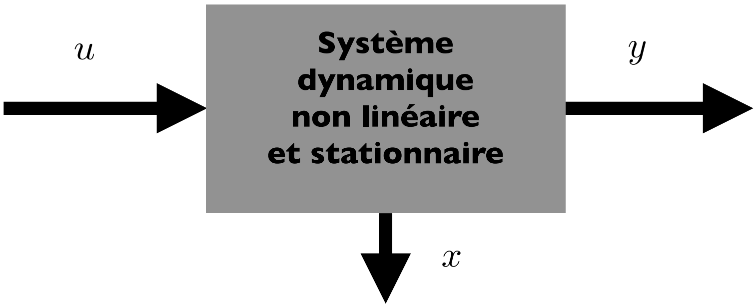
3.2 Grandeurs Nominales



Non linéaire	$\dot{x}(t) = f[x(t), u(t), t]$ $y(t) = g[x(t), u(t), t]$	$x(k+1) = f[x(k), u(k), k]$ $y(k) = g[x(k), u(k), k]$
Station-naire	$x(k+1) = f[x(k), u(k)]$ $y(k) = g[x(k), u(k)]$	$\dot{x}(t) = f[x(t), u(t)]$ $y(t) = g[x(t), u(t)]$
Trajectoire nominale	$\dot{\bar{x}}(t) = f[\bar{x}(t), \bar{u}(t)]$ $\bar{y}(t) = g[\bar{x}(t), \bar{u}(t)]$	$\bar{x}(k+1) = f[\bar{x}(k), \bar{u}(k)]$ $\bar{y}(k) = g[\bar{x}(k), \bar{u}(k)]$

3.2 Grandeurs Nominales: Exemple

Profil de position souhaité
Trajectoire de fonctionnement
Trajectoire nominale

$$\bar{\theta}(t) = \bar{x}(t) = A\sin(\omega t)$$

Modèle d'état valable pour tout $x(t)$, donc aussi pour $\bar{x}(t)$

$$\dot{\bar{x}}(t) = f[\bar{x}(t), \bar{u}(t)]$$

$$\dot{\bar{x}}(t) = \omega A\cos(\omega t)$$

$$\omega A\cos(\omega t) = f[A\sin(\omega t), \bar{u}(t)]$$
$$\bar{y}(t) = g[A\sin(\omega t), \bar{u}(t)]$$

Equations algébriques

Cas général:

n + p équations pour trouver n+p+r inconnues $\bar{x}(t), \bar{u}(t), \bar{y}(t)$

donc r degrés de liberté (pas forcément des entrées)

$\bar{u}(t)$ **est appelé la commande a priori**

En l'absence de perturbation et d'erreur de modèle, la commande a priori permet de suivre les trajectoires nominales

3.2.3 Point de fonctionnement

Point de fonctionnement = état nominal = point d'équilibre

Si le système se trouve à son point de fonctionnement et en l'absence de perturbation, il y reste

La trajectoire nominale n'évolue plus au cours du temps

$$\bar{x}(t) = \bar{x}; \quad \bar{u}(t) = \bar{u}; \quad \bar{y}(t) = \bar{y}$$

Analogique

$$\dot{\bar{x}}(t) = \dot{\bar{x}} = 0$$

$$0 = f[\bar{x}, \bar{u}]$$
$$\bar{y} = g[\bar{x}, \bar{u}]$$

Discret

$$\bar{x}(k+1) = \bar{x}(k) = \bar{x} \qquad \forall k$$

$$\bar{x} = f[\bar{x}, \bar{u}]$$
$$\bar{y} = g[\bar{x}, \bar{u}]$$

n+p équations algébriques à résoudre

Moteur commandé en vitesse

$$G(s) = \frac{\omega(s)}{U(s)} = \frac{A}{1 + s\tau}$$

$$\omega(s) + s\omega(s)\tau = AU(s)$$
$$s\omega(s)\tau = -\omega(s) + AU(s)$$
$$\dot{\omega}(t) = -\tfrac{1}{\tau}\omega(t) + \tfrac{A}{\tau}u(t)$$

$$x(t) = \omega(t) = y(t)$$

$$\dot{x}(t) = -\tfrac{1}{\tau}x(t) + \tfrac{A}{\tau}u(t)$$
$$y(t) = x(t)$$

$$n = 1; \quad p = 1; \quad r = 1$$

Un degré de liberté $\bar{y} = 10 \text{ rad/s}$

Point de fonctionnement $0 = -\tfrac{1}{\tau}\bar{x} + \tfrac{A}{\tau}\bar{u} \longrightarrow \bar{u} = \dfrac{\bar{x}}{A}$

$\bar{y} = \bar{x} \longrightarrow \bar{x} = \bar{y}$

Linéarisation par la tangente

Analogique

$$\dot{x}(t) = f[x(t), u(t), t]$$
$$y(t) = g[x(t), u(t), t]$$

Discret

$$x(k+1) = f[x(k), u(k), k]$$
$$y(k) = g[x(k), u(k), k]$$

Forme commune des fonctions non linéaires

$$f[x, u]$$
$$g[x, u]$$

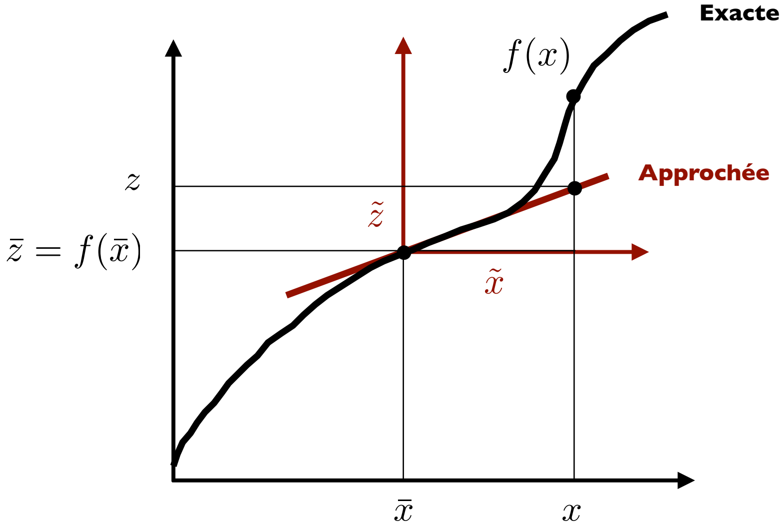
On cherche une approximation linéaire valable autour d'une trajectoire nominale ou d'un point de fonctionnement

$$\left.\begin{array}{l} \{\ \bar{x}(t) \quad \bar{u}(t) \quad \bar{y}(t)\ \} \\ \{\ \bar{x}(k) \quad \bar{u}(k) \quad \bar{y}(k)\ \} \end{array}\right\} \rightarrow \{\ \bar{x} \quad \bar{u} \quad \bar{y}\ \}$$

Linéarisation par la tangente

Ecarts

$$\tilde{x} = x - \bar{x}$$
$$\tilde{y} = y - \bar{y}$$
$$\tilde{u} = u - \bar{u}$$



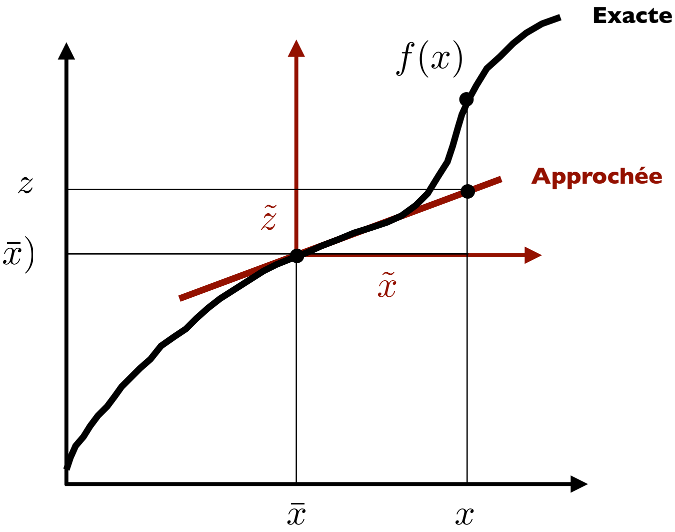
Fonction de x uniquement

$$\tilde{z} = \frac{d}{dx} f(x)\Big|_{x=\bar{x}} \tilde{x} = \frac{df}{dx}(\bar{x})\tilde{x}$$

Linéarisation par la tangente

Ecarts

$\tilde{x} = x - \bar{x}$
$\tilde{y} = y - \bar{y}$
$\tilde{u} = u - \bar{u}$



Fonction de x et de u (n=r=1)

$$f(x,u) - f(\bar{x},\bar{u}) \cong z - \bar{z} = \tilde{z} = \frac{\partial}{\partial x} f(\bar{x},\bar{u})\tilde{x} + \frac{\partial}{\partial u} f(\bar{x},\bar{u})\tilde{u}$$

$$g(x,u) - g(\bar{x},\bar{u}) \cong y - \bar{y} = \tilde{y} = \frac{\partial}{\partial x} g(\bar{x},\bar{u})\tilde{x} + \frac{\partial}{\partial u} g(\bar{x},\bar{u})\tilde{u}$$

Analogique $\quad \tilde{z}(t) \cong \dot{x}(t) - \dot{\bar{x}}(t) = \dot{\tilde{x}}(t)$

Discret $\quad \tilde{z}(k) \cong x(k+1) - \bar{x}(k+1) = \tilde{x}(k+1)$

Linéarisation par la tangente

$$\tilde{z}_1 = \frac{\partial f_1}{\partial x_1}(\bar{x},\bar{u})\tilde{x}_1 + \ldots + \frac{\partial f_1}{\partial x_n}(\bar{x},\bar{u})\tilde{x}_n + \frac{\partial f_1}{\partial u_1}(\bar{x},\bar{u})\tilde{u}_1 + \ldots + \frac{\partial f_1}{\partial u_r}(\bar{x},\bar{u})\tilde{u}_r$$
$$\vdots$$
$$\tilde{z}_n = \frac{\partial f_n}{\partial x_1}(\bar{x},\bar{u})\tilde{x}_1 + \ldots + \frac{\partial f_n}{\partial x_n}(\bar{x},\bar{u})\tilde{x}_n + \frac{\partial f_n}{\partial u_1}(\bar{x},\bar{u})\tilde{u}_1 + \ldots + \frac{\partial f_n}{\partial u_r}(\bar{x},\bar{u})\tilde{u}_r$$

$$\tilde{y}_1 = \frac{\partial g_1}{\partial x_1}(\bar{x},\bar{u})\tilde{x}_1 + \ldots + \frac{\partial g_1}{\partial x_n}(\bar{x},\bar{u})\tilde{x}_n + \frac{\partial g_1}{\partial u_1}(\bar{x},\bar{u})\tilde{u}_1 + \ldots + \frac{\partial g_1}{\partial u_r}(\bar{x},\bar{u})\tilde{u}_r$$
$$\vdots$$
$$\tilde{y}_p = \frac{\partial g_p}{\partial x_1}(\bar{x},\bar{u})\tilde{x}_1 + \ldots + \frac{\partial g_p}{\partial x_n}(\bar{x},\bar{u})\tilde{x}_n + \frac{\partial g_p}{\partial u_1}(\bar{x},\bar{u})\tilde{u}_1 + \ldots + \frac{\partial g_p}{\partial u_r}(\bar{x},\bar{u})\tilde{u}_r$$

$$\tilde{z} = \frac{\partial f}{\partial x}(\bar{x},\bar{u})\tilde{x} + \frac{\partial f}{\partial u}(\bar{x},\bar{u})\tilde{u}$$

$$\tilde{y} = \frac{\partial g}{\partial x}(\bar{x},\bar{u})\tilde{x} + \frac{\partial g}{\partial u}(\bar{x},\bar{u})\tilde{u}$$

Linéarisation par la tangente

$$\frac{\partial f}{\partial x}(\bar{x},\bar{u}) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1}(\bar{x},\bar{u}) & \frac{\partial f_1}{\partial x_2}(\bar{x},\bar{u}) & \dots & \frac{\partial f_1}{\partial x_n}(\bar{x},\bar{u}) \\ \frac{\partial f_2}{\partial x_1}(\bar{x},\bar{u}) & \frac{\partial f_2}{\partial x_2}(\bar{x},\bar{u}) & \dots & \frac{\partial f_2}{\partial x_n}(\bar{x},\bar{u}) \\ \vdots & \vdots & & \vdots \\ \frac{\partial f_n}{\partial x_1}(\bar{x},\bar{u}) & \frac{\partial f_n}{\partial x_2}(\bar{x},\bar{u}) & \dots & \frac{\partial f_n}{\partial x_n}(\bar{x},\bar{u}) \end{bmatrix} = A$$

$$\frac{\partial f}{\partial u}(\bar{x},\bar{u}) = \begin{bmatrix} \frac{\partial f_1}{\partial u_1}(\bar{x},\bar{u}) & \frac{\partial f_1}{\partial u_2}(\bar{x},\bar{u}) & \dots & \frac{\partial f_1}{\partial u_r}(\bar{x},\bar{u}) \\ \frac{\partial f_2}{\partial u_1}(\bar{x},\bar{u}) & \frac{\partial f_2}{\partial u_2}(\bar{x},\bar{u}) & \dots & \frac{\partial f_2}{\partial u_r}(\bar{x},\bar{u}) \\ \vdots & \vdots & & \vdots \\ \frac{\partial f_n}{\partial u_1}(\bar{x},\bar{u}) & \frac{\partial f_n}{\partial u_2}(\bar{x},\bar{u}) & \dots & \frac{\partial f_n}{\partial u_r}(\bar{x},\bar{u}) \end{bmatrix} = B$$

Linéarisation par la tangente

$$\frac{\partial g}{\partial x}(\bar{x},\bar{u}) = \begin{bmatrix} \frac{\partial g_1}{\partial x_1}(\bar{x},\bar{u}) & \frac{\partial g_1}{\partial x_2}(\bar{x},\bar{u}) & \dots & \frac{\partial g_1}{\partial x_n}(\bar{x},\bar{u}) \\ \frac{\partial g_2}{\partial x_1}(\bar{x},\bar{u}) & \frac{\partial g_2}{\partial x_2}(\bar{x},\bar{u}) & \dots & \frac{\partial g_2}{\partial x_n}(\bar{x},\bar{u}) \\ \vdots & \vdots & & \vdots \\ \frac{\partial g_p}{\partial x_1}(\bar{x},\bar{u}) & \frac{\partial g_p}{\partial x_2}(\bar{x},\bar{u}) & \dots & \frac{\partial g_p}{\partial x_n}(\bar{x},\bar{u}) \end{bmatrix} = C$$

$$\frac{\partial g}{\partial u}(\bar{x},\bar{u}) = \begin{bmatrix} \frac{\partial g_1}{\partial u_1}(\bar{x},\bar{u}) & \frac{\partial g_1}{\partial u_2}(\bar{x},\bar{u}) & \dots & \frac{\partial g_1}{\partial u_r}(\bar{x},\bar{u}) \\ \frac{\partial g_2}{\partial u_1}(\bar{x},\bar{u}) & \frac{\partial g_2}{\partial u_2}(\bar{x},\bar{u}) & \dots & \frac{\partial g_2}{\partial u_r}(\bar{x},\bar{u}) \\ \vdots & \vdots & & \vdots \\ \frac{\partial g_p}{\partial u_1}(\bar{x},\bar{u}) & \frac{\partial g_p}{\partial u_2}(\bar{x},\bar{u}) & \dots & \frac{\partial g_p}{\partial u_r}(\bar{x},\bar{u}) \end{bmatrix} = D$$

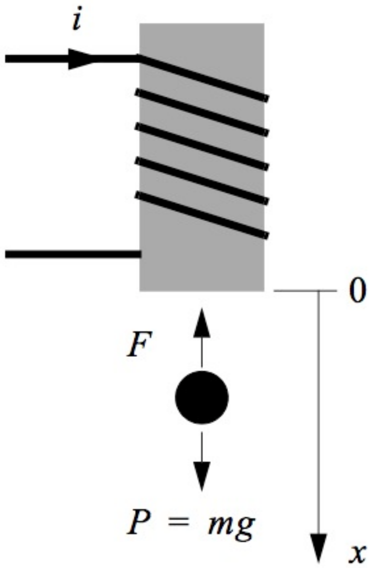
Résumé linéarisation par la tangente

	Analogique	Discret
Non linéaire	$\dot{x}(t) = f[x(t), u(t), t]$ $y(t) = g[x(t), u(t), t]$	$x(k+1) = f[x(k), u(k), k]$ $y(k) = g[x(k), u(k), k]$
Trajectoire nominale	$\tilde{x}(t) = x(t) - \bar{x}(t)$ $\tilde{y}(t) = y(t) - \bar{y}(t)$ $\tilde{u}(t) = u(t) - \bar{u}(t)$ $\dot{\bar{x}}(t) = f[\bar{x}(t), \bar{u}(t)]$ $\bar{y}(t) = g[\bar{x}(t), \bar{u}(t)]$	$\tilde{x}(k) = x(k) - \bar{x}(k)$ $\tilde{y}(k) = y(k) - \bar{y}(k)$ $\tilde{u}(k) = u(k) - \bar{u}(k)$ $\bar{x}(k+1) = f[\bar{x}(k), \bar{u}(k)]$ $\bar{y}(k) = g[\bar{x}(k), \bar{u}(k)]$
Point de fonctionnement	$\dot{\bar{x}}(t) = \dot{\bar{x}} = 0$ $0 = f[\bar{x}, \bar{u}]$ $\bar{y} = g[\bar{x}, \bar{u}]$	$\bar{x}(k+1) = \bar{x}(k) = \bar{x} \qquad \forall k$ $\bar{x} = f[\bar{x}, \bar{u}]$ $\bar{y} = g[\bar{x}, \bar{u}]$
Modèle linéarisé	$\dot{\tilde{x}}(t) = \left.\frac{\partial f}{\partial x}\right\|_{\bar{x},\bar{u}} \tilde{x}(t) + \left.\frac{\partial f}{\partial u}\right\|_{\bar{x},\bar{u}} \tilde{u}(t)$ $\tilde{y}(t) = \left.\frac{\partial g}{\partial x}\right\|_{\bar{x},\bar{u}} \tilde{x}(t) + \left.\frac{\partial g}{\partial x}\right\|_{\bar{x},\bar{u}} \tilde{u}(t)$	$\tilde{x}(k+1) = \left.\frac{\partial f}{\partial x}\right\|_{\bar{x},\bar{u}} \tilde{x}(k) + \left.\frac{\partial f}{\partial u}\right\|_{\bar{x},\bar{u}} \tilde{u}(k)$ $\tilde{y}(k) = \left.\frac{\partial g}{\partial x}\right\|_{\bar{x},\bar{u}} \tilde{x}(k) + \left.\frac{\partial g}{\partial x}\right\|_{\bar{x},\bar{u}} \tilde{u}(k)$

Systèmes intrinsèquement linéaires

	Analogique	Discret
	$\dot{x}(t) = Ax(t) + Bu(t)$ $y(t) = Cx(t) + Du(t)$	$x(k+1) = \Phi x(k) + \Gamma u(k)$ $y(k) = Cx(k) + Du(k)$
	$\tilde{x}(t) = x(t) - \bar{x}(t)$ $\tilde{y}(t) = y(t) - \bar{y}(t)$ $\tilde{u}(t) = u(t) - \bar{u}(t)$	$\tilde{x}(k) = x(k) - \bar{x}(k)$ $\tilde{y}(k) = y(k) - \bar{y}(k)$ $\tilde{u}(k) = u(k) - \bar{u}(k)$
Point de fonctionnement	$\dot{\bar{x}}(t) = \dot{\bar{x}} = 0$ $0 = A\bar{x} + B\bar{u}$ $\bar{y} = C\bar{x} + D\bar{u}$	$\bar{x}(k+1) = \bar{x}(k) = \bar{x} \qquad \forall k$ $\bar{x} = \Phi\bar{x} + \Gamma\bar{u}$ $\bar{y} = C\bar{x} + D\bar{u}$
	$\{\bar{u}, \bar{x}, \bar{y}\} = \{0, 0, 0\}$	
	$\bar{x} = -A^{-1}B\bar{u}$	$\bar{x} = (I - \Phi)^{-1}\Gamma\bar{u}$
Modèle "linéarisé"	$\dot{\tilde{x}}(t) = A\tilde{x}(t) + B\tilde{u}(t)$ $\tilde{y}(t) = C\tilde{x}(t) + D\tilde{u}(t)$	$\tilde{x}(k+1) = \Phi\tilde{x}(k) + \Gamma\tilde{u}(k)$ $\tilde{y}(k) = C\tilde{x}(k) + D\tilde{u}(k)$

Sustentation: Linéarisation par la tangente



Modèle physique

$$F(x,i) = \frac{1}{2}\frac{L}{(1+x)^2}i^2$$

$$m\ddot{x} = mg - F(x,i)$$

$$\ddot{x} = g - \frac{1}{2m}\frac{L}{(1+x)^2}i^2$$

Modèle d'état

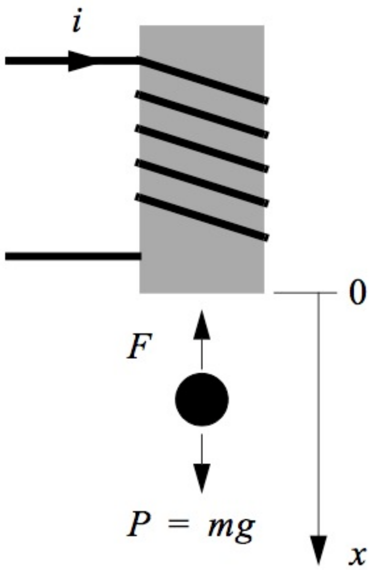
$$u = i, \quad x_1 = x \quad x_2 = \dot{x}$$

$$\dot{x}_1 = \dot{x} = x_2 = f_1(x,u)$$

$$\dot{x}_2 = \ddot{x} = g - \tfrac{L}{2m(1+x_1)^2}u^2 = f_2(x,u)$$

$$y = x_1 = g_1(x,u)$$

Sustentation: Linéarisation par la tangente



Modèle d'état

$$\dot{x}_1 = \dot{x} = x_2 = f_1(x,u)$$

$$\dot{x}_2 = \ddot{x} = g - \tfrac{L}{2m(1+x_1)^2}u^2 = f_2(x,u)$$

$$y = x_1 = g_1(x,u)$$

Point de fonctionnement $\bar{y} = y_o$

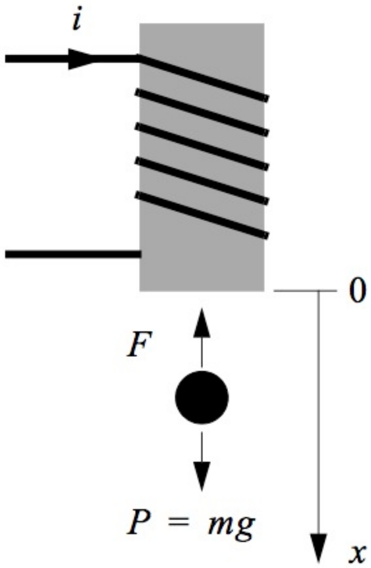
$$0 = \bar{x}_2$$

$$0 = g - \tfrac{L}{2m(1+\bar{x}_1)^2}\bar{u}^2$$

$$\bar{y} = \bar{x}_1 = y_o$$

$$\bar{u} = \sqrt{\frac{2mg}{L}}(1+y_o)$$

Sustentation: Linéarisation par la tangente



Modèle d'état non linéaire

$$\dot{x}_1 = \dot{x} = x_2 = f_1(x,u)$$

$$\dot{x}_2 = \ddot{x} = g - \frac{L}{2m(1+x_1)^2}u^2 = f_2(x,u)$$

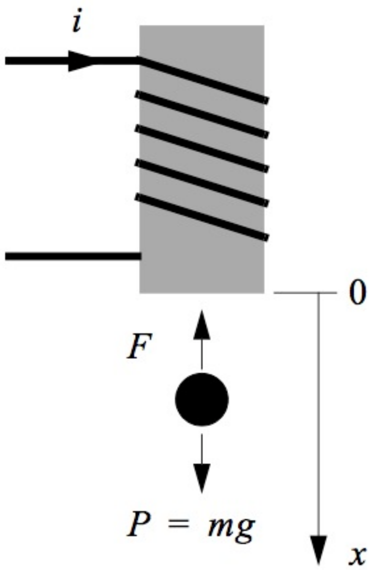
$$y = x_1 = g_1(x,u)$$

Modèle linéarisé $\bar{y} = y_o$

$$\dot{\tilde{x}} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1}(\bar{x},\bar{u}) & \frac{\partial f_1}{\partial x_2}(\bar{x},\bar{u}) \\ \frac{\partial f_2}{\partial x_1}(\bar{x},\bar{u}) & \frac{\partial f_2}{\partial x_2}(\bar{x},\bar{u}) \end{bmatrix} \tilde{x} + \begin{bmatrix} \frac{\partial f_1}{\partial u}(\bar{x},\bar{u}) \\ \frac{\partial f_2}{\partial u}(\bar{x},\bar{u}) \end{bmatrix} \tilde{u}$$

$$\tilde{y} = \begin{bmatrix} \frac{\partial g_1}{\partial x_1}(\bar{x},\bar{u}) & \frac{\partial g_1}{\partial x_2}(\bar{x},\bar{u}) \end{bmatrix} \tilde{x} + \left[\frac{\partial g_1}{\partial u}(\bar{x},\bar{u})\right] \tilde{u}$$

Sustentation: Linéarisation par la tangente



Modèle d'état non linéaire

$$\dot{x}_1 = \dot{x} = x_2 = f_1(x,u)$$

$$\dot{x}_2 = \ddot{x} = g - \frac{L}{2m(1+x_1)^2}u^2 = f_2(x,u)$$

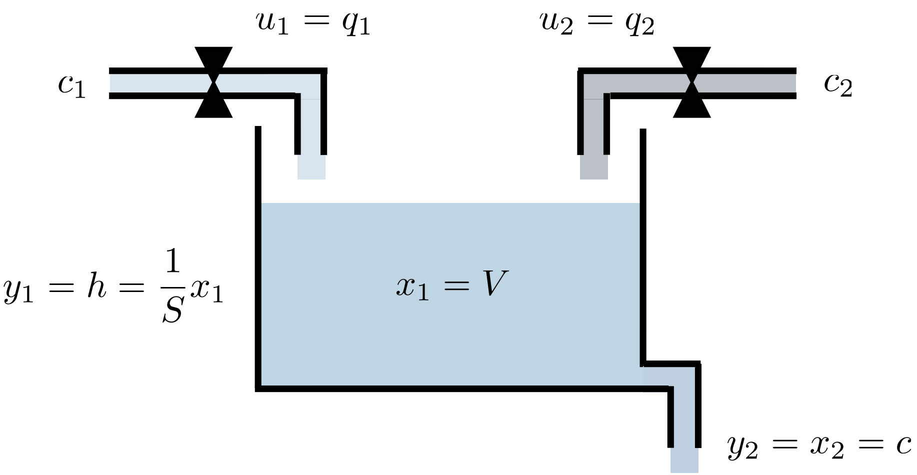
$$y = x_1 = g_1(x,u)$$

Modèle linéarisé $\bar{y} = y_o$

$$\dot{\tilde{x}} = \begin{bmatrix} 0 & 1 \\ \frac{L\bar{u}^2}{m(1+\bar{x}_1)^3} & 0 \end{bmatrix} \tilde{x} + \begin{bmatrix} 0 \\ \frac{L\bar{u}}{m(1+\bar{x}_1)^2} \end{bmatrix} \tilde{u}$$

$$\tilde{y} = \begin{bmatrix} 1 & 0 \end{bmatrix} \tilde{x}$$

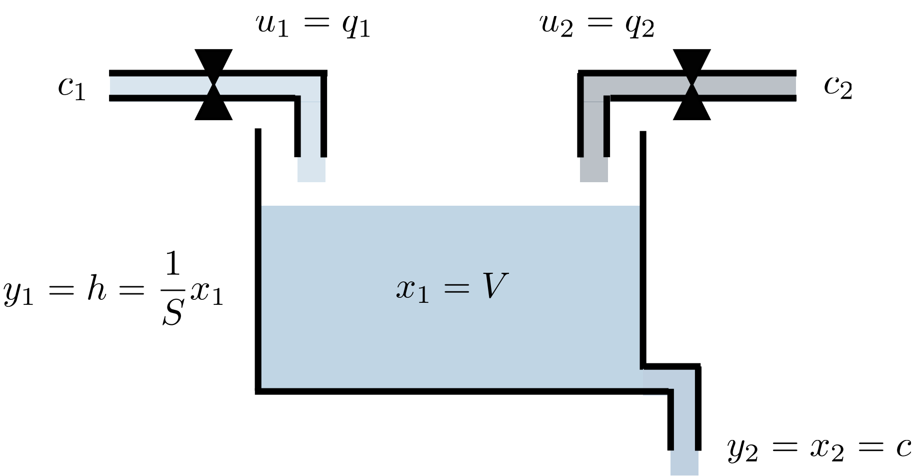
Cuve de mélange: Modèle d'état



$$\left| \begin{aligned} \dot{x}_1(t) &= u_1(t) + u_2(t) - K\sqrt{\tfrac{x_1(t)}{S}} \\ \dot{x}_2(t) &= \tfrac{1}{x_1(t)} \left\{ \left[c_1 - x_2(t)\right] u_1(t) + \left[c_2 - x_2(t)\right] u_2(t) \right\} \end{aligned} \right.$$

$$\left| \begin{aligned} y_1(t) &= \tfrac{1}{S} x_1(t) \\ y_2(t) &= x_2(t) \end{aligned} \right.$$

Cuve de mélange: Point de fonctionnement



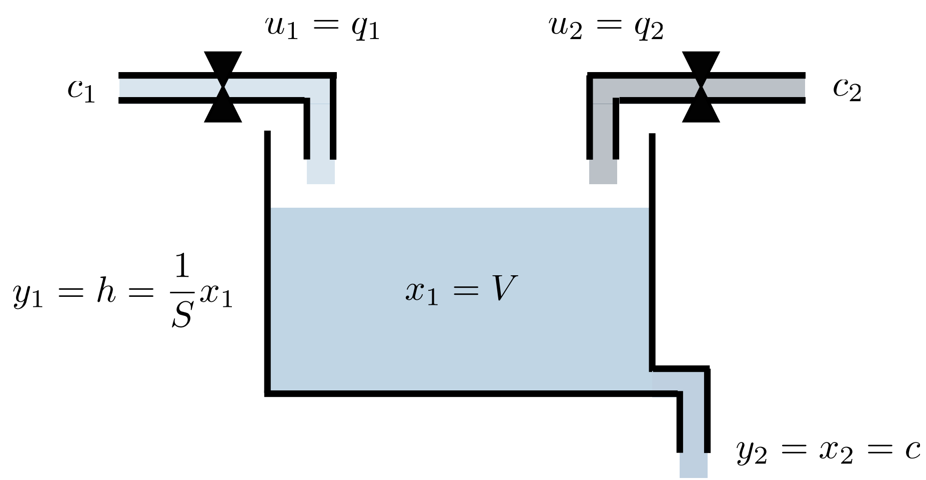
$$\left| \begin{aligned} \dot{x}_1(t) &= u_1(t) + u_2(t) - K\sqrt{\tfrac{x_1(t)}{S}} \\ \dot{x}_2(t) &= \tfrac{1}{x_1(t)} \left\{ \left[c_1 - x_2(t)\right] u_1(t) + \left[c_2 - x_2(t)\right] u_2(t) \right\} \end{aligned} \right.$$

$$\left| \begin{aligned} y_1(t) &= \tfrac{1}{S} x_1(t) \\ y_2(t) &= x_2(t) \end{aligned} \right.$$

$$\left| \begin{aligned} 0 &= \bar{u}_1 + \bar{u}_2 - K\sqrt{\tfrac{\bar{x}_1}{S}} \\ 0 &= \tfrac{1}{\bar{x}_1} \left\{ \left[c_1 - \bar{x}_2\right] \bar{u}_1 + \left[c_2 - \bar{x}_2\right] \bar{u}_2 \right\} \end{aligned} \right.$$

$$\left| \begin{aligned} \bar{y}_1 &= \tfrac{1}{S} \bar{x}_1 \\ \bar{y}_2 &= \bar{x}_2 \end{aligned} \right.$$

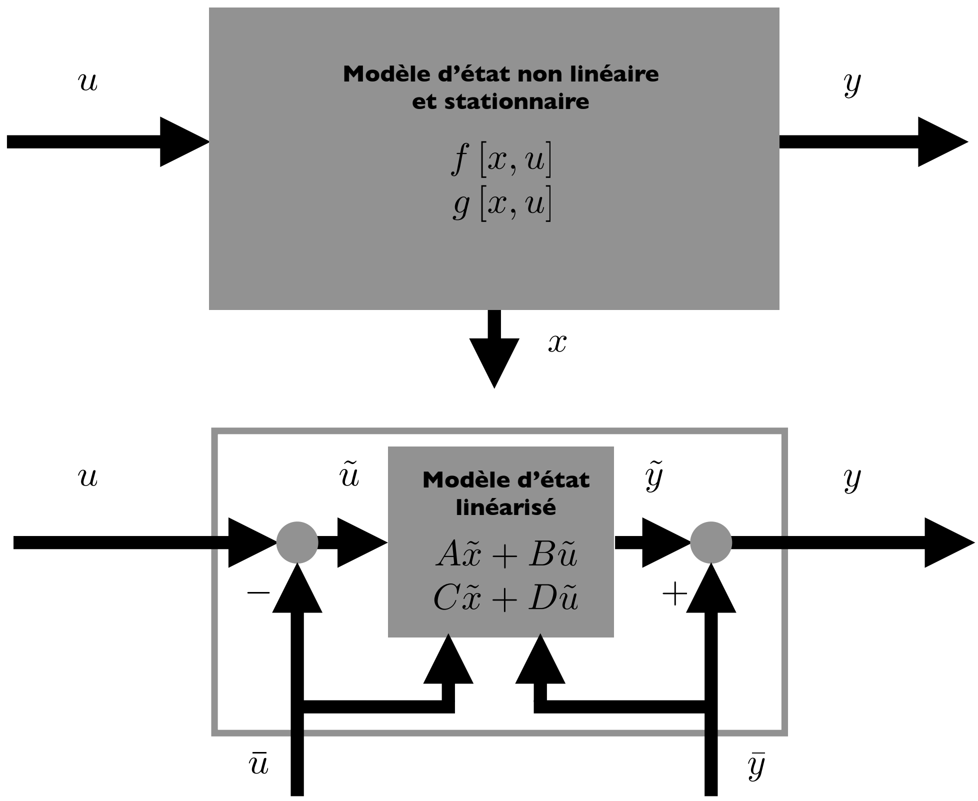
Cuve de mélange: Modèle d'état linéarisé



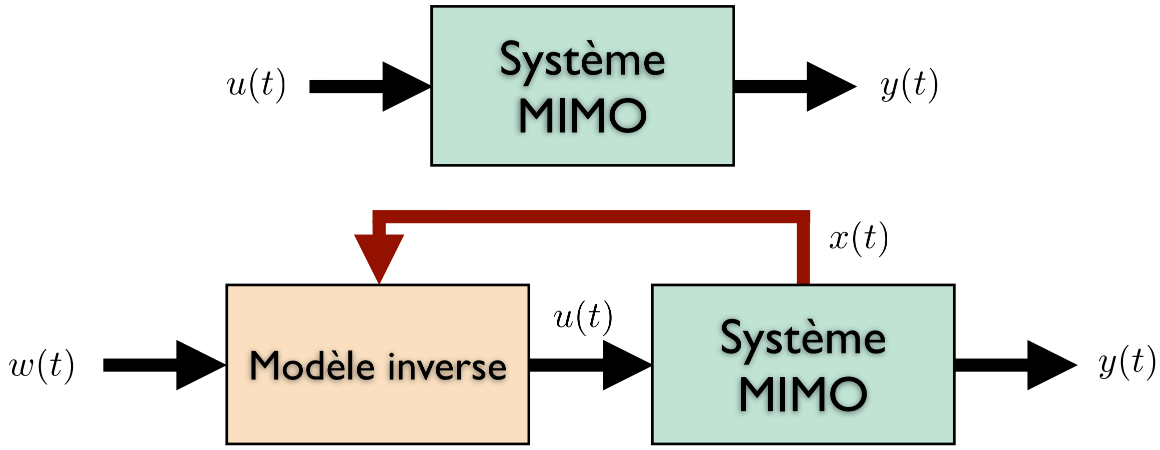
$$\begin{bmatrix}\dot{\tilde{x}}_1\\ \dot{\tilde{x}}_2\end{bmatrix} = \underbrace{-\frac{1}{\bar{x}_1}\begin{bmatrix}\frac{K}{2}\sqrt{\frac{\bar{x}_1}{S}} & 0\\ 0 & \bar{u}_1+\bar{u}_2\end{bmatrix}}_{\begin{bmatrix}a & 0\\ 0 & b\end{bmatrix}} \begin{bmatrix}\tilde{x}_1\\ \tilde{x}_2\end{bmatrix} + \underbrace{\begin{bmatrix}1 & 1\\ \frac{(c_1-\bar{x}_2)}{\bar{x}_1} & \frac{(c_2-\bar{x}_2)}{\bar{x}_1}\end{bmatrix}}_{\begin{bmatrix}1 & 1\\ p & q\end{bmatrix}} \begin{bmatrix}\tilde{u}_1\\ \tilde{u}_2\end{bmatrix}$$

$$\begin{bmatrix}\tilde{y}_1\\ \tilde{y}_2\end{bmatrix} = \begin{bmatrix}\frac{1}{S} & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}\tilde{x}_1\\ \tilde{x}_2\end{bmatrix}$$

Résumé linéarisation par la tangente



Linéarisation par contre-réaction

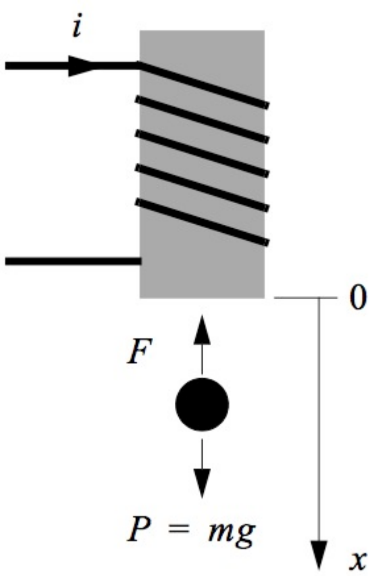


Globale

Prix à payer: Besoin de l'état

Pas toujours possible !

Sustentation: Modèle inverse



Modèle physique

$$F(x,i) = \frac{1}{2}\frac{L}{(1+x)^2}i^2$$

$$m\ddot{x} = mg - F(x,i)$$

$$\ddot{x} = g - \frac{1}{2m}\frac{L}{(1+x)^2}i^2$$

Modèle d'état

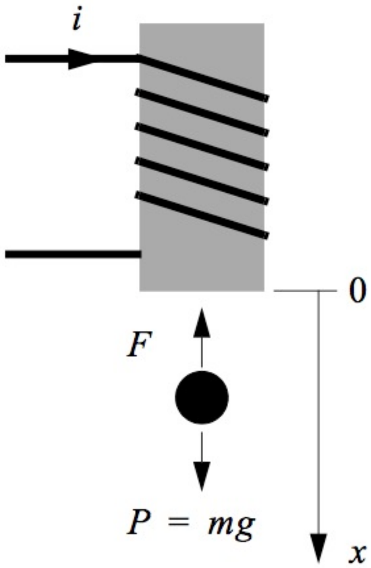
$$u = i, \quad x_1 = x \quad x_2 = \dot{x}$$

$$\dot{x}_1 = \dot{x} = x_2 = f_1(x,u)$$

$$\dot{x}_2 = \ddot{x} = g - \tfrac{L}{2m(1+x_1)^2}u^2 = f_2(x,u)$$

$$y = x_1 = g_1(x,u)$$

Sustentation: Modèle inverse



Modèle d'état

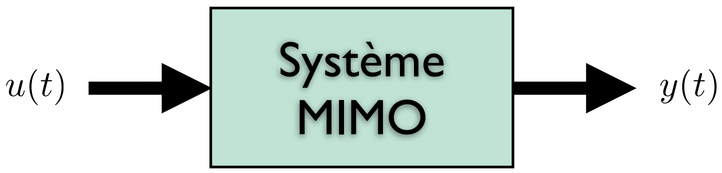
$$\begin{aligned} x_1 &= x = y \\ \dot{x}_1 &= \dot{x} = x_2 \\ \dot{x}_2 &= \ddot{x} = \ddot{y} = g - \tfrac{L}{2m(1+x_1)^2}u^2 \\ y &= x_1 \end{aligned}$$

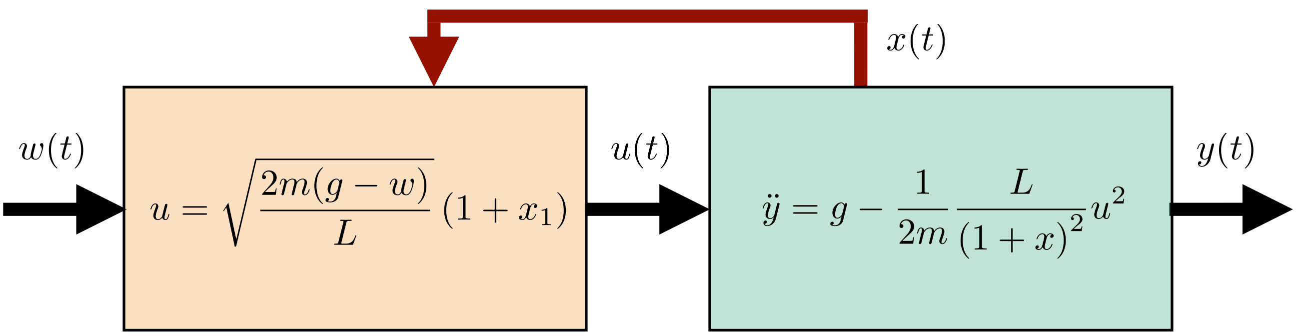
Modèle inverse

$$\begin{aligned} y &= x_1 \\ \dot{y} &= \dot{x}_1 = x_2 \\ \ddot{y} &= \dot{x}_2 = g - \tfrac{1}{2m}\tfrac{L}{(1+x_1)^2}u^2 \equiv w \end{aligned}$$

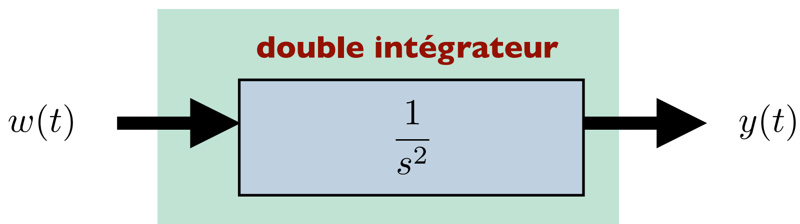
$$u = \sqrt{\frac{2m(g-w)}{L}}(1+x_1)$$

Sustentation: Modèle inverse

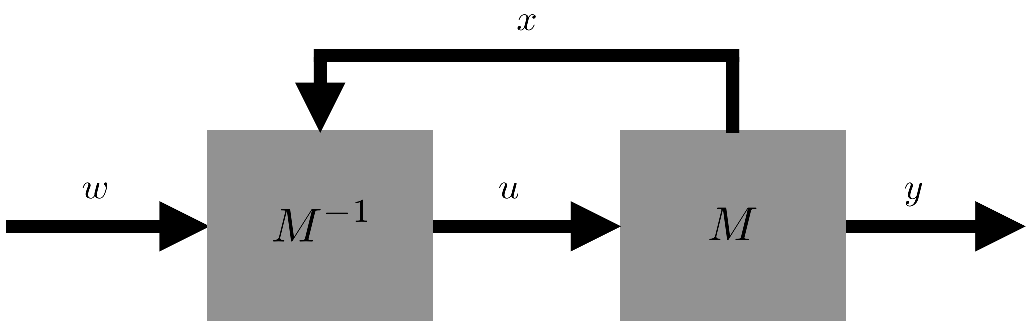




$\ddot{y} = w$

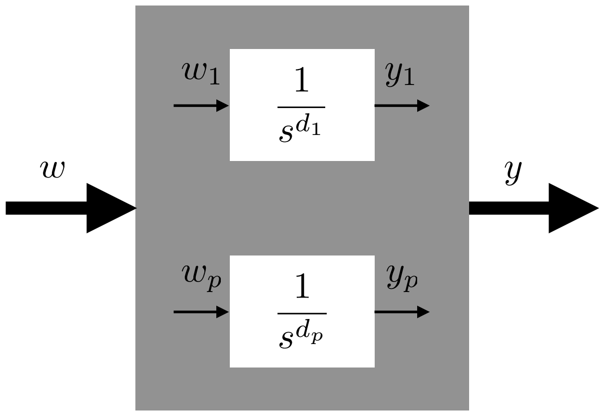


Linéarisation par contre-réaction

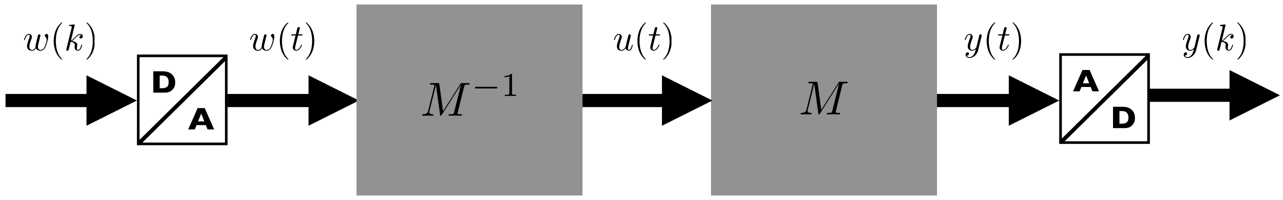


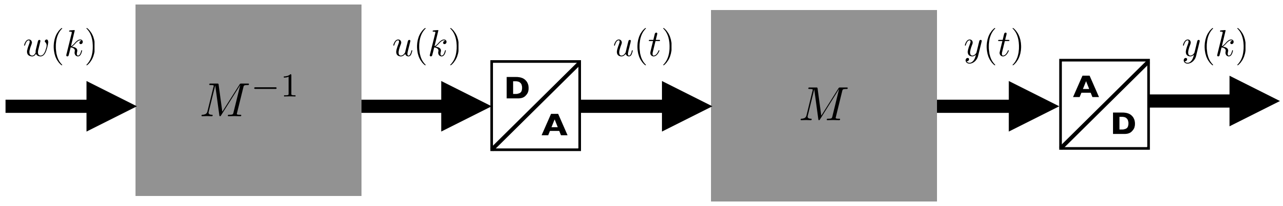
$$y_i = g_i(x) \qquad i = 1, \ldots, p$$
$$\vdots$$
$$y_i^{(d_i)} = g_i^*(x,u) \quad d_i = 0, \ldots, \bar{d}_i$$

$$y_i^{(\bar{d}_i)} = w_i$$



Linéarisation par contre-réaction

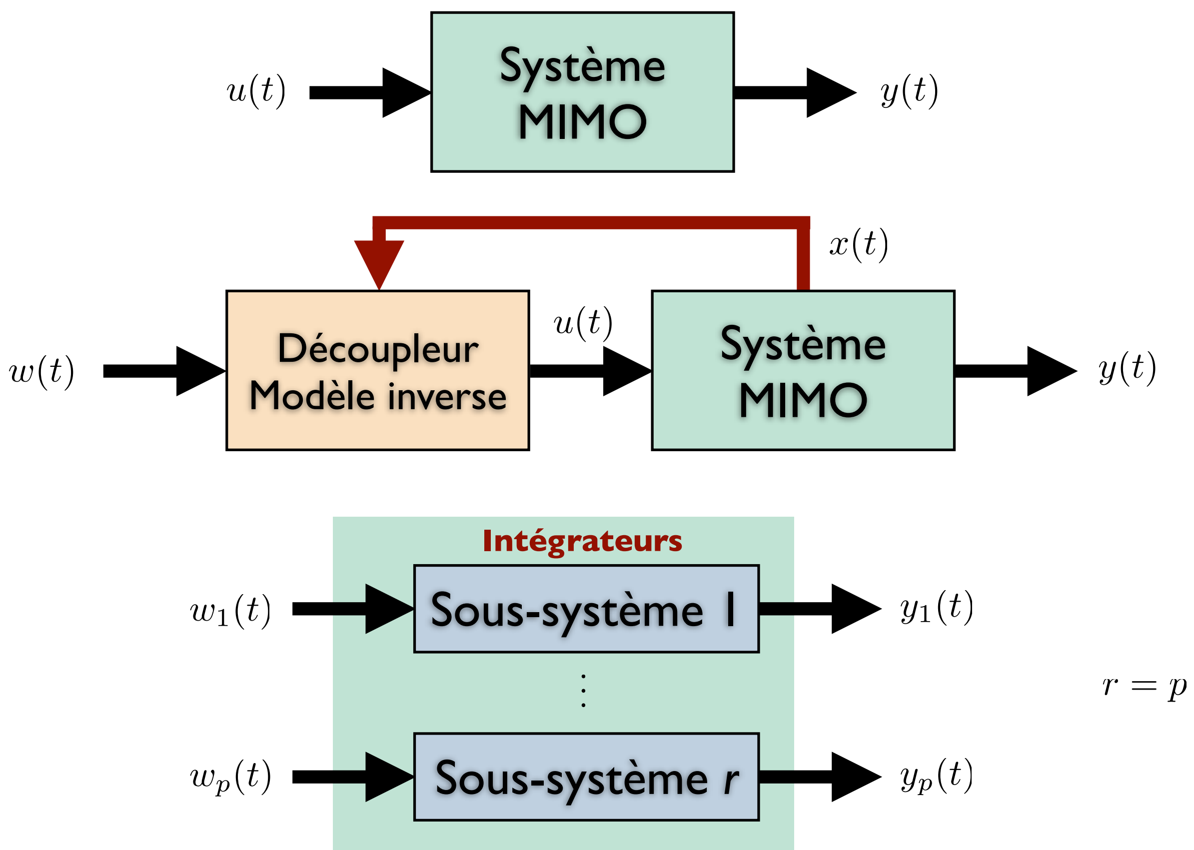




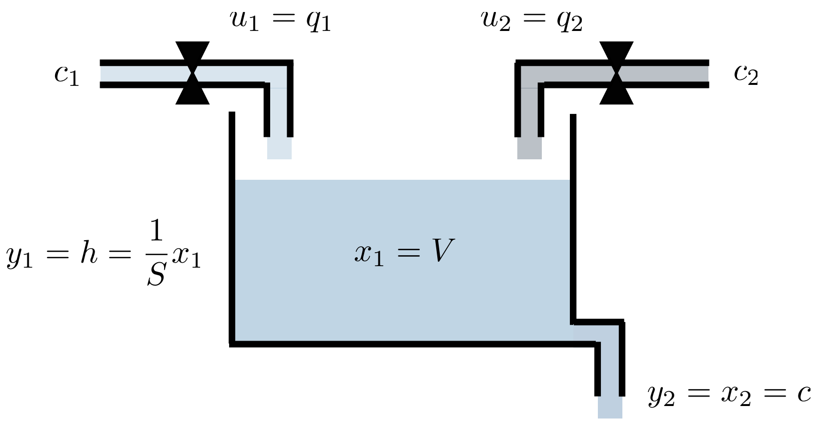
Modèle d'état

	Analogique	Discret
Non linéaire	$\dot{x}(t) = f[x(t), u(t), t]$ $y(t) = g[x(t), u(t), t]$	$x(k+1) = f[x(k), u(k), k]$ $y(k) = g[x(k), u(k), k]$
Linéarisé (tangente)	$\dot{\tilde{x}}(t) = \overbrace{\left.\frac{\partial f}{\partial x}\right\|_{\bar{x},\bar{u}}}^{A} \tilde{x}(t) + \overbrace{\left.\frac{\partial f}{\partial u}\right\|_{\bar{x},\bar{u}}}^{B} \tilde{u}(t)$ $\tilde{y}(t) = \underbrace{\left.\frac{\partial g}{\partial x}\right\|_{\bar{x},\bar{u}}}_{C} \tilde{x}(t) + \underbrace{\left.\frac{\partial g}{\partial x}\right\|_{\bar{x},\bar{u}}}_{D} \tilde{u}(t)$ $\tilde{x} = x - \bar{x}; \quad \tilde{y} = y - \bar{y}; \quad \tilde{u} = u - \bar{u}$	$\tilde{x}(k+1) = \overbrace{\left.\frac{\partial f}{\partial x}\right\|_{\bar{x},\bar{u}}}^{\Phi} \tilde{x}(k) + \overbrace{\left.\frac{\partial f}{\partial u}\right\|_{\bar{x},\bar{u}}}^{\Gamma} \tilde{u}(k)$ $\tilde{y}(k) = \underbrace{\left.\frac{\partial g}{\partial x}\right\|_{\bar{x},\bar{u}}}_{C} \tilde{x}(k) + \underbrace{\left.\frac{\partial g}{\partial x}\right\|_{\bar{x},\bar{u}}}_{D} \tilde{u}(k)$ $\tilde{x} = x - \bar{x}; \quad \tilde{y} = y - \bar{y}; \quad \tilde{u} = u - \bar{u}$
Linéaire ou linéarisé par contre-réaction	$\dot{x}(t) = Ax(t) + Bu(t)$ $y(t) = Cx(t) + Du(t)$	$x(k+1) = \Phi x(k) + \Gamma u(k)$ $y(k) = Cx(k) + Du(k)$

Découplage non linéaire



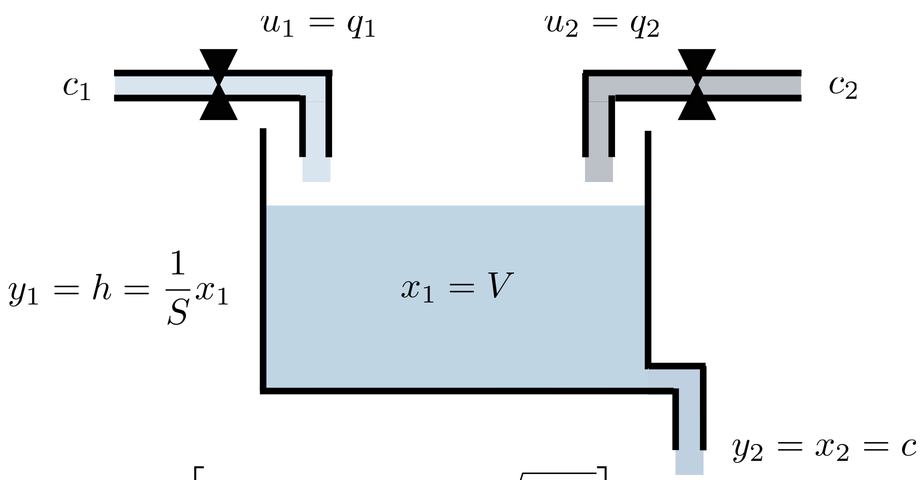
Cuve de mélange: Modèle inverse



$$\left| \begin{array}{l} \dot{x}_1(t) = u_1(t) + u_2(t) - K\sqrt{\frac{x_1(t)}{S}} \\ \dot{x}_2(t) = \frac{1}{x_1(t)} \left\{ [c_1 - x_2(t)]\, u_1(t) + [c_2 - x_2(t)]\, u_2(t) \right\} \end{array} \right. \qquad \left| \begin{array}{l} y_1(t) = \frac{1}{S} x_1(t) \\ y_2(t) = x_2(t) \end{array} \right.$$

$$\left| \begin{array}{l} \dot{y}_1(t) = \frac{1}{S}\dot{x}_1(t) = \frac{1}{S}\left[u_1(t) + u_2(t) - K\sqrt{\frac{x_1(t)}{S}} \right] \equiv w_1(t) \\ \dot{y}_2(t) = \dot{x}_2(t) = \frac{1}{x_1(t)} \left\{ [c_1 - x_2(t)]\, u_1(t) + [c_2 - x_2(t)]\, u_2(t) \right\} \equiv w_2(t) \end{array} \right.$$

Cuve de mélange: Modèle inverse



$$\left| \begin{array}{l} \dot{y}_1(t) = \frac{1}{S}\dot{x}_1(t) = \frac{1}{S}\left[u_1(t) + u_2(t) - K\sqrt{\frac{x_1(t)}{S}} \right] \equiv w_1(t) \\ \dot{y}_2(t) = \dot{x}_2(t) = \frac{1}{x_1(t)} \left\{ [c_1 - x_2(t)]\, u_1(t) + [c_2 - x_2(t)]\, u_2(t) \right\} \equiv w_2(t) \end{array} \right.$$

$$\left| \begin{array}{l} u_1(t) = \frac{1}{c_2 - c_1} \left\{ [c_2 - x_2(t)] \left[w_1(t) S + K\sqrt{\frac{x_1(t)}{S}} \right] - w_2(t) x_1(t) \right\} \\ u_2(t) = \frac{1}{c_2 - c_1} \left\{ w_2(t) x_1(t) - [c_1 - x_2(t)] \left[w_1(t) S + K\sqrt{\frac{x_1(t)}{S}} \right] \right\} \end{array} \right.$$

Cuve de mélange: Modèle inverse