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Consider the functional:

$$F(p_1,p_2)=x(1)$$

$$\text{with: } \dot{x}(t)=-x(t)+p_1; \quad x(0)=p_2$$

1- Calculate the value of F by first solving the ODE analytically; then calculate its partial derivatives w.r.t. p_1 and p_2.
2- Derive the sensitivity equations and their respective initial conditions, then find a solution to these equations; apply the forward sensitivity formula to calculate the partial derivatives of F w.r.t. p_1 and p_2.
3- Derive the adjoint equations and its terminal condition, then find a solution to this equation; apply the adjoint sensitivity formula to calculate the partial derivatives of F w.r.t. p_1 and p_2.

1)

$$\dot{x}(t)=-x(t)+p_1$$

$$x(t)=Ke^{-t}+p_1$$

$$x(0)=p_2=K+p_1 \Rightarrow K=p_2-p_1$$

$$x(t)=(p_2-p_1)e^{-t}+p_1$$

$$F(p)=x(1)=\frac{1}{e}(p_2-p_1)+p_1$$

$$F_{p1}=\frac{-1}{e}+1$$

$$F_{p2}=\frac{1}{e}$$

2)

$$\dot{x}_{p1}(t)=-x_{p1}(t)+1; \quad x_{p1}(0)=0$$

$$\dot{x}_{p2}(t)=-x_{p2}(t); \quad x_{p2}(0)=1$$

$$x_{p1}(t)=K_1e^{-t}+1; \quad x_{p1}(0)=0=K_1+1$$

$$x_{p1}(t)=-e^{-t}+1$$

$$x_{p2}(t)=K_2e^{-t}; \quad x_{p2}(0)=1=K_2$$

$$x_{p2}(t)=e^{-t}$$

$$F_{p1}=x_{p1}(1)=\frac{-1}{e}+1$$

$$F_{p2}=x_{p2}(1)=\frac{1}{e}$$

3)

$$\dot{\boldsymbol{\lambda}}(t) = -\boldsymbol{\lambda}(t).(-1) = \boldsymbol{\lambda}(t); \quad \boldsymbol{\lambda}(1) = 1$$

$$\boldsymbol{\lambda}(t) = Ke^{\mathbf{t}}; \quad \boldsymbol{\lambda}(1) = 1 = Ke$$

$$\boldsymbol{\lambda}(t) = \frac{1}{e}e^{\mathbf{t}}$$

$$F_{\mathbf{p}1} = \int_0^1 \boldsymbol{\lambda}(t)\,dt = \left[\frac{1}{e}e^{\mathbf{t}}\right]_0^1 = \frac{-1}{e} + 1$$

$$F_{\mathbf{p}2} = \boldsymbol{\lambda}(0) = \frac{1}{e}$$