Learning for Adaptive and Reactive Robot Control
Solutions for exercises of lecture 10

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Introduction

INTRO
This part of the course follows *exercises* 11.1 *and* 11.2 and *programming exercise* 11.1 of the book "*Learning for Adaptive and Reactive Robot Control: A Dynamical Systems Approach. MIT Press, 2022*".

1 Theoretical exercises [1h]

1.1

Book correspondence: Ex10.10
Let the nominal task model $f(x)$ be composed of conservative and non-conservative parts:

$$f(x) = f_c(x) + f_r(x)$$

Let the system $M(x)\ddot{x} + C(x,\dot{x})\dot{x} + g(x) = \tau_c + \tau_e$ be controlled by the following:

$$\tau_c = g(x) - D\dot{x} + \lambda_1 f_c(x) + \beta_R(z,s)\lambda_1 f_r(x)$$

where $z = \dot{x}^T f_r(x)$
The storage variable s has the following dynamics

$$\dot{s} = \alpha(s)\dot{x}^T D(x)\dot{x} - \beta_s(s,z)\lambda_1 z$$

and the following properties are satisfied,

$$\begin{aligned}
0 \leq \alpha(s) \leq 1 \qquad & s < \bar{s} \\
\alpha(s) = 0 \qquad & s > \bar{s} \\
\beta_s(z,s) = 0 \qquad & s \leq 0 \;\; and \;\; z \geq 0 \\
\beta_s(z,s) = 0 \qquad & s \geq \bar{s} \;\; and \;\; z \leq 0 \\
0 \leq \beta_s(z,s) \leq 1 \qquad & elsewhere \\
\beta_R(z,s) = \beta_s(z,s) \qquad & z \geq 0 \\
\beta_R(z,s) \geq \beta_s(z,s) \qquad & z < 0
\end{aligned}$$

Consider the storage function $W(x,\dot{x},s) = \frac{1}{2}\dot{x}^T \mathrm{M}\dot{x} + \lambda_1 V_{\mathbf{c}(x)} + s$, where $V_c(x)$ is the potential function associated with $\mathrm{f}_c(x)$

Prove that if $0 < s(0) \leq s$, the resulting closed loop system is passive with respect to the input-output pair $\tau_e, \dot{x}$.

Solution:
First, note that $0 < s(0) \leq \bar{s} \Rightarrow 0 \leq s(t) \leq \bar{s}, \forall t > t_0$. Consider the storage function $W(x, \dot{x}, s) = \frac{1}{2}\dot{x}^T \mathrm{M}\dot{x} + \lambda_1 V_{\mathbf{c}(x)} + s$, where $V_c(x)$ is the potential function associated with $\mathrm{f}_c(x)$. The rate of change of W is:

$$\dot{W}(x, \dot{x}) = x^T \mathrm{M}\ddot{x} + \frac{1}{2}\dot{x}^T \dot{M}\dot{x} + \lambda_1 \nabla V_c^T \dot{x} + \dot{s} \tag{1}$$

Substituting $\mathrm{M}\ddot{x}$ and τ_c using the skew-symmetry of $\dot{\mathrm{M}} - 2\mathrm{C}$ yields:

$$\begin{aligned}\dot{W}(x, \dot{x}) = &- \dot{x}^T \mathrm{D}\dot{x} + \dot{x}^T \tau_e + \beta_R(z, s)\lambda_1 z + \\ &+ \lambda_1 \dot{x}^T \mathrm{f}_c(x) + \lambda_1 \nabla V_c^T \dot{x} + \dot{s}\end{aligned} \tag{2}$$

The second-to-last two terms cancel because $\mathrm{f}_c(x) = -\nabla V_c(x)$. Substituting $\dot{s}$ then yields:

$$\dot{W}(x, \dot{x}) = -\underbrace{(1 - \alpha(s))}_{\geq 0}\dot{x}^T \mathrm{D}\dot{x} + \zeta(z, s)\lambda_1 z + \dot{x}^T \tau_e \tag{3}$$

where $\zeta(z, s) = \beta_R(z, s) - \beta_s(z, s)$ has been introduced to ease the notation. We have $1 - \alpha(s) \geq 0$ and $\zeta(z, s) = 0$ for all $z > 0$ and $\zeta(z, s) \geq 0$ for $z < 0$. Hence, we have:

$$\dot{W}(x, \dot{x}) \leq \dot{x}^T \tau_e \tag{4}$$

which concludes the proof.

1.2

Book correspondence: Ex11.1, p302

Consider the control law :

$$\tau_c = D(x)(\dot{x}_d - \dot{x}) = \lambda_1 \dot{x}_d - D(x)\dot{x} \tag{5}$$

This control law is passive for conservative DSs. However, equation 6

$$\dot{x}_d = f(x) + f_n(x) \tag{6}$$

is not a conservative DS. By using the energy tanks approach, modify equation 6 such that the system stays passive.
Hint 1: define a dynamics for the storage variable s. $\dot{s} = \alpha(s)\dot{x}^T D(x)\dot{x} - \beta_s(s, z)\lambda_1 z$
Hint 2: $\dot{M}(x) - 2C(x, \dot{x})$ is a skew symmetric matrix
As mentioned in Exercise 1.1, following properties are satisfied,

$$\begin{aligned}
0 \leq \alpha(s) \leq 1 &\qquad s < \bar{s} \\
\alpha(s) = 0 &\qquad s > \bar{s} \\
\beta_s(z, s) = 0 &\qquad s \leq 0 \;\; and \;\; z \geq 0 \\
\beta_s(z, s) = 0 &\qquad s \geq \bar{s} \;\; and \;\; z \leq 0 \\
0 \leq \beta_s(z, s) \leq 1 &\qquad elsewhere \\
\beta_R(z, s) = \beta_s(z, s) &\qquad z \geq 0 \\
\beta_R(z, s) \geq \beta_s(z, s) &\qquad z < 0
\end{aligned}$$

Solution:
Let us start by assuming that the nominal dynamical system $f(x)$ is composed of a conservative part $f_c(x)$ and a non-conservative part $f_r(x)$:

$$f(x) = f_c(x) + f_r(x) \tag{7}$$

with $f_c(x)$ deriving from a potential function $V_c(x)$ such that:

$$f_c(x) = -\nabla V_c(x) \tag{8}$$

Let us then consider a storage function $W(x, \dot{x})$ that includes the kinetic energy of the robot and the potential function $V_c(x)$:

$$W(x, \dot{x}) = \frac{1}{2}\dot{x}^T M(x)\dot{x} + \lambda_1 V_c(x) \tag{9}$$

Using (8), the rate of change of $W(x, \dot{x})$ is:

$$\dot{W}(x, \dot{x}) = \dot{x}^T M(x)\ddot{x} + \frac{1}{2}\dot{x}^T \dot{M}(x)\dot{x} - \lambda_1 \dot{x}^T f_c(x) \tag{10}$$

By substituting $M(x)\ddot{x}$ and using the skew-symmetry of $\dot{M}(x) - 2C(x, \dot{x})$, (10) simplifies to:

$$\dot{W}(x, \dot{x}) = \dot{x}^T \tau_c + \dot{x}^T \tau_e - \lambda_1 \dot{x}^T f_c(x) \tag{11}$$

Substituting τ_c finally leads to:

$$\dot{W}(x, \dot{x}) = \lambda_1 \dot{x}^T f_r(x) + \lambda_1 \dot{x}^T f_n(x) - \dot{x}^T D(x)\dot{x} + \dot{x}^T \tau_e \tag{12}$$

which can be rewritten into:

$$\dot{W}(x, \dot{x}) = p_r + p_n - p_d + \dot{x}^T \tau_e \tag{13}$$

$p_d = \dot{x}^T D(x)\dot{x}$, $p_r = \lambda_1 \dot{x}^T f_r(x)$ and $p_n = \lambda_1 \dot{x}^T f_n(x)$ respectively denote the dissipated power, the power due to the non-conservative part of the nominal dynamical system and the power generated by the normal modulation term. Thanks to the definition of $D(x)$, we can ensure that $p_d \geq 0$ while the sign of the first two terms in (13) is undefined. Therefore, we cannot guarantee passivity of the system with respect to the environment. To restore passivity, we consider an approach based on energy tanks.

Let us therefore introduce a virtual tank state s that stores the dissipated energy in the system mainly coming from the damping term p_d. We use this energy to modulate the nominal dynamical system without violating passivity. The resulting energy flow is governed by the tank's dynamics, which is coupled with the robot's state (x and $\dot{x}$) as follows:

$$\dot{s} = \alpha(s)p_d - \beta_r(s, p_r)p_r - \beta_n(s, p_n)p_n \tag{14}$$

The scalar functions $\alpha(s)$, $\beta_r(s, p_r)$ and $\beta_n(s, p_n)$ control the energy flow between the virtual tank and the robot. Let us define s_m as the maximum energy level allowed to be stored in the tank. If the tank is depleted, the controller should not generate the potential non-passive actions. The control law should be corrected accordingly by taking the state of the tank and the power variables into account. To this end, we correct the modulation law as follows:

$$\dot{x}_d = f'(x) + f'_n(x) \tag{15}$$

with:

$$\begin{cases} f'(x) = f_c(x) + \beta'_r(s,p_r)f_r(x) \\ f'_n(x) = \beta'_n(s,p_n)f_n(x) \end{cases} \tag{16}$$

where $\beta'_r(s,p_r)$ and $\beta'_n(s,p_n)$ are scalar functions satisfying:

$$\beta'_i(s,p_i) = \begin{cases} 1 & \text{if } p_i < 0 \\ \beta_i(s,p_i) & \text{otherwise} \end{cases} \qquad i = r,n \tag{17}$$

Let us now define the final storage function $W(x,\dot{x},s)$ taking the tank's dynamics into account:

$$W(x,\dot{x},s) = \frac{1}{2}\dot{x}^T M(x)\dot{x} + \lambda_1 V_c(x) + s \tag{18}$$

Substituting $\dot{s}$ by (14) and $\dot{x}_d$ by (15), the rate of change of $W(x,\dot{x},s)$ becomes:

$$\begin{aligned} \dot{W}(x,\dot{x},s) = &\big(\beta'_r(s,p_r) - \beta_r(s,p_r)\big)p_r \\ &+ \big(\beta'_n(s,p_n) - \beta_n(s,p_n)\big)p_n - \big(1-\alpha(s)\big)p_d + \dot{x}^T F_e \end{aligned} \tag{19}$$

The first two terms are now both non-positives, while the third one remains dissipative since $1-\alpha(s) \geq 0$. As a result, the full system is passive with respect to $\dot{x}^T F_e$, which concludes the proof.

1.3 (Optional)

Book correspondence: Ex11.2, p302

In section 11.1.2, the presented control law 5

$$\tau_c = D(x)(\dot{x}_d - \dot{x}) = \lambda_1 \dot{x}_d - D(x)\dot{x}$$

is used to accomplish a dual-arm scenario. Modify the DS 6

$$\dot{x}_d = f(x) + f_n(x)$$

for each robotic arm, such that the robots reach the object, apply a specific amount of force on the object, and move it.

Solution:
To start, let us consider the scenario illustrated in Figure 1. The main variables used to describe the problem are provided in Table 1. The superscript L and R will refer to the left and right robot respectively.

The robots' center position x^C and distance vector x^D are computed from their tool tip positions x^R and x^L:

$$x^C = \frac{x^L + x^R}{2} \qquad x^D = x^R - x^L \tag{20}$$

from where we can derive the relation below:

$$\dot{x}^R = \dot{x}_C + \frac{\dot{x}^D}{2} \qquad \dot{x}^L = \dot{x}_C - \frac{\dot{x}^D}{2} \tag{21}$$

To reach and manipulate the object during the task, we choose to couple the robots' motion by controlling for a desired robots' centre position x_d^C and distance vector x_d^D, using simple linear dynamics:

$$\begin{cases} \dot{x}_d^C = A_C\left(x_d^C - x^C\right) \\ \dot{x}_d^D = A_D\left(x_d^D - x^D\right) \end{cases} \tag{22}$$

x^L,$\dot{x}^L$,$\dot{x}_d^L$	Left robot tool position, velocity and desired dynamics
x^R,$\dot{x}^R$,$\dot{x}_d^R$	Right robot tool position, velocity and desired dynamics
x_o^C and x_o^D	Measured object center position and dimension vector
x^C and x^D	Measured center position and distance vector between the two robots
x_d^C and x_d^D	Desired center position and distance vector
$\dot{x}_d^C$ and $\dot{x}_d^D$	Desired center position and distance vector dynamics

Table 1: Main variables used to describe the reaching, grasping and manipulation task with two robots.

where A_C and A_D are positive gain diagonal matrices. Basically, $\dot{x}_d^C$ specifies the desired positioning behavior of the robots' center while $\dot{x}_d^D$ defines the desired closing behavior on the object's surface. x_d^C and x_d^D can be set to x_o^C and x_o^D respectively during the reaching phase and modified during the manipulation phase.

To do the grasping part of the task, first, the nominal dynamical system $f^R(x^L, x^R)$ and $f^L(x^L, x^R)$ must be introduced. The nominal dynamical system should bring each robot in contact with the target surface (e.g the object's surface). This role is achieved by $\dot{x}_d^D$. Following (21) the nominal dynamical system are defined such that:

$$f^R(x^L, x^R) = \text{-}f^L(x^L, x^R) = \frac{\dot{x}_d^D}{2} \tag{23}$$

Once the robots reach the object's surface, they should generate the desired contact force profile $F_d(x^L, x^R) \geq 0$ which is assumed to be the same for both of them. To this end, the normal modulation terms are defined as follows:

$$f_n^i(x^L, x^R) = \frac{F_d(x^L, x^R)}{d_1^i} n^i \quad i = L, R \tag{24}$$

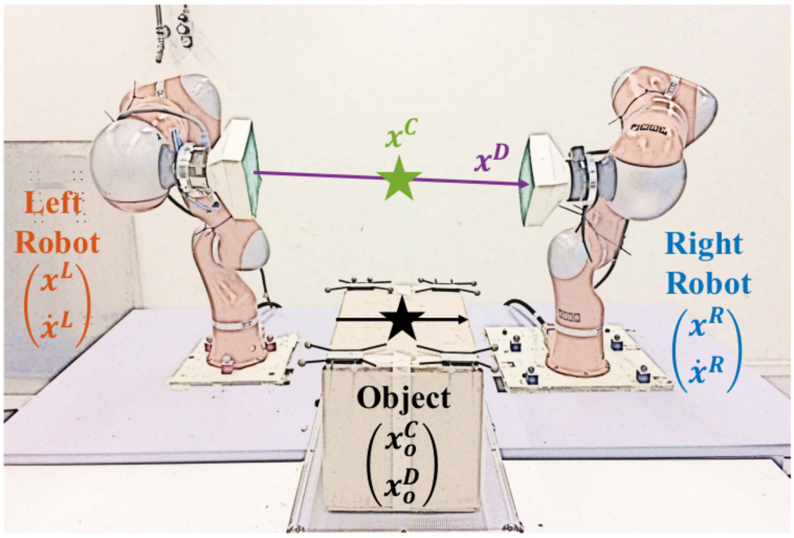

Figure 1: Scenario for reaching, grasping and manipulating an object with two robotic arms

The force application directions n^R and n^L are derived from the desired distance vector. For a box (with two parallel surfaces), they are opposite for the two robots:

$$n^L = -n^R = \frac{x_d^D}{\|x_d^D\|} \tag{25}$$

From there, the desired robots' velocity can be finally expressed:

$$\dot{x}_d^i = f^i(x^L, x^R) + f_n^i(x^L, x^R) + \dot{x}_d^C \quad i = L, R \tag{26}$$

which includes the desired robots' center dynamics $\dot{x}_d^C$, needed to properly positioned the robots' centre. The modulated dynamical system are then tracked with the dynamical system impedance controller.

References

[1] Aude Billard, Sina Mirrazavi, and Nadia Figueroa. *Learning for Adaptive and Reactive Robot Control: A Dynamical Systems Approach.* MIT press, 2022.