IC-32 **Optimal Control** **Winter 2006**

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Problem Set #5

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1. Consider the functional
$$\mathcal{J}(x) \;:=\; \int_0^1 \left([\dot{x}(t)]^2 + 12\; t\; x(t)\right)\; \mathrm{d}t,$$
for $x \in \mathcal{D} := \{x \in \mathcal{C}^1[0,1] : x(0) = 1, x(1) = 2\}$.

   (a) Find stationary functions for this problem, which satisfy the specified end-point conditions.

   (b) What can be said about the resulting stationary functions: Do they give a local minimum? A local maximum? A global minimum? A global maximum? Or, neither of these?

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2. Consider the functional
$$\mathcal{J}(x) \;:=\; \int_{t_1}^{t_2} \left([x(t)]^2 + a x(t)\dot{x}(t) + b[\dot{x}(t)]^2\right)\; \mathrm{d}t,$$
for $x \in \mathcal{D} := \{x \in \mathcal{C}^1[t_1, t_2] : x(t_1) = x_1, x(t_2) = x_2\}$.

   (a) Find stationary functions for this problem, by distinguishing between the cases where $b = 0$, $b > 0$ and $b < 0$. Which stationary functions are candidate (local or global) minimizers for $\mathcal{J}$ on $\mathcal{D}$? Candidate (local or global) maximizers for $\mathcal{J}$ on $\mathcal{D}$?

   (b) How does the parameter a affect the solution? Why?

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3. Consider the problem to
$$\begin{aligned} \text{minimize:} \quad & \mathcal{J}(x) = \int_0^T \exp(-rt)\; x(t)\; \mathrm{d}t \\ \text{subject to:} \quad & x \in \mathcal{D} := \left\{ x \in \mathcal{C}[0,T] : \int_0^T \sqrt{x(t)}\; \mathrm{d}t = A \right\}, \end{aligned}$$
given $A \geq 0$ and $r > 0$.

   (a) Reformulate this constrained problem into a free problem.
   [Hint: consider the new phase variable $y \in \mathcal{C}^1[0,T]$ defined by $y(t) := \int_0^t \sqrt{x(s)}\; \mathrm{d}s$, $0 \leq t \leq T$.]

   (b) Identify candidate solutions to this problem based on Euler's equation and Legendre second-order necessary condition.