Systèmes analogiques non linéaires
Nonlinear continuous time systems

$$\left|\begin{array}{l} \dot{x}_1(t) = f_1\left[x_1(t), \ldots, x_n(t), u_1(t), \ldots u_r(t), t\right] \\ \dot{x}_2(t) = f_2\left[x_1(t), \ldots, x_n(t), u_1(t), \ldots u_r(t), t\right] \\ \vdots \\ \dot{x}_n(t) = f_n\left[x_1(t), \ldots, x_n(t), u_1(t), \ldots u_r(t), t\right] \end{array}\right. \qquad \dot{x}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \vdots \\ \dot{x}_n(t) \end{bmatrix}$$

$$\left|\begin{array}{l} y_1(t) = g_1\left[x_1(t), \ldots, x_n(t), u_1(t), \ldots u_r(t), t\right] \\ y_2(t) = g_2\left[x_1(t), \ldots, x_n(t), u_1(t), \ldots u_r(t), t\right] \\ \vdots \\ y_p(t) = g_p\left[x_1(t), \ldots, x_n(t), u_1(t), \ldots u_r(t), t\right] \end{array}\right. \qquad y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_p(t) \end{bmatrix}$$

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix} \quad u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_r(t) \end{bmatrix} \quad f\left[x(t), u(t), t\right] = \begin{bmatrix} f_1\left[x(t), u(t), t\right] \\ f_2\left[x(t), u(t), t\right] \\ \vdots \\ f_n\left[x(t), u(t), t\right] \end{bmatrix}$$

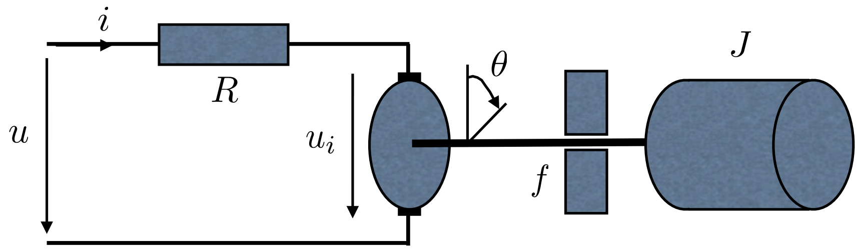
Systèmes analogiques non linéaires
Nonlinear continuous time systems

$$\left|\begin{array}{l} \dot{x}_1(t) = f_1\left[x_1(t), \ldots, x_n(t), u_1(t), \ldots u_r(t), t\right] \\ \dot{x}_2(t) = f_2\left[x_1(t), \ldots, x_n(t), u_1(t), \ldots u_r(t), t\right] \\ \vdots \\ \dot{x}_n(t) = f_n\left[x_1(t), \ldots, x_n(t), u_1(t), \ldots u_r(t), t\right] \end{array}\right.$$

$$\left|\begin{array}{l} y_1(t) = g_1\left[x_1(t), \ldots, x_n(t), u_1(t), \ldots u_r(t), t\right] \\ y_2(t) = g_2\left[x_1(t), \ldots, x_n(t), u_1(t), \ldots u_r(t), t\right] \\ \vdots \\ y_p(t) = g_p\left[x_1(t), \ldots, x_n(t), u_1(t), \ldots u_r(t), t\right] \end{array}\right.$$

$$\begin{array}{l} \dot{x}(t) = f\left[x(t), u(t), t\right] \\ y(t) = g\left[x(t), u(t), t\right] \end{array}$$

Entraînement électrique
Electrical Drive



Modèle physique

$$\dot{\omega}(t) = -\frac{1}{J}\left(\frac{k^2}{R} + f\right)\omega(t) + \frac{k}{JR}u(t)$$
$$\omega(t) = \dot{\theta}(t)$$

Choix des variables d'état

$$x_1(t) = \theta(t)$$
$$x_2(t) = \omega(t)$$
$$y(t) = \theta(t)$$

Modèle d'état: équation d'état

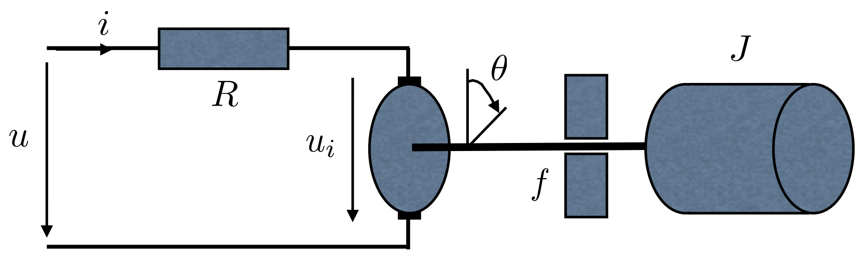
$$\dot{x}_1(t) = x_2(t)$$

$$\dot{x}_2(t) = -\frac{1}{J}\left(\frac{k^2}{R} + f\right)x_2(t) + \frac{k}{JR}u(t)$$

Modèle d'état: équation de sortie

$$y(t) = x_1(t)$$

Entraînement électrique
Electrical Drive



Modèle d'état: équation d'état

$$\dot{x}_1(t) = x_2(t)$$
$$\dot{x}_2(t) = \underbrace{-\frac{1}{J}\left(\frac{k^2}{R} + f\right)}_{a}x_2(t) + \underbrace{\frac{k}{JR}}_{b}u(t)$$

Modèle d'état: équation de sortie

$$y(t) = x_1(t)$$

Modèle d'état: équation d'état

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & a \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ b \end{bmatrix}u(t)$$

Modèle d'état: équation de sortie

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

Systèmes analogiques linéaires
Linear continuous time systems

$$\dot{x}(t) = f\left[x(t), u(t), t\right]$$
$$y(t) = g\left[x(t), u(t), t\right]$$

Non linéaire

$$\dot{x}(t) = A(t)x(t) + B(t)u(t)$$
$$y(t) = C(t)x(t) + D(t)u(t)$$

Linéaire

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$y(t) = Cx(t) + Du(t)$$

Linéaire et
stationnaire

Méthode de Runge-Kutta

$$x\left(kh+h\right) \approx x\left(kh\right) + \frac{1}{6}\left[a(kh) + 2b\left(kh\right) + 2c\left(kh\right) + d\left(kh\right)\right]$$

$$y\left(kh\right) = g\left[x\left(kh\right), u\left(kh\right), kh\right]$$

$$\begin{aligned}
a\left(kh\right) &= hf\left[x\left(kh\right), u\left(kh\right), kh\right] \\
b\left(kh\right) &= hf\left[x\left(kh\right) + \tfrac{a(kh)}{2}, u\left(kh + \tfrac{h}{2}\right), kh + \tfrac{h}{2}\right] \\
c\left(kh\right) &= hf\left[x\left(kh\right) + \tfrac{b(kh)}{2}, u\left(kh + \tfrac{h}{2}\right), kh + \tfrac{h}{2}\right] \\
d\left(kh\right) &= hf\left[x\left(kh\right) + c\left(kh\right), u\left(kh+h\right), kh+h\right]
\end{aligned}$$

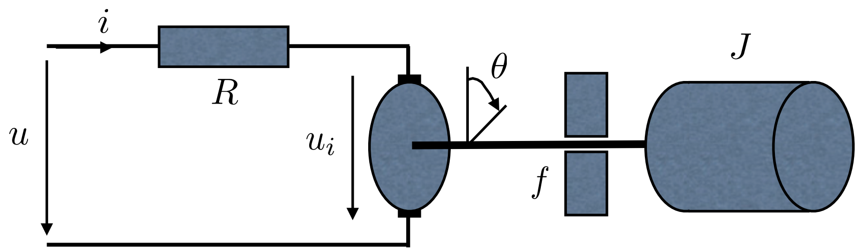
Systèmes discrets non linéaires
Nonlinear discrete time systems

$$\left| \begin{array}{l} x_1(k+1) = f_1\left[x_1(k), \ldots, x_n(k), u_1(k), \ldots, u_r(k), k\right] \\ x_2(k+1) = f_2\left[x_1(k), \ldots, x_n(k), u_1(k), \ldots, u_r(k), k\right] \\ \vdots \\ x_n(k+1) = f_n\left[x_1(k), \ldots, x_n(k), u_1(k), \ldots, u_r(k), k\right] \end{array} \right. \qquad x(k+1) = \begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ \vdots \\ x_n(k+1) \end{bmatrix}$$

$$\left| \begin{array}{l} y_1(k) = g_1\left[x_1(k), \ldots, x_n(k), u_1(k), \ldots, u_r(k), k\right] \\ y_2(k) = g_2\left[x_1(k), \ldots, x_n(k), u_1(k), \ldots, u_r(k), k\right] \\ \vdots \\ y_p(k) = g_p\left[x_1(k), \ldots, x_n(k), u_1(k), \ldots, u_r(k), k\right] \end{array} \right. \qquad y(k) = \begin{bmatrix} y_1(k) \\ y_2(k) \\ \vdots \\ y_p(k) \end{bmatrix}$$

$$x(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \\ \vdots \\ x_n(k) \end{bmatrix} \quad u(k) = \begin{bmatrix} u_1(k) \\ u_2(k) \\ \vdots \\ u_r(k) \end{bmatrix} \quad f\left[x(k), u(k), k\right] = \begin{bmatrix} f_1\left[x(k), u(k), k\right] \\ f_2\left[x(k), u(k), k\right] \\ \vdots \\ f_n\left[x(k), u(k), k\right] \end{bmatrix}$$

Entraînement électrique
Electrical Drive



Modèle analogique

$$G(s) = \frac{\theta(s)}{U(s)} = \frac{A}{s\,(1 + s\tau)} \qquad \tau = -\frac{1}{a} \qquad A = -\frac{b}{a}$$

Modèle discret vu au travers de convertisseurs AD & DA

$$H(z) = \left(1 - z^{-1}\right) Z\left\{L^{-1}\left[\frac{G(s)}{s}\right]\right\}$$

$$\alpha = -e^{-h/\tau}$$
$$b_1 = A\left[h - \tau\,(1 + \alpha)\right]$$
$$b_2 = A\left[\tau\,(1 + \alpha) + h\alpha\right]$$

$$H(z) = \frac{\theta(z)}{U(z)} = \frac{b_1 z^{-1} + b_2 z^{-2}}{\left(1 - z^{-1}\right)\left(1 + \alpha z^{-1}\right)} = \frac{b_1 z^{-1} + b_2 z^{-2}}{1 + (\alpha - 1)\,z^{-1} - \alpha z^{-2}}$$

FT → Modèle d'état

$$H(z) = \frac{Y(z)}{U(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \ldots + b_n z^{-n}}{1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_n z^{-n}}$$

$$Y(z) = b_0 U(z) + \underbrace{\frac{(b_1 - a_1 b_0) z^{-1} + (b_2 - a_2 b_0) z^{-2} + \ldots + (b_n - a_n b_0) z^{-n}}{1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_n z^{-n}} U(z)}_{\tilde{Y}(z)}$$

$$\frac{\tilde{Y}(z)}{U(z)} = \frac{(b_1 - a_1 b_0) z^{-1} + (b_2 - a_2 b_0) z^{-2} + \ldots + (b_n - a_n b_0) z^{-n}}{1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_n z^{-n}}$$

$$\frac{\tilde{Y}(z)}{(b_1 - a_1 b_0) z^{-1} + (b_2 - a_2 b_0) z^{-2} + \ldots + (b_n - a_n b_0) z^{-n}} = \frac{U(z)}{1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_n z^{-n}} \equiv Q(z)$$

FT → Modèle d'état

$$H(z) = \frac{Y(z)}{U(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \ldots + b_n z^{-n}}{1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_n z^{-n}}$$

$$Y(z) = b_0 U(z) + \underbrace{\frac{(b_1 - a_1 b_0) z^{-1} + (b_2 - a_2 b_0) z^{-2} + \ldots + (b_n - a_n b_0) z^{-n}}{1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_n z^{-n}} U(z)}_{\tilde{Y}(z)}$$

$$\frac{\tilde{Y}(z)}{U(z)} = \frac{(b_1 - a_1 b_0) z^{-1} + (b_2 - a_2 b_0) z^{-2} + \ldots + (b_n - a_n b_0) z^{-n}}{1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_n z^{-n}}$$

$$\frac{U(z)}{1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_n z^{-n}} \equiv Q(z)$$

$$U(z) = Q(z) + a_1 z^{-1} Q(z) + a_2 z^{-2} Q(z) + \ldots + a_n z^{-n} Q(z)$$

FT → Modèle d'état

$$H(z) = \frac{Y(z)}{U(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \ldots + b_n z^{-n}}{1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_n z^{-n}}$$

$$Y(z) = b_0 U(z) + \underbrace{\frac{(b_1 - a_1 b_0) z^{-1} + (b_2 - a_2 b_0) z^{-2} + \ldots + (b_n - a_n b_0) z^{-n}}{1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_n z^{-n}} U(z)}_{\tilde{Y}(z)}$$

$$\frac{\tilde{Y}(z)}{U(z)} = \frac{(b_1 - a_1 b_0) z^{-1} + (b_2 - a_2 b_0) z^{-2} + \ldots + (b_n - a_n b_0) z^{-n}}{1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_n z^{-n}}$$

$$\frac{\tilde{Y}(z)}{(b_1 - a_1 b_0) z^{-1} + (b_2 - a_2 b_0) z^{-2} + \ldots + (b_n - a_n b_0) z^{-n}} \equiv Q(z)$$

$$\tilde{Y}(z) = (b_1 - a_1 b_0) z^{-1} Q(z) + (b_2 - a_2 b_0) z^{-2} Q(z) + \ldots + (b_n - a_n b_0) z^{-n} Q(z)$$

FT → Modèle d'état

$$Q(z) = -a_1 z^{-1} Q(z) - a_2 z^{-2} Q(z) - \ldots - a_n z^{-n} Q(z) + U(z)$$

$$\tilde{Y}(z) = (b_1 - a_1 b_0) z^{-1} Q(z) + (b_2 - a_2 b_0) z^{-2} Q(z) + \ldots + (b_n - a_n b_0) z^{-n} Q(z)$$

$$\left| \begin{array}{lcl} W_1(z) & = & z^{-1} Q(z) \\ W_2(z) & = & z^{-2} Q(z) \\ & \vdots & \\ W_n(z) & = & z^{-n} Q(z) \end{array} \right.$$

$$\left| \begin{array}{lcrcc} zW_1(z) & = & Q(z) & = & -a_1 W_1(z) - a_2 W_2(z) - \ldots - a_n W_n(z) + U(z) \\ zW_2(z) & = & z^{-1} Q(z) & = & W_1(z) \\ & \vdots & & & \\ zW_n(z) & = & z^{-n+1} Q(z) & = & W_{n-1}(z) \end{array} \right.$$

$$\tilde{Y}(z) = (b_1 - a_1 b_0) W_1(z) + (b_2 - a_2 b_0) W_2(z) + \ldots + (b_n - a_n b_0) W_n(z)$$

$$Y(z) = b_0 U(z) + \tilde{Y}(z)$$

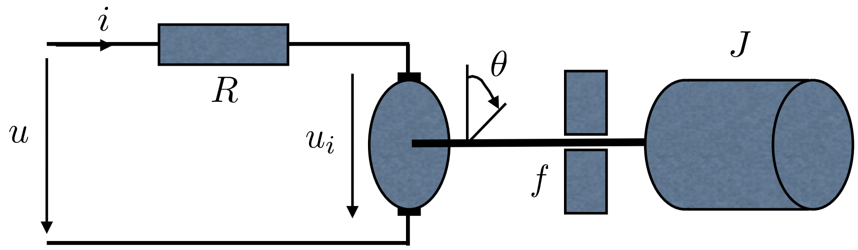
FT → Modèle d'état

$$w(k+1) = \underbrace{\begin{bmatrix} -a_1 & -a_2 & \dots & -a_{n-1} & -a_n \\ 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & & \vdots & \vdots \\ \vdots & & \ddots & 0 & 0 \\ 0 & \dots & 0 & 1 & 0 \end{bmatrix}}_{\Phi_w} w(k) + \underbrace{\begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}}_{g_w} u(k)$$

$$y(k) = \underbrace{\begin{bmatrix} b_1 - a_1 b_0 & b_2 - a_2 b_0 & \dots & b_n - a_n b_0 \end{bmatrix}}_{c_w^T} w(k) + b_0 u(k)$$

$$H(z) = \frac{Y(z)}{U(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_n z^{-n}}{1 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_n z^{-n}}$$

Entraînement électrique
Electrical Drive



Modèle discret vu au travers de convertisseurs AD & DA

$$H(z) = \frac{\theta(z)}{U(z)} = \frac{b_1 z^{-1} + b_2 z^{-2}}{(1 - z^{-1})(1 + \alpha z^{-1})} = \frac{b_1 z^{-1} + b_2 z^{-2}}{1 + (\alpha - 1) z^{-1} - \alpha z^{-2}}$$

$$\alpha = -e^{-h/\tau} \qquad b_1 = A\left[h - \tau(1 + \alpha)\right] \qquad b_2 = A\left[\tau(1 + \alpha) + h\alpha\right]$$

Modèle d'état discret artificiel correspondant

$$\begin{bmatrix} w_1(k+1) \\ w_2(k+1) \end{bmatrix} = \begin{bmatrix} -(\alpha - 1) & \alpha \\ 1 & 0 \end{bmatrix} \begin{bmatrix} w_1(k) \\ w_2(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(k)$$

$$y(k) = \theta(k) = \begin{bmatrix} b_1 & b_2 \end{bmatrix} \begin{bmatrix} w_1(k) \\ w_2(k) \end{bmatrix} + [0]\, u(k)$$

Modèle d'état/State-space Model

	Analogique/Continuous	Discret/Discrete
Non linéaire Nonlinear	$\dot{x}(t) = f[x(t), u(t), t]$ $y(t) = g[x(t), u(t), t]$	$x(k+1) = f[x(k), u(k), k]$ $y(k) = g[x(k), u(k), k]$
Linéaire Linear	$\dot{x}(t) = A(t)x(t) + B(t)u(t)$ $y(t) = C(t)x(t) + D(t)u(t)$	$x(k+1) = \Phi(k)x(k) + \Gamma(k)u(k)$ $y(k) = C(k)x(k) + D(k)u(k)$
Linéaire et stationnaire Linear and time-invariant	$\dot{x}(t) = Ax(t) + Bu(t)$ $y(t) = Cx(t) + Du(t)$	$x(k+1) = \Phi x(k) + \Gamma u(k)$ $y(k) = Cx(k) + Du(k)$