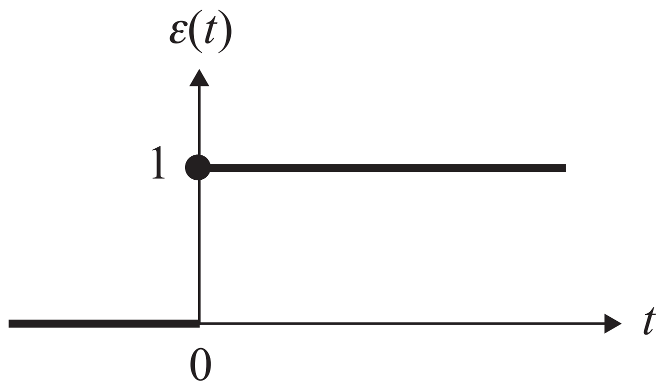
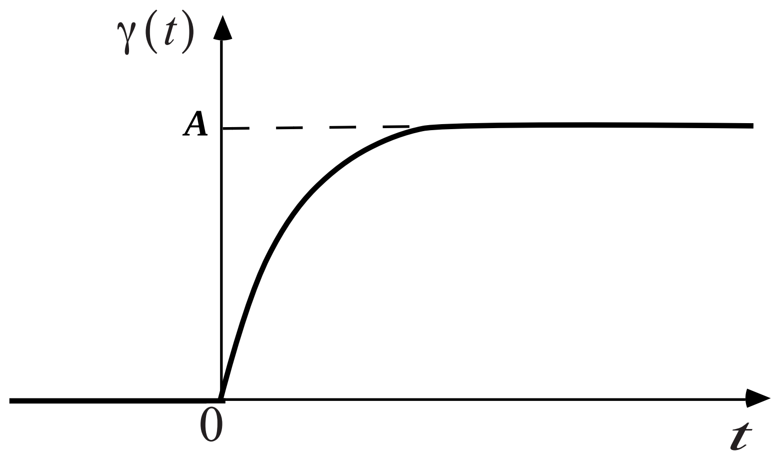
Echelon unité



$$\varepsilon(t) = \begin{cases} 0 & \text{pour } t < 0 \\ 1 & \text{pour } t \geq 0 \end{cases}$$

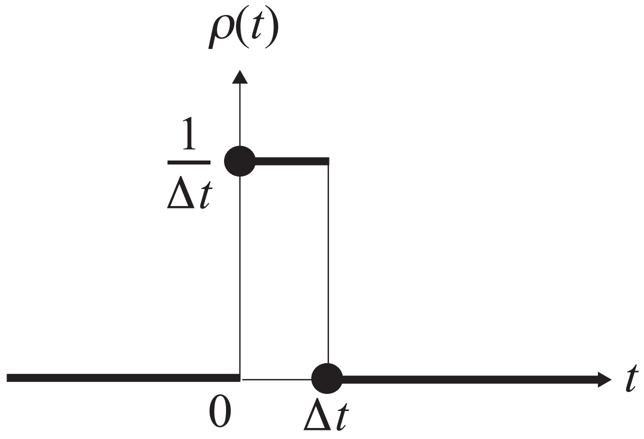
Réponse indicielle



Impulsion de Dirac

Impulsion rectangulaire

$$\rho(t) = \begin{cases} 0 & t < 0 \\ 1/\Delta t & t \in [0, \Delta t) \\ 0 & t \geq \Delta t \end{cases}$$

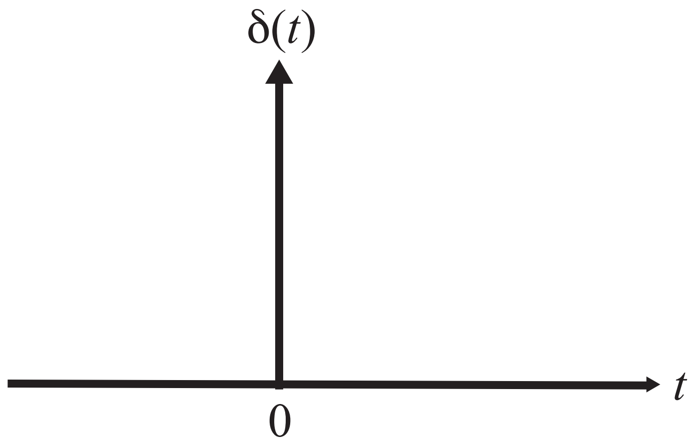


Impulsion de Dirac

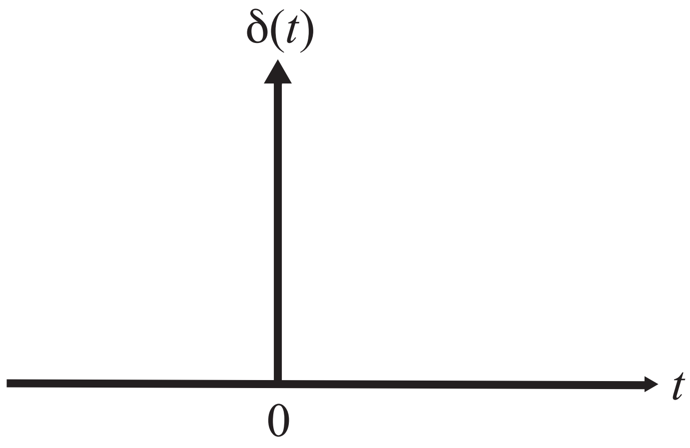
$$\delta(t) = \lim_{\Delta t \to 0} \rho(t)$$

$$\delta(t) = 0 \quad \text{pour} \quad t \neq 0$$

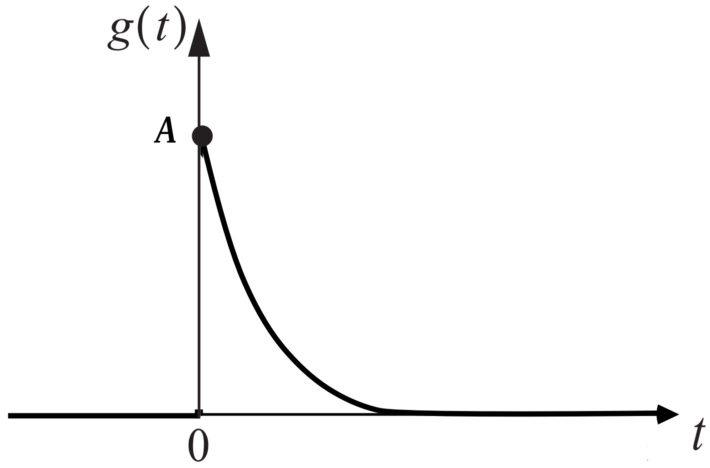
$$\int_{-\infty}^{\infty} \delta(t) \mathrm{d}t = 1$$



Impulsion de Dirac

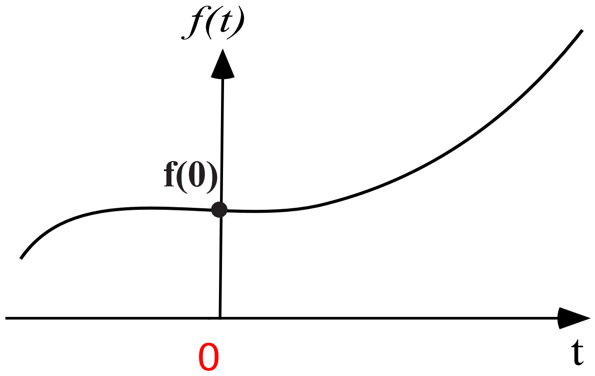


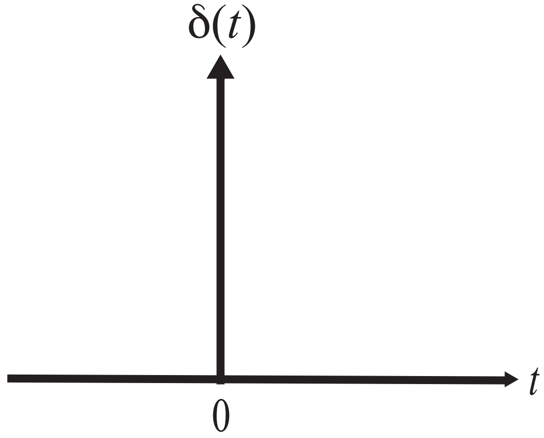
Réponse impulsionelle



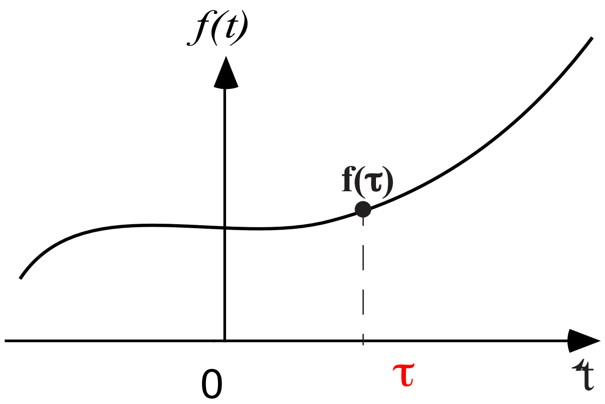
Propriétés de l'impulsion de Dirac

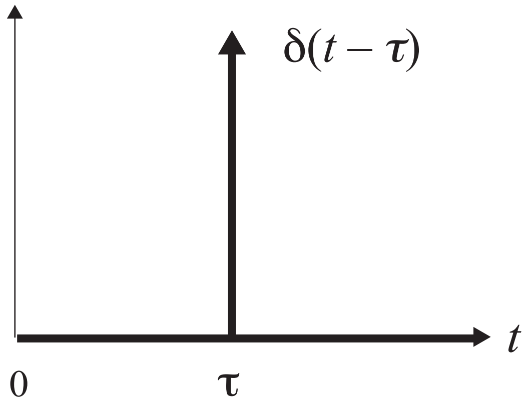
$$\int_{-\infty}^{\infty} \delta(t) f(t)\,\mathrm{d}t = f(0)$$





$$\int_{-\infty}^{\infty} \delta(t-\tau) f(t)\,\mathrm{d}t = f(\tau)$$

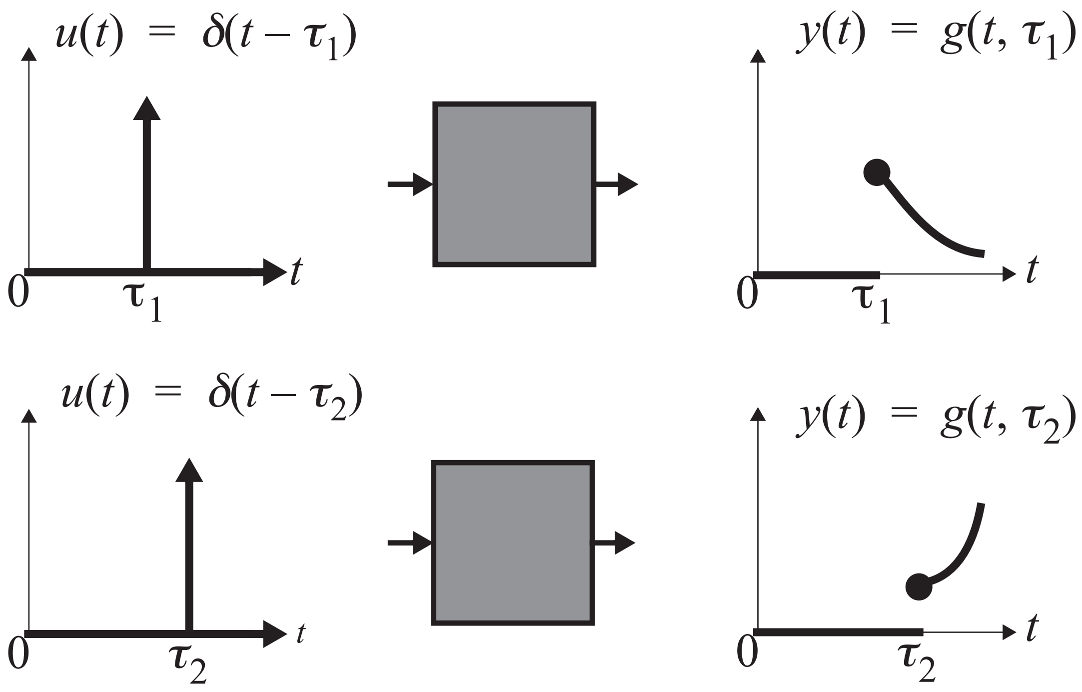




Réponse impulsionnelle

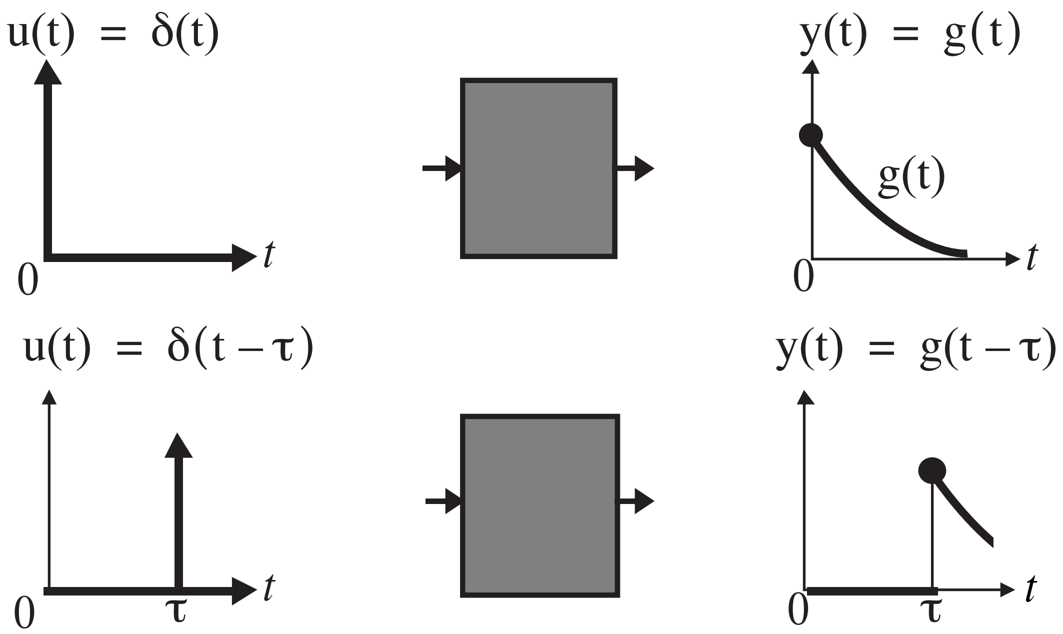
La réponse d'un système (au repos) à l'impulsion de Dirac $\delta(t-\tau)$ est notée $g(t, \tau)$

Système non stationnaire



Réponse impulsionnelle (suite)

Système stationnaire



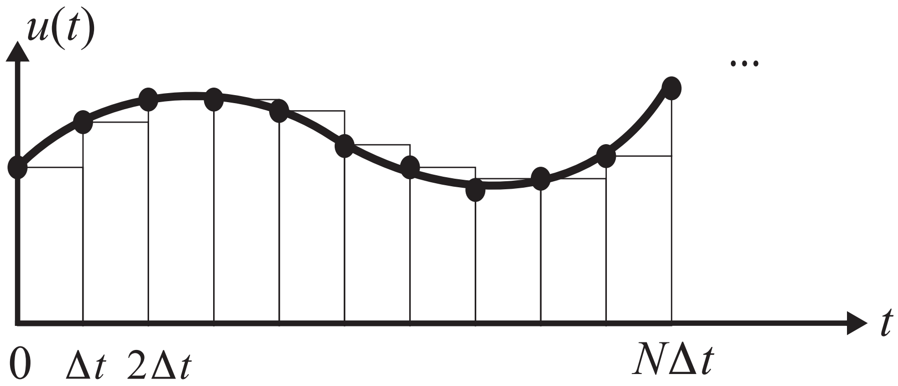
$$g(t, \tau) = g(t + \alpha, \tau + \alpha) \qquad \forall\alpha$$

Pour $\alpha = -\tau$

$$g(t, \tau) = g(t - \tau, 0) \equiv g(t - \tau)$$

Représentation d'un signal quelconque

$$u(t) \cong \sum_{i=0}^{N-1} u(i\Delta t)\rho(t - i\Delta t)\Delta t$$



Pour $\Delta t \to 0 \qquad (N \to \infty)$

$$u(t) = \lim_{\Delta t \to 0} \sum_{i=0}^{(t^*/\Delta t)-1} u(i\Delta t)\rho(t - i\Delta t)\Delta t$$

$$= \int_0^{t^*} u(\tau)\delta(t - \tau)d\tau$$

Si $u(t)$ est défini pour $t \in (-\infty, \infty)$ $\longrightarrow$ $u(t) = \int_{-\infty}^{\infty} u(\tau)\delta(t - \tau)d\tau$

Produit de convolution

- $u(t) = \int_{-\infty}^{\infty} u(\tau)\delta(t-\tau)\ d\tau$
- Réponse à l'impulsion $u(\tau)\delta(t-\tau)$: $u(\tau)g(t,\tau)$
- Système linéaire --> sommation : $y(t) = \int_{-\infty}^{\infty} u(\tau)g(t,\tau)\ d\tau$

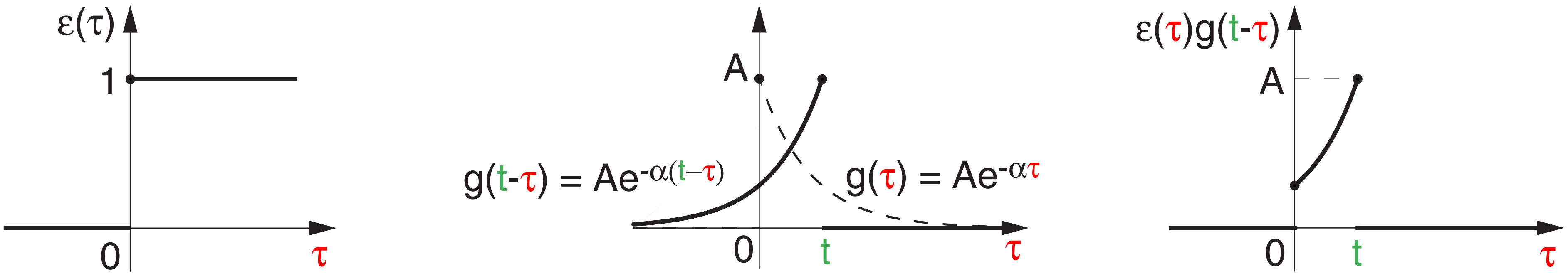
Pour un système linéaire $y(t) = \int_{-\infty}^{\infty} u(\tau)g(t,\tau)d\tau \equiv u(t)*g(t) \equiv g(t)*u(t)$

Pour un système lscr $y(t) = \int_{0}^{t} u(\tau)\,g(t-\tau)\,d\tau$

Exemple de convolution

Système dynamique lscr : $g(t)$

Réponse indicielle : $\gamma(t) = \varepsilon(t) * g(t) = \int_0^t \varepsilon(\tau) g(t-\tau) d\tau$

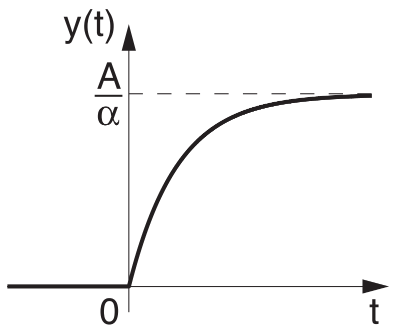


Exemple de convolution

***t* < 0** $$\int_0^t 0\,d\tau = 0$$

***t* ≥ 0** $$\int_0^t A e^{-\alpha(t-\tau)} d\tau = A e^{-\alpha t} \int_0^t e^{\alpha\tau} d\tau = A e^{-\alpha t} \frac{1}{\alpha} e^{\alpha\tau} \Big|_0^t$$

$$= \frac{A}{\alpha} e^{-\alpha t}(e^{\alpha t} - 1) = \frac{A}{\alpha}(1 - e^{-\alpha t})$$



Représentation entrée-sortie

$$y(t) = \int_0^t u(\tau) g(t - \tau) d\tau$$

Réponse indicielle : $u(t) = \varepsilon(t)$

$$\gamma(t) = \int_0^t \varepsilon(\tau) g(t-\tau) d\tau = \int_0^t g(t-\tau) d\tau = \int_0^t g(t') dt'$$

$t' = t - \tau$

$$\Rightarrow \qquad g(t) = \frac{d}{dt}\gamma(t)$$