5.4 Comportements en BF

Commande qui ramène le système perturbé à son point d'équilibre en *n* périodes d'échantillonnage: Réponse pile

$$x(n) = \Phi^n x(0) + \Phi^{n-1}\Gamma u(0) + \Phi^{n-2}\Gamma u(1) + \ldots + \Phi\Gamma u(n-2) + \Gamma u(n-1) \equiv 0$$

$$-\Phi^n x_0 = \begin{bmatrix} \Gamma & \Phi\Gamma & \ldots & \Phi^{n-1}\Gamma \end{bmatrix} \begin{bmatrix} u(n-1) \\ u(n-2) \\ \vdots \\ u(0) \end{bmatrix}$$

$$\begin{bmatrix} u(n-1) \\ u(n-2) \\ \vdots \\ u(0) \end{bmatrix} = -\underbrace{\begin{bmatrix} \Gamma & \Phi\Gamma & \ldots & \Phi^{n-1}\Gamma \end{bmatrix}^{-1}}_{G^{-1}} \Phi^n x_0$$

$$u(0) = \begin{bmatrix} 0 & \ldots & 0 & 1 \end{bmatrix} \begin{bmatrix} u(n-1) \\ u(n-2) \\ \vdots \\ u(0) \end{bmatrix} = -\begin{bmatrix} 0 & \ldots & 0 & 1 \end{bmatrix} G^{-1}\Phi^n x_0$$

$$u(0) = -Kx(0) = -\begin{bmatrix} 0 & \ldots & 0 & 1 \end{bmatrix} G^{-1}\Phi^n x_0$$

$$K = \begin{bmatrix} 0 & \ldots & 0 & 1 \end{bmatrix} G^{-1}\Phi^n$$

Choix correspondant des valeurs propres $\lambda_i = 0, \quad \forall i$

5.4 Comportements en BF

Choix de pôles (valeurs propres) en analogiques et conversion en discret

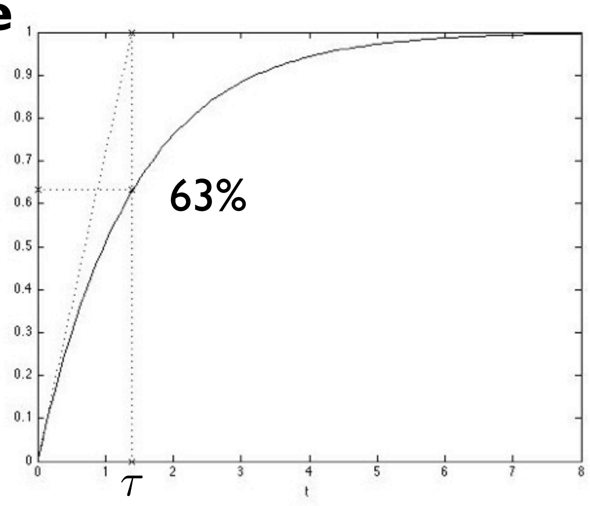
$$z_i = e^{s_i h}$$

Réponse non oscillante

Un pôle dominant, les autres au centre

$$G = \frac{K}{1 + s_1\tau}, \quad s_1 = -\frac{1}{\tau}$$

$$z_i = 0, \quad \forall i, \quad i \neq 1$$



5.4 Comportements en BF

Réponse oscillante

Un paire de pôles complexes conjugués dominants,
les autres au centre

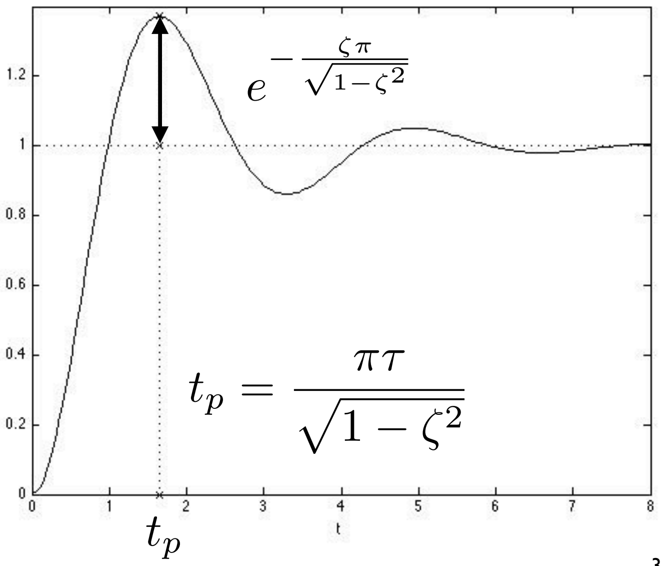
$$0 \leq \zeta < 1$$

$$G = \frac{K}{\tau^2 s^2 + 2\zeta\tau s + 1}, \quad \omega_0 = \frac{1}{\tau}$$

$$s_{1,2} = -\frac{1}{\tau}\left(\zeta \pm \sqrt{\zeta^2 - 1}\right)$$

$$z_i = 0, \quad \forall i, \quad i \neq 1, 2$$

$$z_i = e^{s_i h}$$



5.4 Comportements en BF

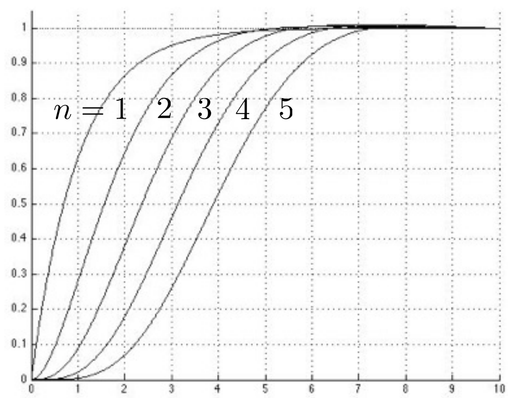
Réponse oscillante

Fréquence de coupure (bande passante en BF) $\omega_n = 1$

$$H_N(s) = \frac{K}{\prod(s - s_i)} = \frac{K}{B_N(s)} B_N(0)$$

$$B_0(s) = 1$$
$$B_1(s) = s + 1$$
$$\vdots$$
$$B_N(s) = (2N-1)B_{N-1}(s) + s^2 B_{N-2}(s)$$

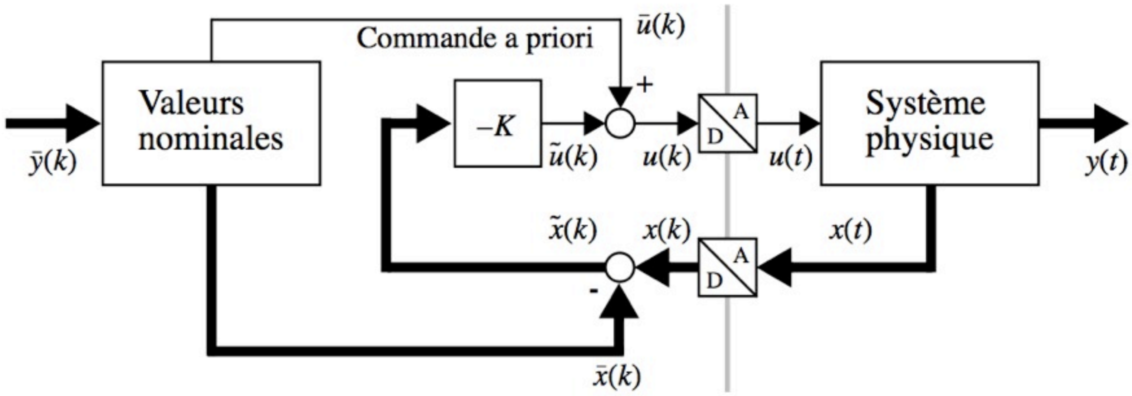
$$z_i = e^{s_i h}$$



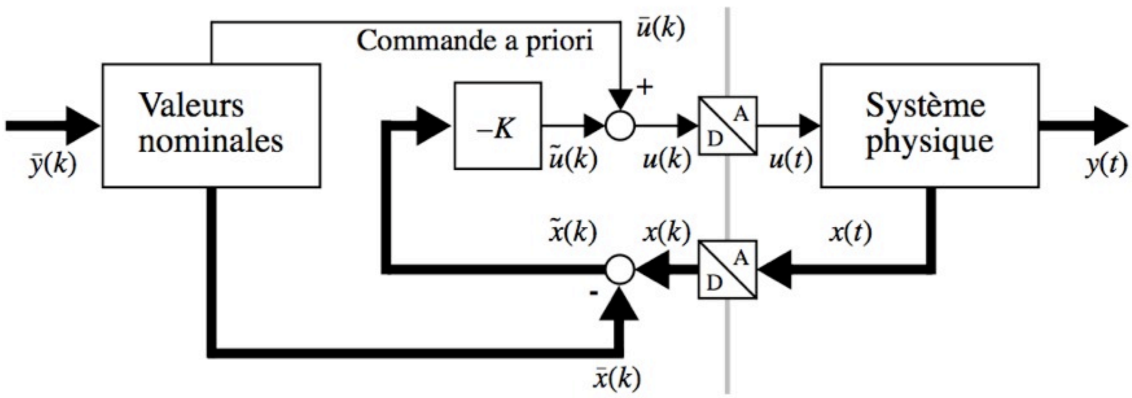
n	Pôles • K = 1
1	-1
2	-0.866 ± 0.5i
3	-0.7456 ± 0.7114i, -0.9416
4	-0.9048 ± 0.2709i, -0.6572 ± 0.8302i
5	-0.5906 ± 0.9072i, -0.8516 ± 0.4427i, -0.9264

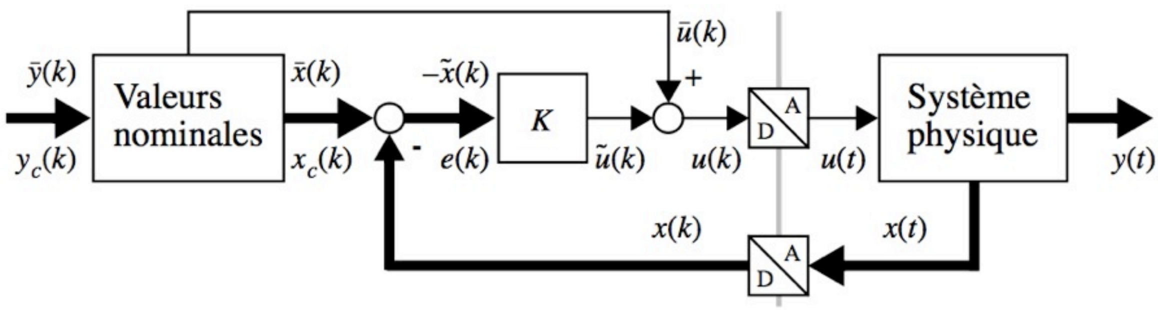
5.5 Commande d'état avec commande a priori

$$\begin{aligned}\tilde{x}(k+1) &= \Phi\tilde{x}(k) + \Gamma\tilde{u}(k)\\ \tilde{y}(k) &= C\tilde{x}(k) + D\tilde{u}(k)\\ \tilde{u}(k) &= -K\tilde{x}(k)\end{aligned}$$



Commande d'état avec "consigne"





Résumé commande d'état SISO

Système à régler
$$\begin{aligned} x(k+1) &= \Phi x(k) + \Gamma u(k) \\ y(k) &= Cx(k) + Du(k) \end{aligned}$$

Commande
$$u(k) = -Kx(k)$$

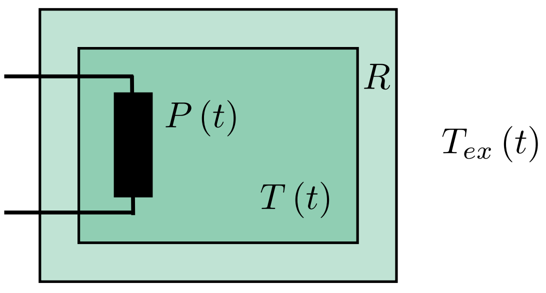
Système en boucle fermée (BF)
$$\begin{aligned} x(k+1) &= (\Phi - \Gamma \mathrm{K})\, x(k) &= \Phi_{bf} x(k) \\ y(k) &= (C - DK)\, x(k) &= C_{bf} x(k) \end{aligned}$$

$$\begin{aligned} \alpha_c(\lambda) = \det(\lambda I - \Phi_{bf}) &= (\lambda - \lambda_1)(\lambda - \lambda_2)\ldots(\lambda - \lambda_n) \\ &= \lambda^n + \alpha_1 \lambda^{n-1} + \ldots \lambda a_{n-1} + \alpha_n = 0 \end{aligned}$$

Formule d'Ackermann

$$\begin{aligned} K &= \begin{bmatrix} 0 & \ldots & 0 & 1 \end{bmatrix} G^{-1} \alpha_c(\Phi) \\ &= \begin{bmatrix} 0 & \ldots & 0 & 1 \end{bmatrix} \begin{bmatrix} \Gamma & \Phi\Gamma & \ldots & \Phi^{n-1}\Gamma \end{bmatrix}^{-1} \alpha_c(\Phi) \end{aligned}$$

5.9 Exemple de synthèse: Enceinte de chauffe



Entrée: $u(t) = P(t)$, sortie: $y(t) = T(t)$, perturbation: $q(t) = T_{ex}(t)$

Modèle analogique: $\underbrace{mc}_{cte=1} \dot{T}(t) = P(t) - R\left[T(t) - T_{ex}(t)\right]$, pour $x(t) = T(t)$

$$\left| \begin{array}{l} \dot{x}(t) = -Rx(t) + u(t) + Rq(t) \\ y(t) = x(t) \end{array} \right.$$

5.9 Exemple de synthèse: Enceinte de chauffe

$$\left| \begin{array}{l} \dot{x}(t) = -Rx(t) + u(t) + Rq(t) \\ y(t) = x(t) \end{array} \right.$$

Point de fonctionnement

$$\bar{y} = y_c \qquad \left| \begin{array}{l} 0 = -R\bar{x} + \bar{u} + Rq \\ \bar{y} = \bar{x} \end{array} \right.$$

$$\left| \begin{array}{l} 0 = -R\bar{x} + \bar{u} + Rq \rightarrow \bar{u} = R\left(\bar{x} - q\right) \\ \bar{x} = \bar{y} = y_c \rightarrow \bar{u} = R\left(y_c - q\right) \end{array} \right.$$

Modèle d'état en variables écart

$$\begin{array}{l} (\dot{\bar{x}} + \dot{\tilde{x}}) = -R(\bar{x} + \tilde{x}) + (\bar{u} + \tilde{u}) + Rq \\ (0 + \dot{\tilde{x}}) = -R(y_c + \tilde{x}) + (Ry_c - Rq + \tilde{u}) + Rq \\ \dot{\tilde{x}} = -R\tilde{x} + \tilde{u} \\ \tilde{y} = \tilde{x} \end{array}$$

5.9 Exemple de synthèse: Enceinte de chauffe

$$\left| \begin{array}{l} \dot{\tilde{x}}(t) = -R\tilde{x}(t) + \tilde{u}(t) \\ \tilde{y}(t) = \tilde{x}(t) \end{array} \right.$$

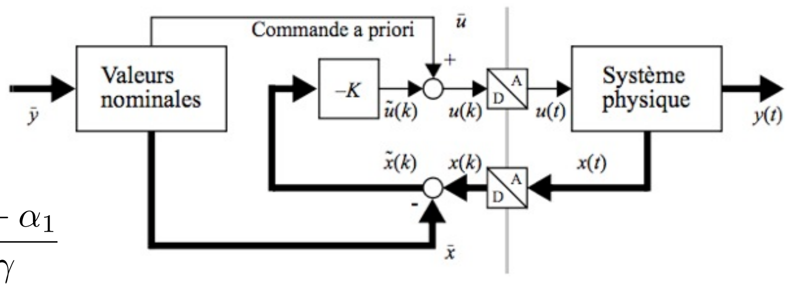
Discrétisation

$$\Phi = e^{Ah} = e^{-Rh} \equiv \phi$$

$$\Gamma = \left\{ \int_0^h e^{-R\eta} d\eta \right\} B = -\frac{1}{R}\left(e^{-Rh} - 1\right) = \frac{1}{R}\left(1 - \phi\right) \equiv \gamma$$

$$\left| \begin{array}{rcl} \tilde{x}(k+1) & = & \phi\tilde{x}(k) + \gamma\tilde{u}(k) \\ \tilde{y}(k) & = & \tilde{x}(k) \end{array} \right.$$

Commande



$$\alpha_c(\lambda) = \lambda + \alpha_1 = 0$$

$$K = [1]\left[\gamma\right]^{-1}\alpha_c(\phi) = \frac{\phi + \alpha_1}{\gamma}$$