Commande de procédés, Test May 2015

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Problem 1 (Modeling) [2 points]

Consider a continuous stirred tank reactor of constant volume V in which the reaction 2A→B takes place. The inlet and outlet volumetric flowrates are equal to q. The inlet concentrations of the chemical species A and B are denoted as $c_{A,e}$ and $c_{B,e}$, respectively, whereas the inlet temperature is denoted as T_e. Moreover, c_A and c_B represent the concentrations of A and B in the reactor and in the outlet, respectively, and T represents the temperature in the reactor and in the outlet.

The density ρ and heat capacity c_p of the reaction mixture are constant. The reactor exchanges heat with its wall, which has an temperature T_w and is characterized by a heat transfer coefficient U (per unit of area) and an exchange area A. Besides, a heating coil supplies a heating power P to the reactor. The enthalpy of reaction (per number of moles of product) is equal to ΔH_1.

The reaction rate (per unit of volume) is $k_1 c_A^2$. According to the Arrhenius' law, the rate constant k_1 in a nonisothermal reactor is given by $A_1 \exp\left(\frac{-E_{a,1}}{RT}\right)$, where A_1 is the pre-exponential factor, $E_{a,1}$ is the activation energy, and R is the ideal gas constant.

1. Write the dynamic model for this reactor.
2. Identify the dependent and independent variables.
3. What is the order of this system? What is its *minimal* order?

Solution:

1. From the mole balance for A

$$V\dot{c}_A = q\left(c_{A,e} - c_A\right) + V\left(-2A_1 \exp\left(\frac{-E_{a,1}}{RT}\right) c_A^2\right),$$

one can obtain the dynamic equation for c_A:

$$\dot{c}_A = \frac{q}{V}\left(c_{A,e} - c_A\right) - 2A_1 \exp\left(\frac{-E_{a,1}}{RT}\right) c_A^2.$$

A similar mole balance for B yields the following dynamic equation for c_B:

$$\dot{c}_B = \frac{q}{V}\left(c_{B,e} - c_B\right) + A_1 \exp\left(\frac{-E_{a,1}}{RT}\right) c_A^2.$$

Finally, from the heat balance

$$V\rho c_p \dot{T} = q\rho c_p\left(T_e - T\right) + V\left(-\Delta H_1 A_1 \exp\left(\frac{-E_{a,1}}{RT}\right) c_A^2\right) + UA\left(T_w - T\right) + P,$$

it is possible to write the dynamic equation for T:

$$\dot{T} = \frac{q}{V}\left(T_e - T\right) + \frac{(-\Delta H_1)A_1 \exp\left(\frac{-E_{a,1}}{RT}\right) c_A^2}{\rho c_p} + \frac{UA}{V\rho c_p}\left(T_w - T\right) + \frac{P}{V\rho c_p}.$$

2. The dependent variables are c_A, c_B and T, whereas the independent variables are q, $c_{A,e}$, $c_{B,e}$, T_e, T_w, and P.

3. The order of this system is 3, because there are 3 state variables. The minimal order of the system is also 3, because there is one independent reaction, one independent inlet/outlet flowrate and one independent heat exchange.

Problem 2 (Linearization) [1 point]

Consider the following dynamic equations that describe the concentrations of substrate S and of product P in a continuous stirred tank reactor:

$$\dot{c}_S = \frac{1}{\theta}\left(c_{S,e} - c_S\right) - V_{max}\frac{c_S}{K_m + c_S},$$
$$\dot{c}_P = -\frac{1}{\theta}c_P + V_{max}\frac{c_S}{K_m + c_S},$$

with the following numerical values: $\theta = 10$ min, $V_{max} = 0.8$ kmol m^{-3} min^{-1}, $K_m = 2$ kmol m^{-3}.

1. Calculate the values $\bar{c}_S$ and $\bar{c}_P$ at steady state, knowing that $\bar{c}_{S,e} = 6$ kmol m^{-3}.
2. Linearize the system around the point calculated in point 1.

Solution:

1. At steady state, $\dot{c}_S = \dot{c}_P = 0$. If the constant values are replaced in the dynamic equations, the system of equations to be solved is

$$\begin{cases} 0 = 0.1\left(6 - \bar{c}_S\right) - 0.8\frac{\bar{c}_S}{2+\bar{c}_S} \\ 0 = -0.1\bar{c}_P + 0.8\frac{\bar{c}_S}{K_m+\bar{c}_S}, \end{cases} \Leftrightarrow \begin{cases} 0 = 0.1\left(6 - \bar{c}_S\right)\left(2 + \bar{c}_S\right) - 0.8\bar{c}_S = 1.2 - 0.4\bar{c}_S - 0.1\bar{c}_S^2 \\ \bar{c}_P = 8\frac{\bar{c}_S}{2+\bar{c}_S}, \end{cases}$$

   whose solution is the following:

$$\begin{cases} \bar{c}_S = \frac{0.4\pm\sqrt{0.4^2+4\times0.1\times1.2}}{-0.2} = -2 \pm 4 \\ \bar{c}_P = 8\frac{\bar{c}_S}{2+\bar{c}_S} \end{cases} \Rightarrow \begin{cases} \bar{c}_S = 2 \text{ kmol m}^{-3} \\ \bar{c}_P = 8\frac{2}{2+2} = 4 \text{ kmol m}^{-3} \end{cases}.$$

2. In the dynamic model given above, the only (nonlinear) term that needs to be linearized is $\frac{c_S}{K_m+c_S}$. It is known that

$$\left[\frac{\partial}{\partial c_S}\left(\frac{c_S}{K_m + c_S}\right)\right]_{c_S=\bar{c}_S} = \frac{K_m}{\left(K_m + \bar{c}_S\right)^2}.$$

   Then, if the dynamic model is linearized and written in deviation variables, it becomes

$$\begin{cases} \delta\dot{c}_S = \frac{1}{\theta}\left(\delta c_{S,e} - \delta c_S\right) - V_{max}\frac{K_m}{(K_m+\bar{c}_S)^2}\delta c_S \\ \delta\dot{c}_P = -\frac{1}{\theta}\delta c_P + V_{max}\frac{K_m}{(K_m+\bar{c}_S)^2}\delta c_S \end{cases}.$$

Problem 3 (Laplace transform) [1 point]

Consider the following dynamic system:

$$\ddot{y} + 6\dot{y} + 9y \quad = \quad u(t)$$

with the input $u(t) = e^{-3t}$ and the initial conditions $y(0) = 1, \dot{y}(0) = -3$.

1. Compute $y(t)$ using the Laplace transform.
2. What is the static gain of this system?

Solution:

1. Applying the Laplace transforms to the two equations:

$$s^2Y(s) - s + 3 + 6[sY(s) - 1] + 9Y(s) = U(s)$$

therefore,

$$\begin{aligned}(s^2 + 6s + 9)Y(s) &= U(s) + (s + 3), \\ Y(s) &= \frac{1}{(s+3)^2}U(s) + \frac{1}{s+3}.\end{aligned}$$

$$U(s) = \frac{1}{s+3}.$$

Therefore,

$$Y(s) = \frac{1}{(s+3)^3} + \frac{1}{s+3}.$$

Hence,

$$y(t) = e^{-3t}\left[\frac{t^2}{2} + 1\right]\varepsilon(t).$$

2.

$$G(s) = \frac{Y(s)}{U(s)} = \frac{1}{(s+3)^2},$$

The static gain is

$$K = \lim_{s \to 0} G(s) = \frac{1}{9}.$$

Problem 4 (Time responses and control) [1 point]

Consider the systems described by the following transfer functions:

(i) $G(s) = \frac{\exp(-1.5s)}{5s}$

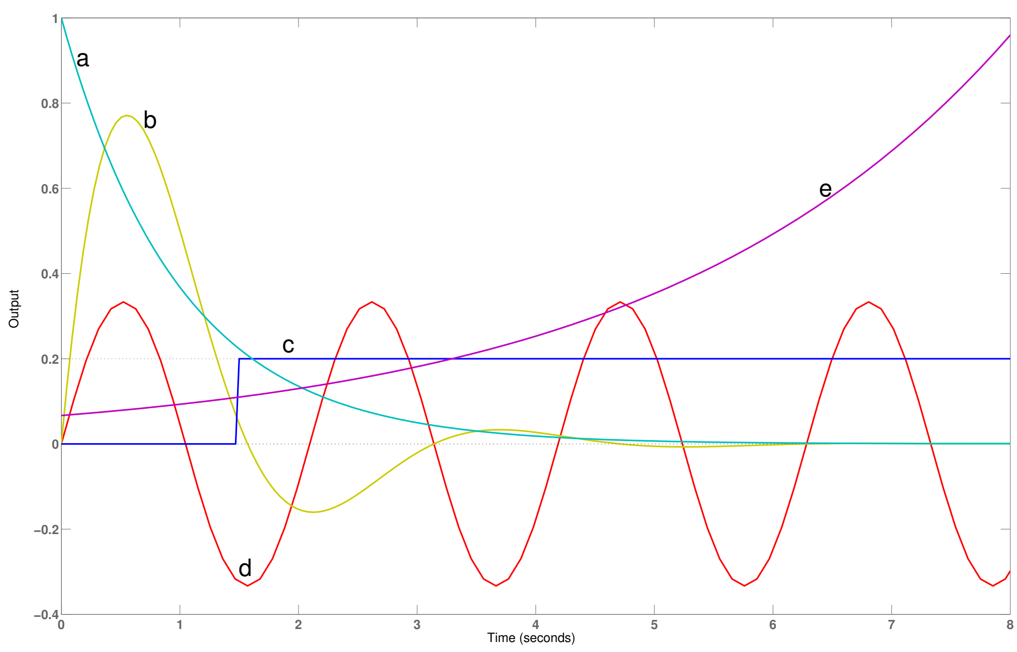
(ii) $G(s) = \frac{1}{s^2+9}$

(iii) $G(s) = \frac{1}{s+1}$

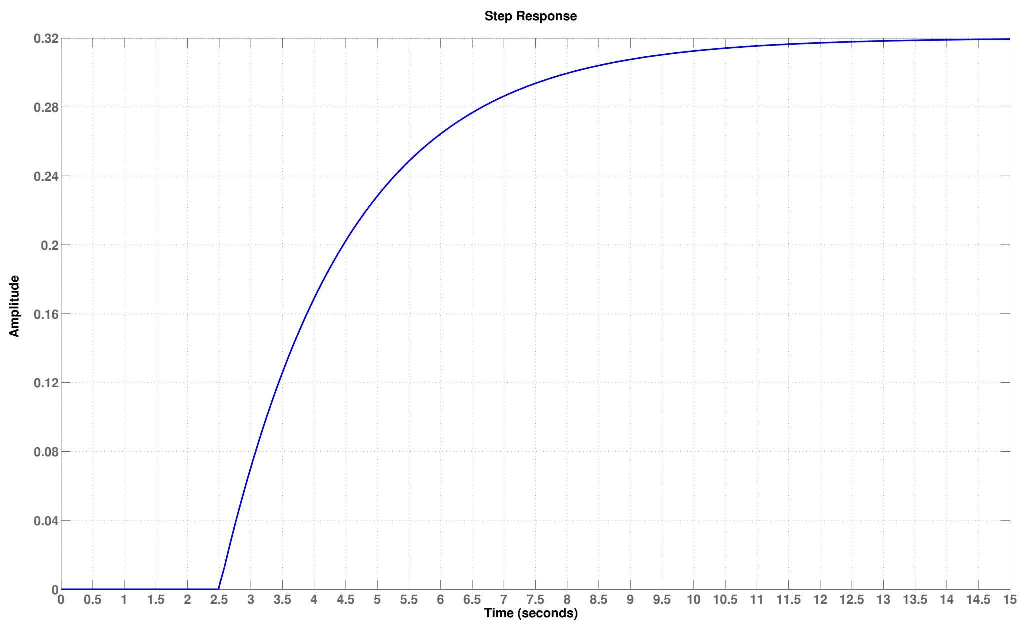
(iv) $G(s) = \frac{0.3(s-1)}{-3s+1}$

(v) $G(s) = \frac{3}{s^2+2s+5}$

1. Match these transfer functions with the impulse responses shown in the following figure:



2. The graph below describes the unit step response for a system. Find the transfer function and design a PID controller for this system.



Solution:

1) We can match the impulse response based on the computed poles.

a)The impulse response is decaying to zero with no delay and no oscillations. This corresponds to a first order system with a stable pole, namely, $G(s) = \frac{1}{s+1}$

b) The impulse response is oscillatory, therefore, the poles of the system have non-zero imaginary part. The maximum amplitude of oscillations decay with time. Hence, the poles of the system have negative real part . $G(s) = \frac{3}{s^2+2s+5}$

c) The impulse response has a constant amplitude with time. This behaviour is typical of the system with pole at the origin. Also, there is delay of 1.5 seconds in the response. $G(s) = \frac{\exp(-1.5)}{5s}$

d) The impulse response is oscillatory, therefore, the poles of the system have non-zero imaginary part. The maximum amplitude of oscillations does not decay with time and remains constant. Hence, the system has no real part. $G(s) = \frac{1}{s^2+9}$

e) The impulse response has no oscillations and the amplitude increases with time. This behaviour corresponds to the system with unstable pole with no imaginary part. $G(s) = \frac{0.3(s-1)}{-3s+1}$

2) The step response corresponds to the first order system with time delay.

$$G(s) = \frac{Ke^{-\theta s}}{\tau s+1}$$

Next, we find the parameters K, θ, and τ.

The value of final response is 0.32. Therefore,

$$K = 0.32.$$

The system responds after 2.5 seconds. Hence,

$$\theta = 2.5.$$

The value of final response is 0.32. Therefore, $0.63 * 0.32 = 0.2$. The amplitude reaches this value at time 4.5 seconds. Therefore,

$$\tau = 4.5 - \theta = 2.$$

Design of PID controller:

We can use the ZN - method for designing the controller.

PID Controller:

$$\begin{aligned} K_R &= 1.2\frac{\tau}{\theta K} &&= 3, \\ \tau_I &= 2\theta &&= 5, \\ \tau_D &= 0.5\theta &&= 1.25. \end{aligned}$$