf{K}(\tilde{x})$ and $\mathbf{D}(\tilde{x})$ is not chosen wisely.

2 Programming exercises [1h]

2.1 Programming Exercise : Impedance controller

1. Section 1:

   (a) The robot should converge well starting from any position or velocity. Here the simulation and controllers consider that the acceleration are instantaneously applied from the joint torque, so the initial joint acceleration has no effect.

   (b) Increasing the stiffness will reduce the tracking error, but also increase the required joint torque. One limit is thus the maximum torque of the robot. In a real robot, increasing the stiffness can also lead to oscillations due to un-modelled effects (communication delay, friction, etc ...)

   (c) If the system is under-damped, the robot can accelerate to very high speed and overshoot its target, resulting in oscillations. Over-damping will slow the control correction, resulting in higher tracking error. Note also that a numerical implementation of damping can lead to divergence if the damping term is too high.

2. Section 2:

   (a) This controller doesn't have a steady-state error like with the first controller, as the added feed-forward terms allow the controller to match the dynamic of the system

   (b) Reducing the stiffness allow to reduce joint torque. Thus a better controller can compensate for limiting hardware.

3. Section 3:

   (a) The controller has similar tracking performances with the provided gains

   (b) Increasing the virtual inertia generates a system that is less subjected to perturbations, but also has more difficulty tracking the moving target.
   Reducing the virtual inertia makes the system more reactive, but at a cost of higher joint torques

2.2 Programming Exercise : Variable Impedance controller

1. Section 1:

   (a) The system is more robust to perturbations when the gains are higher, as can be seen when comparing the the peak in joint position for each perturbations; the first one is much smaller due to the gains being much higher. In real life scenarios, it can be very useful to have different gains for different tasks, which can be encoded here as joint positions. However, one must be careful not to increase gains too much as it can lead to oscillations or high tracking error.

(b) The scheduling position parameters set the joint positions at which our system will reach each desired set of gains. Here, since both joints move in circles of radius 1 with an constant offset, the initially set $\mu_1 = [-1; 1]$ and $\mu_2 = [1; 1]$ can never be reached. Hence, we never reach the desired gains and the switch to the other set of gains happens in the joint position closest to the scheduling parameters.

2. Section 2:

   (a) The system is more robust to perturbations when the gains are higher, as can be seen when comparing the the peak in joint position for each perturbations; the first one is smaller due to the gains being higher.

   (b) The impedance profile now depends on both the position and velocity of the joints, hence more parameters can contribute to changing the gains. By computing the velocity in the impedance profile, the gains are now more sensible to perturbations and should better adjust to them.

3. Section 3:

   (a) The system is more robust to perturbations when the gains are higher, as can be seen when comparing the the peak in joint position for each perturbations; the second one is smaller due to the gains being higher.

References

[1] Aude Billard, Sina Mirrazavi, and Nadia Figueroa. *Learning for Adaptive and Reactive Robot Control: A Dynamical Systems Approach.* MIT press, 2022.