Robust Loop Shaping Controller Design for Spectral Models by Quadratic Programming

Gorka Galdos, Alireza Karimi and Roland Longchamp

***Abstract*— A quadratic programming approach is proposed to tune fixed-order linearly parameterized controllers for stable LTI plants represented by spectral models. The method is based on the shaping of the open-loop or closed-loop frequency functions in the Nyquist diagram. The quadratic error between a desired open loop transfer function and the actual open loop frequency function is minimized in the frequency domain subject to linear constraints guaranteeing stability and robustness margins by quadratic programming. Moreover, it is shown that the H infinity mixed sensitivity robust performance problem can be approximated by linear constraints and be integrated in the control design method. The method can directly consider multi-model as well as frequency-domain uncertainties. An application to a difficult benchmark problem illustrates the effectiveness of the proposed approach.**

I. Introduction

Most controller design methods are based on plant parametric models. A parametric model can be obtained either by first principle modeling or by parameter estimation techniques using measured data. However, it is usually too difficult or time consuming to obtain a parametric model based on physical laws. On the other hand, identification of parametric models is based on several a priori information and user choices like sampling period, time-delay, number of parameters in numerator and denominator of plant and noise model, optimal excitation etc. As a result, identification of frequency response functions or spectral models that need less a priori knowledge of plant has attracted the attention of many researchers in recent years [1]. In this type of models the information is not condensed into a small set of parameters thus avoiding errors of unmodeled dynamics that appear in parametric models.

Although identification of spectral models has been largely considered in literature, methods for controller design based on this type of models are rather limited. The first systematic controller design methods were based on graphical tools in the Bode diagram and are discussed in the classical textbooks for the design and analysis of control systems. The well-known Ziegler-Nichols tuning method based on only one point on the frequency response of the plant model (critical frequency) is still used to tune PID controllers in many practical situations. This method gives typically good responses to load disturbances for systems that can be approximated by a first order model with relatively small delay. However, for more complicated systems the results are not satisfactory. There are some attempts to modify the Ziegler-Nichols tuning algorithm which are reported in [2].

Recently, some iterative methods using specific points of the frequency response function have been developed. A PI controller tuning method achieving specified maximum sensitivity and phase margin using a Phase Locked Loop (PLL) identifier module for measuring some frequency points is presented in [3]. A PID controller tuning technique based on the minimization of the sum of square errors between the desired and measured specifications (gain margin, phase margin, maximum sensitivity and crossover frequency) has been proposed in [4], [5] based on simple relay experiments. A linear quadratic control criterion in frequency domain is minimized iteratively using only the spectral models of the closed-loop system in [6]. At each iteration the closed-loop system (with the controller from previous iteration) is excited with a reference signal and the gradient and Hessian of the criterion are estimated using the spectral models identified by the measured data. It should be noted that all of the mentioned iterative methods use the Gauss-Newton algorithm and consequently they converge to a local optimum of their criteria. Moreover, they need many experiments on the real system and cannot consider the multimodel uncertainty.

With new progress in numerical methods for solving convex optimization problems, new approaches for controller design with convex objectives and constraints have been developed. In [7] a convex optimization method for PID controller tuning by open-loop shaping in the frequency-domain is proposed. The infinity-norm of the difference between the desired open-loop transfer function and the achieved one weighted by a so-called target sensitivity function is minimized. It is shown based on the small gain theorem that if the infinity norm is less than 1 the nominal closed-loop system is stable. This is a sufficient condition and depends on the choice of the target sensitivity function. The condition for the stability of multiple models becomes more conservative as for each model a reasonable target sensitivity function should be available. In [8] a robust fixed-order controller design using linear programming is proposed. The main feature of this method is that the stability and some robustness margins are guaranteed by linear constraints in the Nyquist diagram and the method is applicable to multiple models as well. However, the performance specifications are limited to the choice of a lower bound for crossover frequency and minimization of the integral of the tracking error.

In this paper, the method proposed in [8] is improved.

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This research work is financially supported by the Swiss National Science Foundation under Grant No. 200020-107872.

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First, it is shown that by minimizing the two norm of the difference between the actual open-loop transfer function and the desired one under the stability constraints, the closed-loop performance can be ameliorated. Secondly the H infinity mixed sensitivity function shaping for robust performance is represented by linear constraints in the Nyquist diagram and then solved by quadratic programming. In this approach frequency-domain uncertainty originating from noise and numerical error in spectral models and multiple model uncertainty can be directly taken into account. The proposed method can be used for PID controllers as well as for higher order linearly parametrized controllers in discrete or continuous time. The effectiveness of the proposed approach is illustrated by application to a benchmark problem for robust digital controller design [9].

This paper is organized as follows: In Section II the class of models, controllers and the control objectives are defined. Section III introduces the linear stability margin and presents the open-loop shaping method. New linear constraints assuring the robust performance and the closed-loop shaping method are presented in Section IV. A solution to a benchmark problem satisfying all of the specifications is given in Section V. Finally, Section VI gives some concluding remarks.

II. Problem Formulation

A. Class of models

The class of linear time-invariant SISO systems with no pole in the right half plane (RHP) is considered. The model belongs to a set $\mathcal{P}$ that is the convex combination of m spectral models with a finite number of frequency points N:

$$\mathcal{P} = \left\{ \sum_{i=1}^{m} \lambda_i G_i(j\omega_k) : \sum_{i=1}^{m} \lambda_i = 1; k = 1, N \right\} \tag{1}$$

where λ_i are real positive numbers. This set represents multimodel uncertainty as well as unstructured frequency-domain uncertainty. Consider, for example, the following multiplicative uncertainty model :

$$G(j\omega_k) = \big[1 + \Delta(j\omega_k) W_2(j\omega_k)\big] G_0(j\omega_k) \ ; \quad k = 1, N \tag{2}$$

where G_0 is the nominal model, W_2 a weighting filter defined in N frequency points (not necessarily represented by a transfer function) and Δ an unknown stable system satisfying $\|\Delta\|_\infty < 1$. This uncertainty can be approximated by a polytopic model which takes the form of (1). The multiplicative uncertainty model at each frequency defines a circle of radius $|W_2 G_0|$ in the Nyquist diagram. At each frequency a convex polygon circumscribing this circle can be defined. Each vertex of these polygons define a model in the set $\mathcal{P}$ (see Fig. 2) .

B. Class of controllers

Linearly parameterized controllers are used in the proposed method :

$$K(s) = \rho^T \phi(s) \tag{3}$$

where

$$\rho^T = [\rho_1, \rho_2, \ldots, \rho_n] \tag{4}$$

$$\phi^T(s) = [\phi_1(s), \phi_2(s), \ldots, \phi_n(s)] \tag{5}$$

n is the number of controller parameters and $\phi_i(s), i = 1, n$ are transfer functions with no RHP pole chosen from a set of orthogonal basis functions. The main property of this parameterization is that every point on the Nyquist diagram of $K(j\omega)G_i(j\omega)$ can be written as a linear function of the controller parameters ρ:

$$\begin{aligned} K(j\omega_k)G_i(j\omega_k) = &\ \rho^T \phi(j\omega_k) G_i(j\omega_k) \\ = &\ \rho^T \mathcal{R}_i(\omega_k) + j\rho^T \mathcal{I}_i(\omega_k) \end{aligned} \tag{6}$$

where $\mathcal{R}_i(\omega_k)$ and $\mathcal{I}_i(\omega_k)$ are respectively the real and imaginary parts of $\phi(j\omega_k)G_i(j\omega_k)$.

C. Design Specifications

Some classical methods for controller tuning are based on either open-loop or closed-loop shaping. Open-loop shaping methods tune the controller so that the open-loop transfer function $L(s) = K(s)G(s)$ has an appropriate shape. Normally high magnitudes at low frequencies are required for load disturbance attenuation. A -20dB/decade slope is desired near the crossover frequency for appropriate robustness. Finally, small magnitudes at high frequencies is advantageous for a good attenuation of measurement noise or unmodeled dynamics perturbations. Practically, a pure integrator is a recommended shape satisfying these specifications.

On the other hand, closed-loop shaping methods tune the controller giving appropriate shapes to closed-loop transfer functions:

output sensitivity function	$\mathcal{S}(s) = [1 + L(s)]^{-1}$
input sensitivity function	$\mathcal{U}(s) = K(s)\mathcal{S}(s)$
complementary sensitivity function	$\mathcal{T}(s) = L(s)\mathcal{S}(s)$

Commonly, lower and upper bounds are fixed in these closed-loop transfer functions. For example, $\mathcal{S}(s)$ should be small at low frequencies and its supremum should not exceed a certain value to have good performance and robustness.

III. Robust Open Loop Shaping using Quadratic Programming

A linear programming approach has been recently proposed in [8]. In this approach a new stability margin is defined that guarantees a lower bound on classical margins (gain, phase and modulus margins). This linear margin can be ensured for all models in $\mathcal{P}$ by linear constraints in the parameters of a linearly parameterized controller. This margin is defined as follows. A straight line d_1 in the complex plane crossing the negative real axis between 0 and -1 with an angle α is considered (see Fig. 1). The new linear stability margin ℓ is the distance between the critical point -1 and the intersection point of d_1 with the negative real axis. If the Nyquist curve of the open-loop transfer function, $L(j\omega)$, lies on the right hand side of d_1, a lower bound on the classical robustness margins is ensured. Furthermore, in

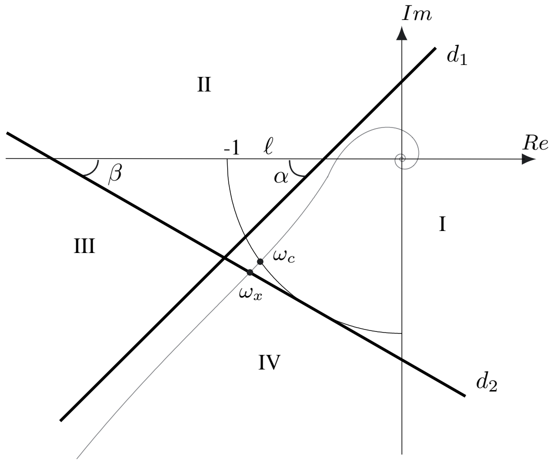

Fig. 1. Linear constraints for robustness and performance. New linear margin assuring classical robustness margins

order to consider a lower bound for the crossover frequency, we plot a second straight line d_2 with an angle β with the real axis tangent to the unit circle centered at the origin. The intersection of d_2 and $L(j\omega)$ defines ω_x, the lower bound for crossover frequency. The lines (d_1, d_2) divide the complex plane in 4 regions I, II, III and IV. By restricting the open-loop transfer function to lie in one of the regions for a frequency interval, closed-loop performance and robustness can be achieved by a set of linear constraints.

Two optimization problems are proposed and solved by linear programming in [8].

1) Minimizing the integrated error (IE) in order to optimize load disturbance rejection. This minimization leads to a linear objective function and therefore the control problem can be solved by linear programming. However, it should be used with caution as it may lead to oscillatory response because a norm of error is not minimized.
2) Maximizing the linear margin ℓ for robustness subject to a lower bound for the crossover frequency.

In this paper we introduce the open-loop shaping in the framework of the controller design method proposed in [8]. In this section, a desired open-loop transfer function $L_d(s)$ corresponding to an appropriate closed-loop behavior is chosen. $L_d(s)$ is typically chosen as an integrator, however, more complicated transfer functions also can be considered. The 2-norm of the difference between the open-loop transfer function $L(j\omega)$ and $L_d(j\omega)$ is minimized subject to the linear constraints assuring the stability margins. Minimizing the 2-norm leads to a quadratic cost function which can be solved by standard quadratic programming algorithms. When $L(j\omega)$ contains one integrator, $L(j\omega)$ should be in Region I or IV for all ω_k. This can be achieved by a simple optimization problem:

$$\begin{aligned}&\text{Minimize } \|\rho^T\phi(j\omega_k)G(j\omega_k) - L_d(j\omega_k)\|_2^2\\&\text{Subject to:}\\&\rho^T(\cot\alpha\,\mathcal{I}(\omega_k) - \mathcal{R}(\omega_k)) + \ell \le 1 \qquad \text{for all } \omega_k\end{aligned} \tag{7}$$

where $k = 1, \ldots, N$.

If $L(j\omega)$ contains two integrators, the constraints should be modified such that for low frequencies $L(j\omega)$ can be located in Region III. The optimization problem can be formulated as:

$$\begin{aligned}&\text{Minimize } \|\rho^T\phi(j\omega_k)G(j\omega_k) - L_d(j\omega_k)\|_2^2\\&\text{Subject to:}\\&\rho^T(\cot\alpha\,\mathcal{I}(\omega_k) - \mathcal{R}(\omega_k)) + \ell \le 1 \quad \text{for } \omega_k > \omega_x\\&\rho^T(\cos\beta\,\mathcal{I}(\omega_k) + \sin\beta\mathcal{R}(\omega_k)) \le 1 \quad \text{for } \omega_k \le \omega_x\end{aligned} \tag{8}$$

for $k = 1, \ldots, N$. This optimization problem can be easily extended to the polytopic models represented by $\mathcal{P}$ in (1). For example, in the case that $L(j\omega)$ contains only one integrator we have :

$$\begin{aligned}&\text{Minimize } \quad \sum_{i=1}^{m} \left\|\rho^T\phi(j\omega_k)G_i(j\omega_k) - L_{d_i}(j\omega_k)\right\|_2^2\\&\text{Subject to:}\\&\rho^T\big(\cot\alpha\,\mathcal{I}_i(\omega_k) - \mathcal{R}_i(\omega_k)\big) + \ell \le 1 \quad \text{for all } \omega_k\end{aligned} \tag{9}$$

for $i = 1, m$, where L_{d_i} is the desired open-loop transfer function for G_i.

IV. Robust Closed-Loop shaping using linear constraints

In the H_∞ control framework, nominal performance is represented by $\|W_1\mathcal{S}\|_\infty < 1$ where $W_1(s)$ is a known weighting filter. On the other hand, robust stability for multiplicative uncertainty is assured iff $\|W_2\mathcal{T}\|_\infty < 1$. It is shown in [10] that, for uncertain systems, the necessary and sufficient condition for robust performance is :

$$\||W_1\mathcal{S}| + |W_2\mathcal{T}|\|_\infty < 1 \tag{10}$$

The graphical interpretation of this constraint in Nyquist diagram is shown in Figure 2. For all ω_k, the plant's uncertainty circle of radius $|W_2(j\omega_k)L_0(j\omega_k)|$, with $L_0 = K(j\omega_k)G_0(j\omega_k)$, should not intersect the circle with radius $|W_1(j\omega_k)|$ centered at -1. This constraint is a non-convex constraint on the controller parameters. However, the circle of radius $|W_1(j\omega_k)|$ centered at -1 can be approximated by a tangential d_k line with an angle α_k between the negative real axis and d_k. This gives a convex constraint for each frequency ω_k. If the circle of radius $|W_2(j\omega_k)L_0(j\omega_k)|$ centered at $L_0(j\omega_k)$ is located on the right hand side of d_k for all frequencies ω_k, then robust performance is assured. This can be presented by a linear constraint for $\alpha_k \in [0, 90°]$:

$$\rho^T\big(\cos\alpha_k\mathcal{I}(\omega_k) - \sin\alpha_k\mathcal{R}(\omega_k)\big) + |W_1(j\omega_k)| \le \sin\alpha_k$$

For low frequencies ($\omega_k < \omega_c$) where typically high values for $|W_1(j\omega_k)|$ are given, $\alpha_k = 0°$ can be chosen (d_k is thus horizontal). For high frequencies ($\omega_k > \omega_c$) $|W_1(j\omega_k)|$ is usually small, and $\alpha_k = 90°$ is a good choice (d_k being vertical). The use of a progressive variation for α_k

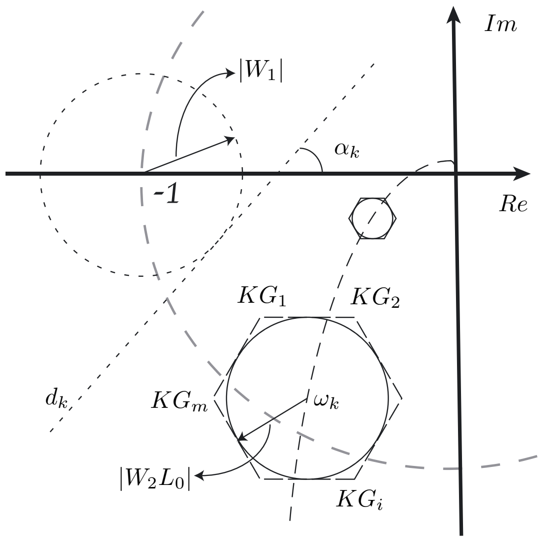

Fig. 2. Multiplicative uncertainty model approximation with a set of discrete models. Graphical interpretation of the robust performance constraint for a given frequency ω_k. Linear robustness performance constraints for low frequencies and high frequencies.

in the neighborhood of ω_c could be useful. This adds less conservatism but the solution could be quite sensitive to the variation rate.

As mentioned in Section II the multiplicative uncertainty is approximated by a set of models at each frequency ω_k giving a linear constraint for each $L_i(j\omega_k) = K(j\omega_k)G_i(j\omega_k)$ point. If all $K(j\omega_k)G_i(j\omega_k)$ points for all ω_k lie at the right hand side of it's corresponding d_k line in the Nyquist diagram, then robust performance is assured.

The objective function of the optimization is to minimize the 2-norm of the error between $L_0(j\omega)$ and $L_d(j\omega)$ subject to linear robust performance constraints. $L_d(s)$ can be defined as an integrator or from the given $W_1(s)$ by $L_d(s) = W_1(s) - 1$ since $W_1(s)$ is typically chosen as the inverse of the target output sensitivity function. When $L_0(j\omega)$ contains only one integrator, the optimization can be defined as follows:

$$\begin{array}{ll} \text{Minimize } \|\rho^T\phi(j\omega_k)G(j\omega_k) - L_d(j\omega_k)\|_2^2 & \\ \text{Subject to:} & \\ \rho^T\mathcal{I}(\omega_k) \leq -|W_1(j\omega_k)| & \text{for } \omega_k \leq \omega_c \\ \rho^T\mathcal{R}(\omega_k) \geq |W_1(j\omega_k)| - 1 & \text{for } \omega_k > \omega_c \end{array} \tag{11}$$

This optimization problem can be applied to the case that the system has both multimodel uncertainty and multiplicative uncertainty. In this case, only the number of constraints is increased and the cost function will change to sum of the two norms of difference between desired and actual open-loop transfer functions. In the next section, the proposed approach is applied to a benchmark problem with multimodel uncertainty.

V. Solution to the flexible transmission benchmark

The design procedure is applied to the flexible transmission benchmark proposed in [9]. A number of researchers have addressed this benchmark problem. Some of the solutions were presented at the 1995 European Control Conference in Rome and published in a special issue (Vol. 1, No. 2, 1995) of the European Journal of Control. Many other contributions to this benchmark problem can be found in literature. To the best of our knowledge, only two controllers meet 100% of specifications up to now. The first one is a QFT controller with 20 parameters in the controller [11] which requires a good expertise for designing different lead-lag filters involved in the QFT method. The second one uses convex optimization methods for pole placement and sensitivity function shaping [12] and ends up with a controller with 16 parameters. The parametric or multimodel uncertainty cannot be considered in this approach. Several H_∞ controllers are proposed for this benchmark [13], [14], [15] that result in very high-order controllers and do not satisfy all of the benchmark specifications. The multimodel uncertainty is considered as unstructured uncertainty and the weighting filters are designed iteratively by a trial and error approach.

The benchmark problem consists of designing a robust digital controller for a flexible transmission system in three different operating points (no load, half load and full load). Three discrete time transfer functions are provided in the benchmark :

$$G_i(q^{-1}) = \frac{q^{-d}B_i(q^{-1})}{A_i(q^{-1})} \qquad i = 1, 2, 3$$

where q^{-1} is the backward shift operator and $d = 2$ is the integer number of sampling periods contained in the plant pure time delay with 20Hz as sampling frequency. The corresponding identified and validated models are:

Unloaded model:

$$\begin{aligned} A_1(q^{-1}) &= 1 - 1.14833q^{-1} + 1.58939q^{-2} \\ &\quad -1.31608q^{-3} + 0.88642q^{-4} \\ B_1(q^{-1}) &= 0.28261q^{-1} + 0.50666q^{-2} \end{aligned}$$

Half loaded model:

$$\begin{aligned} A_2(q^{-1}) &= 1 - 1.99185q^{-1} + 2.20265q^{-2} \\ &\quad -1.84083q^{-3} + 0.89413q^{-4} \\ B_2(q^{-1}) &= 0.1027q^{-1} + 0.18123q^{-2} \end{aligned}$$

Fully loaded model:

$$\begin{aligned} A_3(q^{-1}) &= 1 - 2.09679q^{-1} + 2.31962q^{-2} \\ &\quad -1.93353q^{-3} + 0.87129q^{-4} \\ B_3(q^{-1}) &= 0.06408q^{-1} + 0.10407q^{-2} \end{aligned}$$

A controller should be designed to satisfy the following specifications: rise time (90% of the final value in less than 1s), overshoot (less than 10%), rejection of 90% of the output disturbance filtered by $1/A_i$ in less than 1.2s, perfect rejection of a constant disturbance, disturbance attenuation at low frequency, maximum output sensitivity function $\mathcal{S}$ less than 6dB (modulus margin greater than 0.5), a delay margin of at least 40 ms and a maximum value of less than 10dB for the input sensitivity function $\mathcal{U}$ at high frequencies (between 8 to 10 Hz).

A discrete-time two-degree of freedom polynomial form RST controller is to be designed. The canonical form of the RST controller is given by:

$$S(q^{-1})u(t) = T(q^{-1})r(t) - R(q^{-1})y(t) \qquad (12)$$

where $u(t)$ is the plant input, $y(t)$ the plant output and $r(t)$ the desired reference.

We consider the following linearly parameterized controller :

$$\begin{aligned} R(q^{-1}) &= (\rho_0 + \rho_1 q^{-1} + \ldots + \rho_6 q^{-6})(1 + q^{-1}) \\ S(q^{-1}) &= (1 - q^{-1}) \\ T(q^{-1}) &= t_0 \end{aligned}$$

The fixed term $(1 - q^{-1})$ in $S(q^{-1})$ satisfies the integral action of the controller and disturbance attenuation at low frequencies and the fixed term $(1 + q^{-1})$ in $R(q^{-1})$ reduces the input sensitivity function $\mathcal{U}$ at high frequencies. In addition, $\alpha = 80°$ and $\ell = 0.5/\sin\alpha$ are chosen in order to assure the specified bound on the maximum of the output sensitivity function (modulus margin of 0.5). The time-domain performances (rise time, overshoot and disturbance rejection time) are tuned using the desired open-loop transfer functions $L_{d_i}(s) = \omega_{c_i}/s$. Note that since the design is carried out in frequency domain the desired open-loop transfer function can be defined either in continuous- or in discrete-time. The delay margin cannot be transformed into a linear constraint. However, this specification is naturally met when other specifications are satisfied.

The controller is tuned in two steps. In the first step the feedback controller is tuned based on the proposed method. In the second step the unique parameter in $T(q^{-1})$ is taken equal to sum of the parameters of $R(q^{-1})$ giving a unit gain to the closed-loop system. The frequency response of the three models are computed at N=8000 equally spaced frequency points between 0 and 10Hz (Nyquist frequency).

The desired crossover frequency $\omega_{c_i} = 1.2$ rad/s was chosen for all models giving a controller satisfying all the specifications except the disturbance rejection time. Looking at the frequency response of the closed-loop transfer function between disturbance and output, $\mathcal{S}_i/A_i$, a large peak greater than 35dB is observed for three closed-loop systems. Limiting the maximum of $\mathcal{S}_i/A_i$ can help reducing the disturbance rejection time. This constraint can be represented by :

$$\left\| \frac{\mathcal{S}_i}{A_i} \right\|_\infty < \gamma$$

and be considered in the proposed approach by taking $W_{1_i} = 1/(\gamma A_i)$. The design variable γ is fixed to $10^{27/20}$ (equal to 27dB). Then the following optimization problem is formulated:

$$\text{Minimize} \quad \sum_{i=1}^{3} \left\| \rho^T \phi(j\omega_k) G_i(j\omega_k) - \omega_{c_i}/(j\omega_k) \right\|_2^2$$

Subject to:

$$\begin{aligned} &\rho^T \mathcal{I}_i \leq -|\gamma A_i^{-1}(j\omega_k)| && \text{for } \omega_k \leq \omega_{c_i} \\ & && i = 1,2,3 \\ &\rho^T \mathcal{R}_i \geq |\gamma A_i^{-1}(j\omega_k)| - 1 && \text{for } \omega_k > \omega_{c_i} \\ & && i = 1,2,3 \\ &\rho^T (\cot\alpha\, \mathcal{I}_i - \mathcal{R}_i) + \ell \leq 1 && \text{for all } \omega_k \\ & && i = 1,2,3 \end{aligned}$$

where

$$\begin{aligned} \mathcal{R}_i &= Re[\phi(j\omega_k) G_i(j\omega_k)] \\ \mathcal{I}_i &= Im[\phi(j\omega_k) G_i(j\omega_k)] \end{aligned}$$

and

$$\phi(j\omega_k) = \frac{1 + e^{-j\omega_k h}}{1 - e^{-j\omega_k h}} [1, e^{-j\omega_k h}, \ldots, e^{-6j\omega_k h}]$$

and $h = 0.05$ is the sampling period. Choosing $\omega_{c_i} = 1.2$ rad/s for all models gives a controller satisfying almost all the specifications but the disturbance rejection time for the unloaded case (1.24 s instead of less than 1.2 s). Increasing the crossover frequency of the desired open-loop transfer function for the unloaded system to 2.6 rad/s, a controller satisfying 100% of the specifications is obtained with these parameters :

$$\begin{aligned} R(q^{-1}) &= (1 + q^{-1})(0.4485 - 1.7163 q^{-1} \\ &\quad + 2.9159 q^{-2} - 3.2385 q^{-3} + 2.6753 q^{-4} \\ &\quad - 1.4738 q^{-5} + 0.4126 q^{-6}) \\ S(q^{-1}) &= (1 - q^{-1}) \\ T(q^{-1}) &= 0.0474 \end{aligned}$$

Figures 3 and 4 show that the specifications on the input sensitivity function $\mathcal{U}_i$ and output sensitivity function $\mathcal{S}_i$ are satisfied for the three models. Figure 5 shows the step the disturbance rejection responses. The details of achieved specifications are shown in Table I.

TABLE I
SPECIFICATIONS OF THE CONTROLLER

Specification	No load	Half load	Full load
Rise Time [s]	0.923	0.895	0.802
Overshoot[%]	6.1	7.8	6.7
Disturbance reject. [s]	1.16	1.17	1.00
Maximum $\mathcal{S}$ [dB]	5.86	5.48	5.96
Delay Margin [ms]	76	159	338
Maximum $\mathcal{U}$ [dB]	9.59	9.04	8.95

VI. CONCLUSION

Robust fixed-order controller design for spectral models is formulated as a quadratic optimization problem. The proposed method is based on frequency loop shaping in the Nyquist diagram. The classical robustness margins and robust performance criterion are represented as linear constraints in the Nyquist diagram. The control objective is

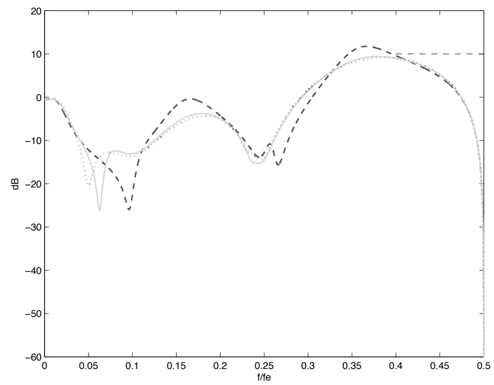

Fig. 3. $\mathcal{U}$ of Unloaded (dashed, blue), Half loaded (solid, green) and Fully loaded (dashed-dotted, red) systems.

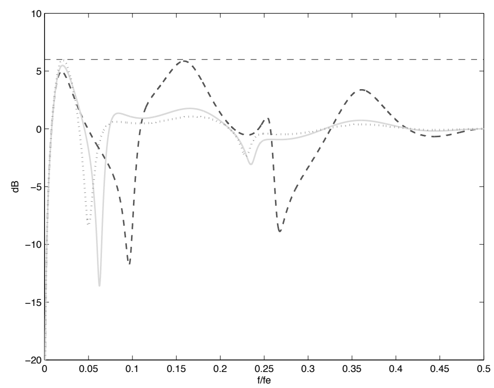

Fig. 4. $\mathcal{S}$ of Unloaded (dashed, blue), Half loaded (solid, green) and Fully loaded (dashed-dotted, red) systems.

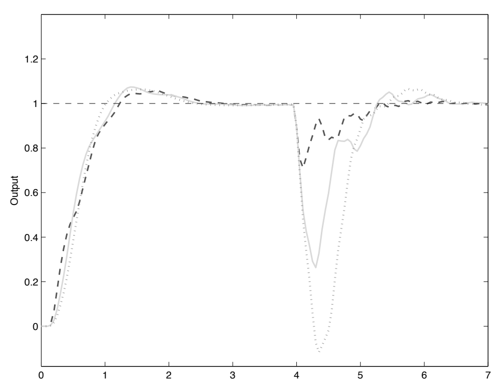

Fig. 5. Step and perturbation rejection of Unloaded (dashed, blue), Half loaded (solid, green) and Fully loaded (dashed-dotted, red) systems. The dashed lines show the limits where the responses should be inside.

to minimize the quadratic error between a desired open-loop transfer function and the achieved one subject to the constraints mentioned before. The few design variables are directly related to robustness (linear margin ℓ) and performance (crossover frequency of the desired transfer function) of the closed-loop system. Multimodel uncertainty can be taken into account straightforwardly. The method is very simple to apply and requires only the frequency response of the plant. The method is very appropriate for PID controller design, as well as being applicable to higher-order linearly parameterized controllers in discrete or continuous time. An application of the proposed method to a challenging benchmark problem illustrates the effectiveness of the proposed method. The multimodel uncertainty of this problem is directly considered in the approach and using a few design parameters a fixed-order robust controller is obtained that satisfies all the specifications with only 10 parameters in the controller. This is the smallest order controller designed that meets 100% of specifications with a very simple and straightforward approach.

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