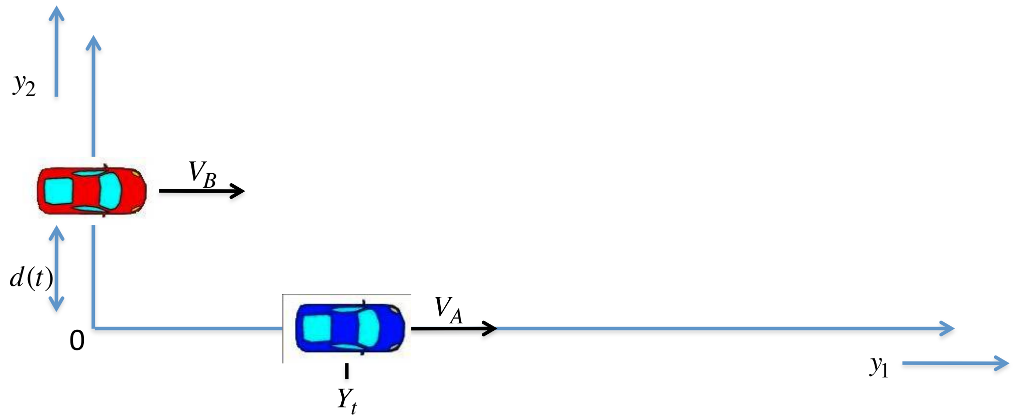
Rendezvous problem

- A target vehicle A is moving in y_1 direction, with constant velocity V_A
- Our vehicle B is moving in y_1 direction with constant velocity V_B ($V_B > V_A$)

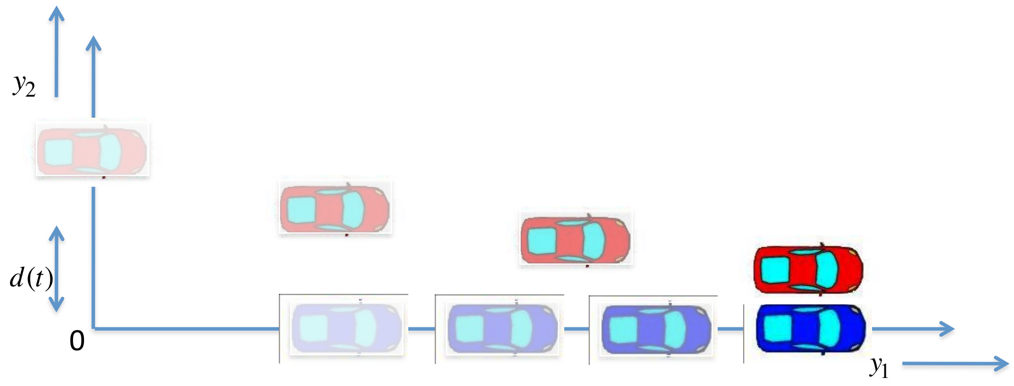


Rendezvous problem

- According to their relative velocity they will be be abreast of each other at time:

$$t_f = \frac{Y_t}{V_B - V_A}$$

- The goal is to design the trajectory for y_2 direction to rendezvous at this time



Dynamic of the vehicle

- The vehicle has constant velocity in y_1 direction
- The dynamic of the vehicle in y_2 direction obeys Newton's Law $m\ddot{d} = u$
- This dynamic can be expressed in state-variable form

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1/m \end{bmatrix} u$$

- Where $x = [\ d \quad v\ ]^T$ with $d(t)$ and $v(t) = \dot{d}(t)$ representing lateral position and velocity

Design of Digital Control Law

- First, we discretize the dynamics (note that $A^2 = 0$)

$$x(k+1) = \begin{bmatrix} 1 & h \\ 0 & 1 \end{bmatrix} x(k) + \begin{bmatrix} h^2/2 \\ h \end{bmatrix} u(k)$$

- Modified cost function with new term for final cost

$$J = \frac{1}{2} x^T(N) Q_0 x(N) + \frac{1}{2} \sum_{k=0}^{N-1} \{x^T(k) Q_1 x(k) + u^T(k) Q_2 u(k)\}$$

- Cost Function for our problem

$$J = \frac{1}{2} x^T(N) \begin{bmatrix} q_0 & 0 \\ 0 & q_0 \end{bmatrix} x(N) + \frac{1}{2} \sum_{k=0}^{N-1} \left\{ x^T(k) \begin{bmatrix} q_1 & 0 \\ 0 & q_1 \end{bmatrix} x(k) + q_2 u^2(k) \right\}$$

LQR extension

- With the new extension to what you saw before
- Optimal control algorithm
  1) Set final conditions $S(N) = Q_0$ and $K(N) = 0$
  2) Start from $k = N$
  3) Determine $R(k) = Q_2 + \Gamma^T S(k)\Gamma$
  4) Determine $M(k) = S(k) - s(k)\Gamma R^{-1}(k)\Gamma^T S(k)$
  5) Determine the gain $K(k-1) = R^{-1}(k)\Gamma^T S(k)\Phi$
  6) save the gain $K(k-1)$
  7) Set $S(k-1) = \Phi^T M(k)\Phi + Q_1$
  8) Set $k = k - 1$
  9) Go back to step 3

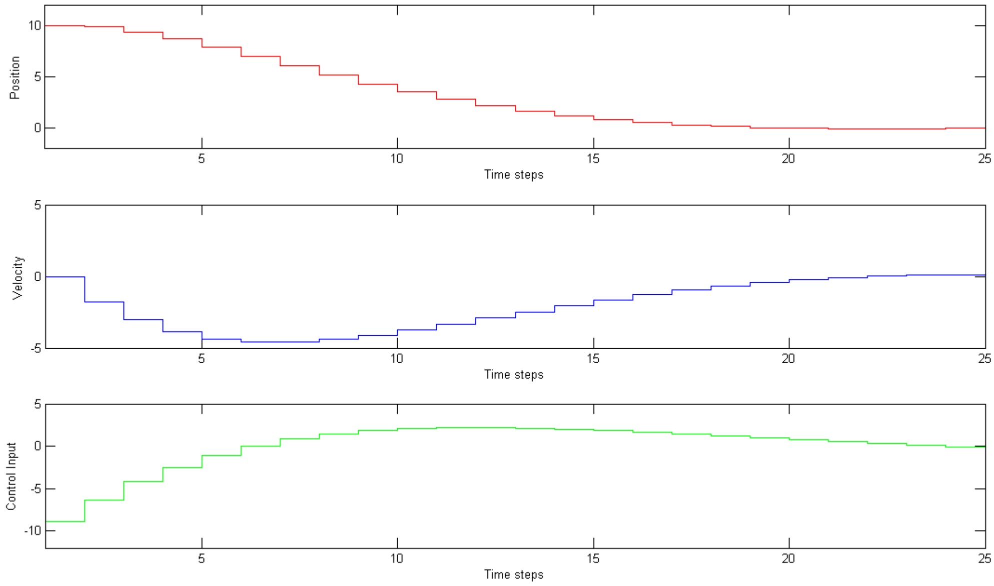
Back to rendezvous problem

- We set time steps as $h = 0.2s$
- Simulation is done for $N = 25$ steps
- In cost function we have :

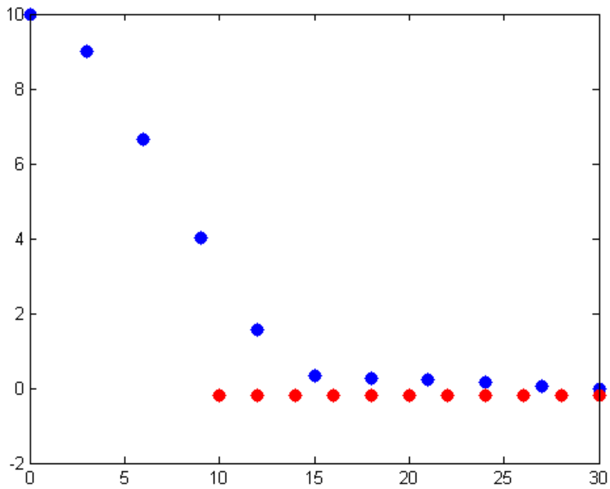
$$Q_0 = \begin{bmatrix} 100 & 0 \\ 0 & 100 \end{bmatrix}, Q_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, Q_2 = [1]$$

- We apply the optimal control steps

Simulation Results

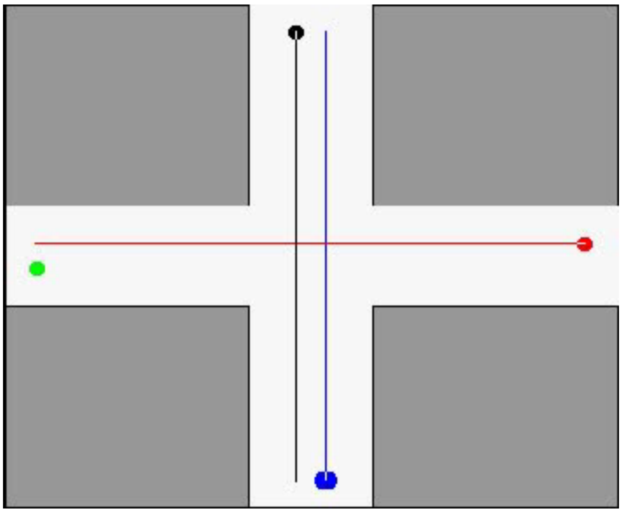


Simulation(Video)



Intersection

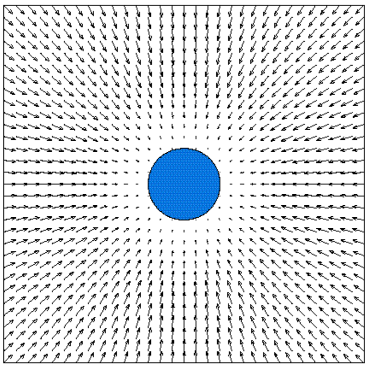
- More complex objectives
- Decentralized versus centralized approaches



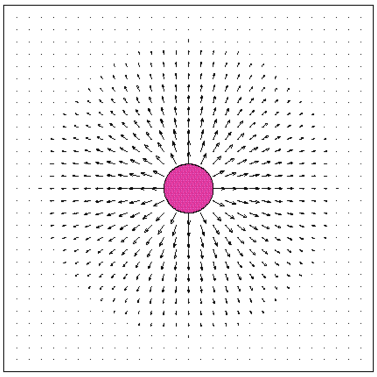
Navigation Function

- Initially proposed for real-time collision avoidance [Khatib 1986]. Hundreds of papers published
- A potential field is a scalar function over the free space
- To navigate, the vehicle applies a force proportional to the negated gradient of the potential field
- A navigation function is an ideal potential field that
  - has global minimum at the goal
  - has no local minima
  - grows to infinity near obstacles
  - is smooth

Potential field

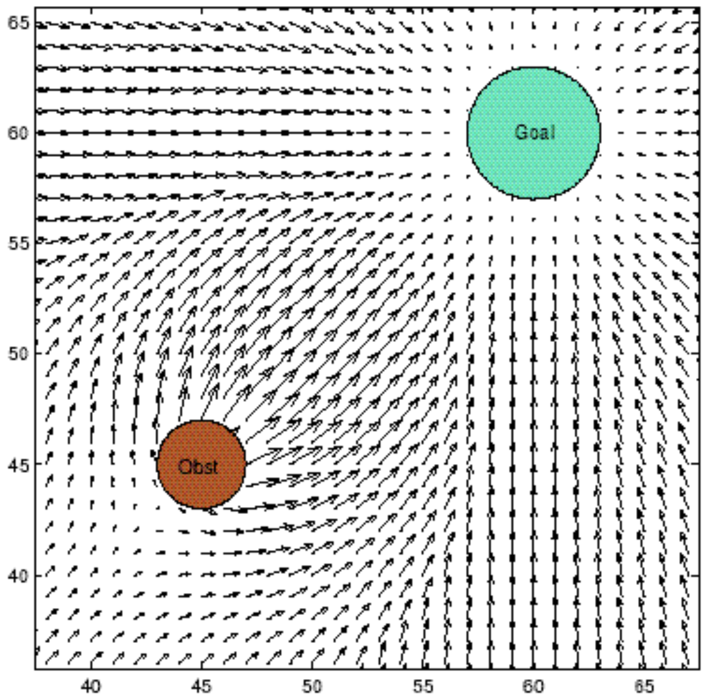


Attractive toward goal



Repulsive from other vehicles and obstacles

Potential Field



Experiments

- Navigation function method applied to the same problem of coordination of vehicles at an intersection

Navigation Function: $\phi_i = \lambda_1 \left\| d_i - d_{ri} \right\|^2 + \lambda_2 \sum_{k=1}^{m} C_k(d_i) + \lambda_3 \sum_{j \neq i} \beta(d_i, d_j)$

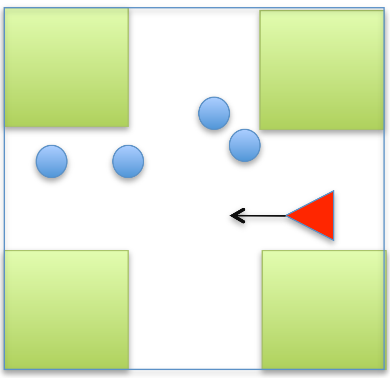
Attraction toward goal

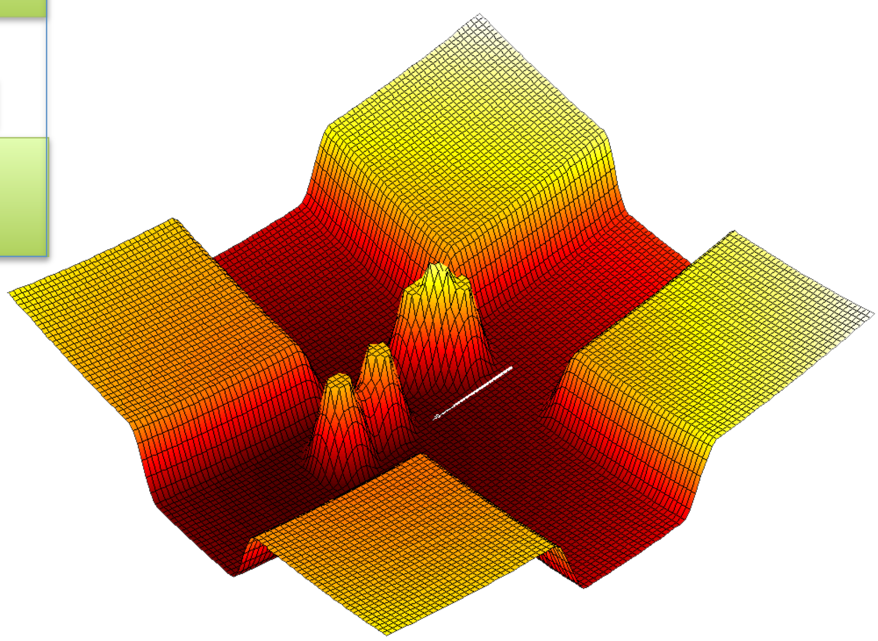
Repulsive force from obstacles and other vehicles

Kinematics model: $\ddot{d}_i = \frac{1}{m_i} u_i$

Control law: $u_i = -\nabla_{d_i} \phi_i$ This control law drive us toward local minimum of the navigation function using gradient descent

Potential Function

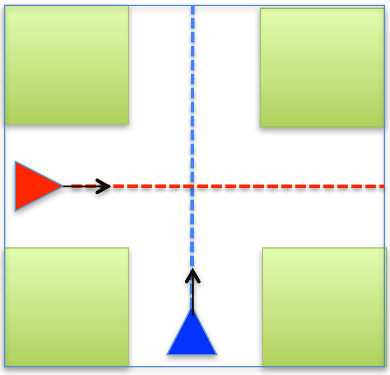




Simulation with two vehicles

- In this case, the vehicles stick to their lane, so we don't need the term for avoiding obstacles
- As every vehicle is moving in one direction, dynamic of the vehicles could be represented in one dimension.
- Modified navigation function is:

$$\phi_i = \lambda_1 \left\| d_i - d_{ri} \right\|^2 + \lambda_2 \sum_{j \neq i} \beta(d_i, d_j)$$



Simulation with two vehicles

$$\begin{cases} \phi_1 = \lambda_1 \left\| d_1 - d_{r1} \right\|^2 + \lambda_2 \beta(d_1, d_2) \\ \phi_2 = \lambda_1 \left\| d_2 - d_{r2} \right\|^2 + \lambda_2 \beta(d_2, d_1) \end{cases}$$

$$\begin{cases} \ddot{d}_1 = -\frac{\partial \phi_1}{\partial d_1} \\ \ddot{d}_2 = -\frac{\partial \phi_2}{\partial d_2} \end{cases}$$

Navigation Equations

This set of differential equations could be solved numerically over time steps, or by choosing appropriate beta function it could be solved analytically

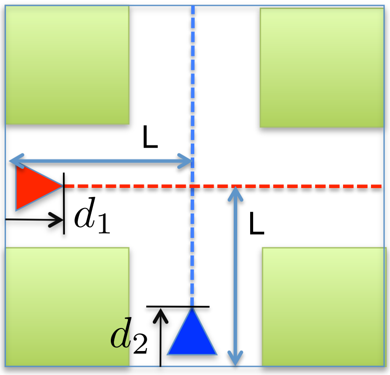
$$\begin{cases} \ddot{d}_1 = -2\lambda_1 (d_1 - d_{r1}) - \lambda_2 \frac{\partial \beta(d_1, d_2)}{\partial d_1} \\ \ddot{d}_2 = -2\lambda_1 (d_2 - d_{r2}) - \lambda_2 \frac{\partial \beta(d_2, d_1)}{\partial d_2} \end{cases}$$

Simulation with two vehicles

- One can define beta function as follows $\beta(d_1, d_2) = -(2L - d_1 - d_2)^2$
- Minimizing beta means maximizing the distance between two vehicles
- Navigation equations could be rewritten:

$$\begin{cases} \ddot{d}_1 = -2\lambda_1(d_1 - d_{r1}) - 2\lambda_2(2L - d_1 - d_2) \\ \ddot{d}_2 = -2\lambda_1(d_2 - d_{r2}) - 2\lambda_2(2L - d_2 - d_1) \end{cases}$$

- Note that the two differential equations are coupled



Simulation with two vehicles

$$Z = d_1 + d_2$$
$$Y = d_1 - d_2$$

$$\ddot{Z} = -2(\lambda_1 - \lambda_2)Z - 8L\lambda_2 + 2\lambda_1(d_{r1} + d_{r2})$$
$$\ddot{Y} = -2\lambda_1 Y + 2\lambda_1(d_{r1} - d_{r2})$$

suppose that $\lambda_2 > \lambda_1$

$$Z = A_2 e^{\sqrt{2(\lambda_2-\lambda_1)}t} + B_2 e^{-\sqrt{2(\lambda_2-\lambda_1)}t} - \frac{1}{2(\lambda_2 - \lambda_1)}(-8L\lambda_2 + 2\lambda_1(d_{r1} + d_{r2}))$$

$$Y = A_1 \sin(\sqrt{2\lambda_1}t) + B_1 \cos(\sqrt{2\lambda_1}t) + (d_{r1} - d_{r2})$$

A_1, A_2, B_1 and B_2 could be computed from initial positions and velocities of two vehicles

Then, we can compute $d_1 = \frac{Z+Y}{2}$ and $d_2 = \frac{Z-Y}{2}$

Simulation with two vehicles

Both vehicles start with zero speeds

$$\lambda_1 = 0.2$$
$$\lambda_2 = 0.3$$
$$d_{goal1} = 16$$
$$d_{goal2} = 18$$
$$L = 5$$

Simulation with more vehicles

- Decentralized coordination of vehicles could be implemented for more vehicles