Lecture 11

Impedance Control with Dynamical Systems – Passive DS

Force Control with Dynamical Systems
DS control makes the system infinitely compliant!
DS control makes the system infinitely compliant!
Robot’s Dynamics, Assumptions and Requirements

Dynamics equation of the robot

\[
M(x) \ddot{x} + C(x, \dot{x}) \dot{x} + g(x) = \tau_c + \tau_e
\]

Inertia Coriolis Gravity \hspace{1cm} Control Input \hspace{1cm} External Forces

Design a control law for generating control torques \( \tau_c \)

For the system to remain stable under external disturbances, we need to show that it remains passive. (see Annexes A.6)

Control torques \( \tau_c \) must be modulated to ensure that the system remains passive.
Stability of the System through Passivity Analysis

\[ \dot{x} = f(x,u), \quad u \in \mathbb{R}^p: \text{input} \]

We must verify that the energy injected by the input \( u \) does not destabilize the system.

\( \rightarrow \) Verify that the system is \textit{closed-loop passive}.

Recall: To study stability of \( f(x) \), we had used \textit{Lyapunov stability}. We defined a way to measure the energy of the system and we verified that it decreased over time before eventually vanishing at the attractor.

Passivity extends the Lyapunov stability concept to systems that are \textit{subjected to an external input} \( u \).

To determine the evolution of the energy of the system, we define a variable:

\[ y = h(x), \quad y \in \mathbb{R}^m \]
Passivity: Definition

**Definition A.8** (Passivity): A system with the form

\[
\begin{align*}
\dot{x} &= f(x, u) \\
y &= h(x)
\end{align*}
\]  

\( (A.9) \)

is passive if there is a lower-bounded storage function \( V: \mathbb{R}^N \rightarrow \mathbb{R}_{0\leq} \) such that

\[
\frac{V(x(t)) - V(x(0))}{\text{Stored energy}} \leq \int_0^t u(s)^T y(s) ds \\ 
\text{Supplied energy}
\]  

\( (A.10) \)

is satisfied for all \( 0 \leq t \), all input functions \( u \), and all initial conditions \( x(0) \in \mathbb{R}^N \).

When is this equivalent to Lyapunov stability?
To show that a system is passive, we must define a storage function and modulate the control input in such a way that passivity remains possible.
Goals for the design of the control torques

Dynamics equation of the robot

\[ M(x) \ddot{x} + C(x, \dot{x}) \dot{x} + g(x) = \tau_c + \tau_e \]

Inertia Coriolis Gravity Control Input External Forces

Design a control law for generating control torques \( \tau_c \)

Goals for the control system:

- The robot should move according to a desired dynamics, set by \( \dot{x} = f(x) \).
- The system should remain passive.
- If \( f(x) \) is Lyapunov stable, the control should dissipate energy solely in directions perpendicular to \( f(x) \).
Format of Control Torques

Dynamics equation of the robot

\[ M(x) \ddot{x} + C(x, \dot{x}) \dot{x} + g(x) = \tau_c + \tau_e \]

Feedback term:

\[ \tau_c = g(x) - D(x) \dot{x} \]

Control torques \( \tau_c \) must be modulated to ensure that the system remains passive.

→ Modulate \( D(x) \).

Internal control system to compensate gravity (gravity compensation mode on standard robots).
Constraints on Control Torques for Passivity

Feedback term:
\[ \tau_c = g(x) - D(x)\dot{x} \]

Gravity compensation
Damping

What is the first constraint we must set for \( D(x) \)?
\[ D(x) > 0, \quad \forall x \]

Robot’s dynamics

We verify that the system remains passive under external disturbances \( \tau_e \).

We set:
\[
\begin{cases}
  u = \tau_e \\
  y = \dot{x}
\end{cases}
\]

We verify that:
\[ \dot{V} \leq \tau_e^T \dot{x} \]
\( V \) : kinetic energy
\[ V = \frac{1}{2} \dot{x}^T M(x) \dot{x} \]
We verify that: \( \dot{V} \leq \tau_e^T \dot{x} \)

\[
\dot{V} = \dot{x}^T M(x) \dot{x} + \frac{1}{2} \dot{x}^T \dot{M}(x) \dot{x}
\]

Replacing with \( M(x) \ddot{x} + C(x, \dot{x}) \dot{x} + g(x) = \tau_c + \tau_e \) and \( \tau_c = g(x) - D(x) \dot{x} \)

\[
\dot{V} = \frac{1}{2} \dot{x}^T \left( \dot{M}(x) - 2C(x, \dot{x}) \right) - \dot{x}^T D(x) \dot{x} + \dot{x} \tau_e \leq \tau_e^T \dot{x}
\]

\( \dot{M}(x) - 2C(x, \dot{x}) = 0 \) \quad \text{Skew-symmetric}

since \( D(x) > 0 \)
Gravity Damping compensation Feedback term:

\[ \tau_c = g(x) - D(x)(\dot{x} - f(x)) \]

The system must follow a desired dynamics \[ \dot{x} = f(x) \]

Traditional Tracking Feedback Control Loop

\[ g(x) - D(\dot{x} - \dot{x}^d) - K(x - x^d) = \tau_c \]

Tracking in position given by the DS

State-dependent Impedance modulation
Shaping the Impedance

\[ g(x) - D(x)(\dot{x} - f(x)) = \tau_c \]

Eigencomposition of \( D(x) \)

\[ D(x) = Q(x) \Lambda(x) Q(x)^T \]

We set \( f(x) \) to be aligned with an eigenvector of \( D(x) \)

\[ Q(x) = [e_1(x), e_2(x)], \quad e_1(x) = \frac{f(x)}{\|f(x)\|}, \quad e_1(x)^T e_2(x) = 0. \]

The eigenvalues will set the impedance

\[ \Lambda(x) = \begin{bmatrix} \lambda_1(x) \\ \lambda_2(x) \end{bmatrix} \]
Shaping the Impedance

\[ g(x) - D(x)(\dot{x} - f(x)) = \tau_c \]

The eigenvalues will set the impedance

\[ \Lambda(x) = \begin{bmatrix} \lambda_1(x) \\ \lambda_2(x) \end{bmatrix} \]

Set \( \lambda_1(x) \) to be very stiff for accurate tracking.

Modulate \( \lambda_2(x) \) to comply with orthogonal disturbances.

\( \lambda_1(x) \sim \text{axes length of ellipse (see impedance class)} \)
Passivity

\[ g(x) - D(x)\left(\dot{x} - f(x)\right) = \tau_c \quad \Rightarrow \quad g(x) + \lambda_1 f(x) - D(x)\dot{x} = \tau_c \]

\[ \dot{x} = f(x) \text{ is an eigenvector of } D: \quad e_1(x) = \frac{f(x)}{\|f(x)\|}. \]

\[ \dot{x} = f(x) \text{ is a Lyapunov stable function} \]

with an associated Lyapunov function \( V_f(x) \).

\( V_f(x) \) is a potential function and we can write:

\[ f(x) = -\nabla V_f(x) \]
Passivity Verification II

We set \( V = \frac{1}{2} \dot{x}^T \dot{M}(x) \dot{x} + \lambda_1 V_f(x) \)

\( \frac{\text{Kinetic Energy}}{\text{Potential Energy of } f(x)} \)

We verify that: \( \dot{V} \leq \tau_e^T \dot{x} \)

\[
\dot{V} = \dot{x}^T M(x) \ddot{x} + \frac{1}{2} \dot{x}^T \dot{M}(x) \dot{x} + \lambda_1 \dot{V}_f(x)
\]

\( \text{Ch} \)

Replacing with \( M(x) \ddot{x} + C(x, \dot{x}) \dot{x} + g(x) = \tau_c + \tau_e \), \( \dot{V}_f(x) = \nabla V_f(x)^T \dot{x} \)

\( \text{and } g(x) - D(x)(\dot{x} - f(x)) = \tau_c \)

\[
\dot{V} = \frac{1}{2} \dot{x}^T (\dot{M}(x) - 2C(x, \dot{x})) - \dot{x}^T D(x) \dot{x} + \lambda_1 \dot{x}^T f(x) + \lambda_1 \dot{V}_f(x) + \dot{x} \tau_e \leq \tau_e^T \dot{x}
\]

\( \dot{M}(x) - 2C(x, \dot{x}) = 0 \)

Skew-symmetric

\( < 0 \)

since \( D(x) > 0 \)

\( = 0 \)

\( f(x) = -\nabla V_f(x) \)
Non-conservative DS – Energy Tank

\[ \dot{x} = f(x) \] is not conservative.  

Decompose \( f \) into a conservative and non-conservative terms:

\[ f(x) = f_c(x) + f_r(x) \]

Conservative part follows:

\[ f_c(x) = -\nabla V_c(x) \]

The energy injection must now be actively controlled as we are left with a uncontrolled term:

\[ \dot{V} = \ldots + f_r(x)^T \dot{x} \]

Introduce a new variable \( s \) to account for energy stored:

\[ V(x, \dot{x}, s) = \frac{1}{2} \dot{x}^T M(x) \dot{x} + \lambda_1 V_c(x) + s \]

Set a tank limit \( \bar{s} \) beyond which we do not allow the system to absorb energy anymore.

Modify the control torque to never exceed the tank limit:

\[ \tau_c = g(x) - D(x) \dot{x} - \lambda_1 f_c(x) - \lambda_1 \beta(s, \bar{s}) f_r(x) \]

See exercise session to determine appropriate function: \( \beta(s, \bar{s}) \)
Open-Loop vs Closed-Loop Control with DS

**DS in feedback configuration**

Given an autonomous DS:

\[ \dot{x} = f(x) \rightarrow \lim_{t \to \infty} \|x - x^*\| = 0 \]

DS can be:

- Linear DS
- Nonlinear DS via LPV-DS or SEDS (Lecture 3)
- Modulated DS (Lecture 6)
- A non conservative DS
Desired Behavior is determined by choice of damping eigenvalues.
Apparent Stiffness

Closed-loop Dynamics with Passive-DS Control-law

\[ M(x) \dot{x} + D(x)(\dot{x} - f(x)) = \tau_e \]

Classical Impedance-like Control

\[ M(x) \dot{x} + D(\dot{x} - x^d) - K(x - x^d) = \tau_e \]

What about the stiffness term?

Equivalent to Damping term in classical impedance control law

Observed/Apparent Stiffness can be derived [2]

\[ \tilde{K}(x) = \frac{\partial \tau_e}{\partial x} = -\lambda_1 \frac{\partial f(x)}{\partial x} \]

\[ e_i^T \tilde{K}(x) e_i = -\lambda_1 e_i^T \frac{\partial f(x)}{\partial x} e_i \]

Unit norm vector

Rayleigh Quotient

Stiffness is dependent on chosen damping values and convergence rate of the DS via the Jacobian!

Passive-DS for Robot Control
Passive-DS for Robot Control

Motion generation with Dynamical Systems and Passive Interaction Control
Passive-DS for Robot Control
Introduction

- Introduced a means to combine impedance control with DS control.
  - The DS acts as a trajectory generator.
  - Impedance control generates torques to track the output of the DS.

- Extended the notion of Lyapunov stability and introduced passivity to characterize a system subjected to disturbance in the form of external forces.
  - Showed that when the nominal DS is conservative (Lyapunov stable), the system is passive.
  - When the DS is not conservative, one must introduce the notion of tank to track energy injected into the system.

- The impedance gains (damping matrix eigenvalues) modulate the response of the system when subjected to external disturbances (external forces).
  - Impedance is directional – aligned with the flow of the DS
  - High impedance in the direction of the DS will force the system to track accurately the DS
  - Low impedance in orthogonal directions allows to dissipate energy.
Force Control with Dynamical Systems

Controlling for force at contact
Controlling for force at contact

Nominal DS $f(x)$
Controlling for force at contact: Principle

Once in contact, moves tangential to the surface
Controlling for force at contact: Principle

\[ f(x)^T n(x) = 0 \quad \text{(in contact)} \]

\[ f(x)^T n(x) > 0 \quad \text{(in free space)} \]
Controlling for force at contact: Principle

The force may vary along the surface $F_d(x)$

**Goal:** Apply a desired force $F_d$

**Idea:** If we can project the control torques onto the surface, we simplify the computation.

→ Need a model of the surface.
We model the surface through a continuous function $\Gamma(x)$, that measures the distance to the surface. $\Gamma(x)$ is continuously differentiable such that we can compute the normal vector $n(x)$ at any position $x$. 

- $\Gamma(x) > 0$
- $\Gamma(x) = 0$ (on the surface)
- $\Gamma(x) < 0$
Nominal DS

We define a linear DS moving downwards towards the surface. \( \dot{x} = Ax \)

We modulate this DS to force it to move along the surface.

To stop the motion at a target on the surface, we can set this as the attractor of the DS.

\[
f(x) = R(x)Ax
\]

\(R(x)\): Rotation to align to the surface once in contact (same as in obstacle avoidance with constant velocity)
Nominal DS

To separate control of force, decompose the nominal DS into two components:

\[
\dot{x}_d = f(x) + f_n(x) \quad f_n(x) = 0 \quad \text{(in free space)}
\]
To generate forces, we need to control the robot’s torques. We use the passive DS approach.

\[ \tau_c = D(x)(\dot{x} - \dot{x}_d) \]

Robot's control torques

\[ \dot{x}_d = f(x) + f_n(x) \]

Reach/Move on the surface  Apply the contact force
Passive-DS for Controlling Forces on the Surface

To generate forces, we need to control the robot’s torques. We use the passive DS approach.

Robot's control torques

\[ \tau_c = D(x)(\dot{x} - \dot{x}_d) \]

where

- \( \tau_c \) are the control torques on the robot.
- \( D(x) \) is the robot's dynamics.
- \( \dot{x} \) is the current velocity.
- \( \dot{x}_d \) is the desired velocity.

Reach/Move on the surface

Apply the contact force

\[ \dot{x}_d = f(x) + F_d(x)n(x) \]

Force controlled direction

Robot's control torques

\( F_d(x) \) is the desired force on the robot.
\( n(x) \) is the normal direction of the surface.
\( f(x) \) is the friction force.
Passive-DS for Controlling Forces on the Surface

To generate forces, we need to control the robot’s torques. We use the passive DS approach.

Robot's control torques

\[ \tau_c = \lambda_1 f(x) + \lambda_1 f_n(x) - D(x) \dot{x} \]

Set \( f_n(x) = \frac{F_d(x)}{\lambda_1} n(x) \)

Eigencomposition of \( D(x) \)

\[ D(x) = E(x) \Lambda(x) E(x)^T \]

Fixed impedance

\[ \Lambda = \begin{bmatrix} \lambda_1 & \lambda_2 \\ \lambda_2 & \lambda_1 \end{bmatrix} \]

Force controlled direction

Apply the contact force
Animation of Principle

No perturbation
Robotic Demonstration

Task: The robot must polish in circular motion the surface applying a constant force of 20N.

Learn a model of the surface using Support Vector Regression

Input dataset to learn the surface model:
**Robotic Demonstration**

**Task:** The robot must polish in circular motion the surface applying a constant force of 20N.

Define a nominal DS that creates a limit cycle on the surface.

Project on the surface.
Robotic Demonstration – Robustness to Various Disturbances
Passivity Analysis

Passivity analysis

\[ W(x, \dot{x}) = \frac{1}{2} \dot{x}^T B(x) \dot{x} + d_1 V(x) \quad \text{with} \quad f(x) = -\nabla V(x) \]

Potential energy function of \( f(x) \)

Introduce energy tank \( s \) to restore passivity

\[ \dot{W}(x, \dot{x}) = \frac{1}{2} \dot{s}^T B(x) \dot{s} + d_1 V(x) \quad \text{with} \quad f(x) = -\nabla V(x) \]

Energy storage function

<table>
<thead>
<tr>
<th>( W(x, \dot{x}) )</th>
<th>Energy storage function</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V(x) )</td>
<td>Potential function</td>
</tr>
</tbody>
</table>

\[ 0 \leq s \leq s_{\text{max}} \quad \text{Energy tank level} \]

\[ 0 \leq \alpha(s) \leq 1 \quad \text{Scalar variables controlling energy flow between robot and tank} \]

\[ 0 \leq \beta(s, u) \leq 1 \]

\[ 0 \leq \gamma(s, v) \leq 1 \]

Energy tank level

Scalar variables controlling energy flow between robot and tank

\[ u = \dot{x}^T f(x) \]

\[ v = \dot{x}^T e_1(x) \]

Fill the tank

Mainly contribute to empty the tank and implement non-passive actions
Passivity Analysis

- Control law adaptation if tank is depleted
  \[ F_u = d_1 \lambda'_a(x)f(x) + \gamma'(s,v)F_d(x)e_1(x) - D(x)\dot{x} + g(x) \]
  Modulation gain adaptation: \[ \lambda'_a(x) = \begin{cases} 1 & \text{if } s < 0 \text{ and } u < 0 \\ \lambda_a(x) & \text{otherwise} \end{cases} \]
  Desired force profile scaling: \[ \gamma'(s,v) = \begin{cases} 1 & \text{if } v < 0 \\ \gamma(s,v) & \text{otherwise} \end{cases} \]

- Passivity analysis with tank dynamics
  \[ W(x,\dot{x}) = \frac{1}{2} \dot{x}^T B(x) \dot{x} + d_1 V(x) + s \]
  \[ \dot{W}(x,\dot{x},s) = d_1 (\lambda'_a(x) - 1)(1 - \beta(s,u))u + F_d(x)(\gamma'(s,v) - \gamma(s,v))v + (1 - \alpha(s)) \dot{x}^T D(x) \dot{x} + \dot{x}^T F_{ext} \leq 0 \]
Extension to Control Bimanual Platform

**Task:** The robots must reach either side of the box and apply enough force to support the box’s weight.
Extension to Control Bimanual Platform
Variables to Control Bimanual Platform

- Robots’ center position and distance vector

\[
\begin{align*}
\mathbf{x}_C &= \frac{\mathbf{x}_L + \mathbf{x}_R}{2} \\
\mathbf{x}_D &= \mathbf{x}_R - \mathbf{x}_L
\end{align*}
\]

- Robots’ nominal dynamics

\[
\begin{align*}
f^R(\mathbf{x}_C, \mathbf{x}_D) &= \dot{\mathbf{x}}_d^C + \frac{\ddot{\mathbf{x}}_d^D}{2} \\
f^L(\mathbf{x}_C, \mathbf{x}_D) &= \dot{\mathbf{x}}_d^C + \left(-\frac{\ddot{\mathbf{x}}_d^D}{2}\right)
\end{align*}
\]

To simplify control and ensure coordination, compute control in relative position.
Nominal Dynamics for Bimanual Platform

- **Desired** robots’ center position and distance vector **dynamics:**

  \[
  \begin{cases}
  \dot{x}_d^C = A_C (x_d^C - x_d^C) \\
  \dot{x}_d^D = A_D (x_d^D - x_d^D)
  \end{cases}
  \]

  Center positioning **dynamics**

  with: \(A_C, A_D \geq 0\)

- **Robots’ nominal dynamics:**

  \[
  \begin{cases}
  f^R(x_L^R, x_R^R) = \dot{x}_d^C + \frac{\dot{x}_d^D}{2} \\
  f^L(x_L^L, x_R^R) = \dot{x}_d^C + \left(-\frac{\dot{x}_d^D}{2}\right)
  \end{cases}
  \]

  Center positioning + Closing **dynamics**
Force Desired for Bimanual Platform

- Robots’ desired modulated dynamics:

\[
\begin{align*}
\dot{x}_R^L &= f_R^R(x^L, x^R) + \frac{F_d(x^L, x^R) n^R}{d^R_1} \\
\dot{x}_d^L &= f_d^L(x^L, x^R) + \frac{F_d(x^L, x^R) n^L}{d^L_1}
\end{align*}
\]

with: \( n^R = -n^L = \frac{x_d^L}{\|x_d^L\|} = \) Grasping direction

\( F_d(x^L, x^R) \): Desired contact force

\( d^L_1, d^R_1 \): Impedance gains

- Ensures the passivity through a tank for energy of both arms
Robot Demonstration
Learning Force Adaptation
Force tracking errors result from:
- Uncertainties in the surface
- Uncertainties in the robot model
- Measurement noises

Goal: Exploit the adaptability and robustness of the time-invariant DS framework to learn force compensation models

An error pattern can be observed
State-Dependent Force Correction

❖ Force generation with DS:
\[ \dot{x}_d = f(x) + f_n(x) \]

Nominal DS (responsible of motion) Modulation term (responsible of contact force)

❖ Introduction of a state-dependent force correction model \( \tilde{F}_d(x, \theta) \):
\[ f_n(x) = \frac{F_d(x) + \tilde{F}_d(x, \theta)}{d_1} n(x) \]

DS in closed-loop configuration

| \( \dot{x}_d \in \mathbb{R}^3 \) | Desired dynamics |
|\( f(x) \in \mathbb{R}^3 \) | Nominal DS |
|\( f_n(x) \in \mathbb{R}^3 \) | Normal modulation term |
|\( F_d(x) \geq 0 \) | Desired force profile |
|\( n(x) \in \mathbb{R}^3 \) | Normal direction to the surface |
|\( d_1 \geq 0 \) | Impedance gain |
State-Dependent Force Correction

- Design $\tilde{F}_d(x, \theta)$ using Gaussian Radial Basis kernel functions (RBFs):

$$\tilde{F}_d(x, \theta) = \frac{\sum_{i=1}^{K} \theta_i \varphi(x - c_i)}{\sum_{j=1}^{K} \varphi(x - c_j)}$$

with:

$$\varphi(x) = \exp \left( -\frac{\|x\|^2}{2\sigma^2} \right)$$

Gaussian kernel

Hyper-parameters:
- $K$: Number of Gaussian
- $c_i$: Center position of Gaussian $i$
- $\sigma$: Kernel width

- Update the weights $\theta$ to minimize the normal force error $F_e$:

$$F_e = F_d(x) - n(x)^T F_m$$

$F_d(x)$: Desired contact force

$n(x)$: Normal vector to the surface

$F_m$: Measured contact forces
An error pattern can be observed.

Correction pattern

Initial error range
Other Examples of Learned-Force Based Control with DS

Learning and adaptive control for robots

Learning Arm Massage

Human recordings

Modeling

Robotic massage

Supported by Samsung
Motion Tracking

- Mannequin on massage chair
- Silicone sheet placed over the mannequin
  - Represent human tissue for the robot

- 7 Optitrack cameras
  - 5 x Prime 17w
  - 2 x S250:E
    - Limited to 125 Hz
- Infrared marker positions (X, Y, Z) captured
• Pressure Profiles Systems, FingerTPS

+ Sensors have multiple sensing areas
+ Multiple sizes
+ Fabric covered sensor
  - Force mapped across entire surface
  - 40 Hz sampling rate
Learning and adaptive control for robots

Demonstration by Massage Therapist

- No Thumb, Hands Together
- Thumb only
Modeling Back Surface
• Cross-validation used for model validation with 5 folds.
• Grid search to find optimal $C$ and $\sigma$ for the SVR.
• Best testing accuracy achieved with a RMSE of 0.52 [cm] for points on the back.
Reconstructed surface & gradients for the trained SVR

Simulated sample trajectories for the trained SVR
Test data from movements on the back of the mannequin
Reconstruction of Motion on Back Surface

- **Surface**
- **Gradients**
- **Linear Trajectory**
- **n(x)**
- **v(x)**
- **Circular Trajectory**
- **Applied Pressure**

Current Pressure [N]: 0
Learning and adaptive control for robots

Learning Arm Massage

Khoramshahi et al, ICRA 2020

Supported by Samsung
Create artificial data for the arm

Train SVR and predict the surface
Step 2:
• Use the model to reconstruct the behavior

Reconstructed Trajectory with Pressure Data

Stacked Radius of Trajectory with Pressure Data

Khoramshahi et al, ICRA 2020
Reconstruction of Motion on Arm Surface
Reconstruction of Motion on Arm Surface
Learning and adaptive control for robots

Diagram:
- Dynamical System
  - Desired motion
  - Learned from human demonstrations
- Task space impedance control
  - Desired Force
  - Fingertip position
  - Fingertip velocity
  - Joint torques
- Joint torque control
Summary

- Introduced a means to use Passive-DS to perform force control.
  - The DS is decomposed into two parts, one controlling for motion along the surface, the other controlling for force.
  - The control is simplified by introducing a function $\Gamma$ that determines the distance and normal to the surface everywhere (akin to principle used in obstacle avoidance) → This allows decoupling control of force from motion along two orthogonal axes.
- Machine learning (e.g. nonlinear regression through SVR) can be used to model the function $\Gamma$.
- A force-based model associated to position is used for modulate the force along the DS
- To show that the system remains passive, as the DS is not necessarily conservative, the tank is required.

- Pattern of force can be learned
  - To compensate for unmodeled interaction forces (nonlinear friction, other dynamics of robot poorly modelled)
  - To learn a position-dependent pattern of force. Showed an example of application to model massage