Theory of Ground State Cooling of a Mechanical Oscillator Using Dynamical Backaction

I. Wilson-Rae,[1] N. Nooshi,[2] W. Zwerger,[1,*] and T. J. Kippenberg[2,†]
[1]*Technische Universität München, D-85748 Garching, Germany*
[2]*Max Planck Institut für Quantenoptik, D-85748 Garching, Germany*
(Received 5 February 2007; published 28 August 2007)

A quantum theory of cooling of a mechanical oscillator by radiation pressure-induced dynamical backaction is developed, which is analogous to sideband cooling of trapped ions. We find that final occupancies well below unity can be attained when the mechanical oscillation frequency is larger than the optical cavity linewidth. It is shown that the final average occupancy can be retrieved directly from the optical output spectrum.

DOI: 10.1103/PhysRevLett.99.093901 PACS numbers: 42.50.Vk, 03.65.Ta, 07.10.Cm

Mesoscopic mechanical oscillators are currently attracting interest due to their potential to enhance the sensitivity of displacement measurements [1] and to probe the quantum to classical transition of a macroscopic degree of freedom [2]. For these applications the capability of initializing an oscillator with a long phonon lifetime in its quantum ground state is highly desirable. So far this has never been demonstrated because the combination of sufficiently high mechanical frequencies ($\omega_m/2\pi$) and Q values in the relevant regime $\hbar\omega_m \gg k_B T$ has not been reached [2]. In contrast, in atomic physics laser cooling has enabled the preparation of motional ground states [3,4]. This has prompted researchers to study means of cooling a single mechanical resonator mode directly using laser radiation [5–11]. Early work demonstrated cooling of a mechanical degree of freedom of a Fabry-Pérot mirror using a radiation pressure force controlled by an electronic feedback scheme [5,6], in analogy to stochastic cooling. Alternatively, the radiation pressure-induced coupling of an optical cavity mode to a mechanical oscillator [cf. Fig. 1(a)] can give rise to self-cooling via *dynamical backaction* [12]. In essence, the cavity delay induces correlations between the radiation pressure force and the Brownian motion that lead to cooling or amplification, depending on the laser detuning. In a series of experiments, these effects have been used to cool a single mechanical mode [8,9,11]. While classical and semiclassical analysis of dynamical backaction have been developed [11,13,14], the question as to whether *ground state cooling* is possible has not been addressed.

Here a quantum theory of cooling via dynamical backaction is presented. We find that final occupancies below unity can indeed be attained when the optical cavity's photon lifetime (τ) is comparable to or exceeds the mechanical oscillation period ($2\pi/\omega_m$). Along these lines, an analogy between this mechanism and the sideband cooling of trapped ions in the Lamb-Dicke regime is elucidated [4]. In our setting the optical cavity mode plays the role of the pseudospin mediating the frequency up-conversion. To unveil this mechanism it is convenient to adopt a "shifted" representation in which the steady state of the laser driven optical cavity mode maps onto the vacuum ($|0\rangle_p$). Denoting by $|n\rangle$ the number states of the mechanical oscillator we have anti-Stokes (Stokes) processes in which the transition $|0\rangle_p|n\rangle \rightarrow |1\rangle_p|n-1\rangle$ ($|0\rangle_p|n\rangle \rightarrow |1\rangle_p|n+1\rangle$) followed by the decay of the cavity photon leads to cooling (amplification) [cf. Fig. 1(b)]. If the cavity photon is sufficiently long-lived (i.e., $\tau \gtrsim 2\pi/\omega_m$) a red detuning of the laser from the cavity mode by an amount ω_m will ensure that the desired anti-Stokes process is resonantly enhanced while the deleterious Stokes process, being off-resonant, is suppressed.

We treat the laser driven optical cavity mode coupled to the mechanical resonator mode as an open quantum system and adopt a rotating frame at the laser frequency ω_L. The system Hamiltonian is given by [15]

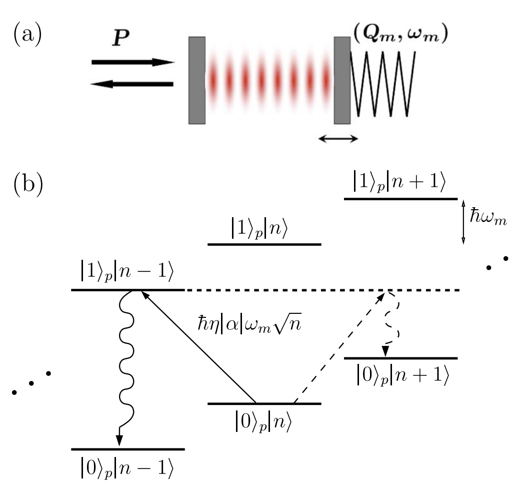

FIG. 1 (color online). (a) Fabry-Pérot equivalent of a mechanical eigenmode (frequency $\omega_m/2\pi$ and Q value Q_m) coupled to an optical cavity mode. (b) Level diagram of the two modes in a shifted representation for perturbative optomechanical coupling η. Light scattering processes induced by the latter can decrease (solid lines) or increase (dashed) the mechanical eigenmode's quantum number n (α is the steady state amplitude in the optical cavity mode and $|0\rangle_p, |1\rangle_p, \ldots$ its Fock states).

$$H' = -\hbar\Delta'_L a_p^\dagger a_p + \hbar\eta\omega_m a_p^\dagger a_p(a_m + a_m^\dagger) + \hbar\frac{\Omega}{2}(a_p + a_p^\dagger) + \hbar\omega_m a_m^\dagger a_m. \quad (1)$$

Here a_p (a_m) is the annihilation operator for the optical (mechanical) oscillator, ω_p (ω_m) is its angular frequency, and Δ'_L is the laser detuning from the optical resonance. We have also introduced the driving amplitude $\Omega \equiv 2\sqrt{P/\hbar\omega_L\tau_{\rm ex}}$, where P is the input laser power and $1/\tau_{\rm ex}$ is the photon decay rate into the associated outgoing modes. The optomechanical coupling via radiation pressure can be characterized by the dimensionless parameter $\eta \equiv (\omega_p/\omega_m)(l_m/L)$, with $l_m = \sqrt{\hbar/2m\omega_m}$ the zero point motion of the mechanical resonator mode, m its effective mass, and L an effective optical cavity length. For typical materials and dimensions [11] one obtains $\eta \sim 10^{-4}$.

The optical cavity losses and the intrinsic dissipation of the mechanical resonator give rise to a dissipative contribution to the Liouvillian $\mathcal{L}'_D$ (i.e., $\mathcal{L}' = -(i/\hbar)[H', \ldots] + \mathcal{L}'_D$) that is of Lindblad form with collapse operators [16]: $\sqrt{1/\tau}a_p$, $\sqrt{\gamma_m n(\omega_m)}a_m^\dagger$, and $\sqrt{\gamma_m[n(\omega_m)+1]}a_m$. Here $\gamma_m = \omega_m/Q_m$ is the mechanical oscillator's natural linewidth and $n(\omega_m)$ its Bose number at the environmental temperature. We will focus on the regime $[n(\omega_m) + 1]\gamma_m \ll \omega_m$, $\gamma_m \ll 1/\tau$ and η, $\eta|\alpha| \ll 1, 1/\omega_m\tau$, where α is the steady state amplitude in the optical cavity mode (see below). The first condition will turn out to be necessary for ground state cooling, the second one is satisfied in all recent experiments [8,9,11], and the last one, given the smallness of η, will hinge on having a sufficiently *low input power*.

To study the dynamics generated by $\mathcal{L}'$, it proves useful to apply a shift to the normal coordinates: $a_p \to a_p + \alpha$, $a_m \to a_m + \beta$ with the c numbers α and β chosen to cancel out all the linear terms in the transformed Liouvillian. To lowest order in the small parameters η and $1/Q_m$ we have the following: $\alpha \approx \Omega\tau/(2\tau\Delta'_L + i)$, $\beta \approx -\eta|\alpha|^2$. We include the radiation pressure-induced optical resonance shift into the effective detuning $\Delta'_L + 2\eta^2|\alpha|^2\omega_m \to \Delta_L$. While the dissipative part of the Liouvillian remains invariant, the Hamiltonian transforms into

$$H = -\hbar\Delta_L a_p^\dagger a_p + \hbar\omega_m a_m^\dagger a_m + \hbar\eta\omega_m(a_p^\dagger a_p + \alpha^* a_p + \alpha a_p^\dagger)(a_m + a_m^\dagger). \quad (2)$$

Henceforth we will refer to the primed representation (1) as the "physical" one and to the unprimed representation (2) as the shifted one.

The smallness of η^2 and $[n(\omega_m) + 1]/Q_m$ imply a wide separation between the time scales for cooling and heating the mechanical oscillator and those characterizing the dynamics of the optical cavity mode and the mechanical oscillation period. Thus, the electromagnetic environment (including the optical cavity) can be regarded as a structured reservoir with which the mechanical mode interacts perturbatively [cf. Fig. 1(b)]. This prompts us to derive a "generalized quantum optical" master equation for the reduced density matrix [16] of the latter: $\mu = {\rm Tr}_p\{\rho\}$. This procedure can also be viewed as an adiabatic elimination [17,18] of the optical cavity in the presence of fast rotating terms ($\propto e^{\pm i\omega_m t}$) in the optomechanical interaction [19]. We note that while in the physical representation the steady state average occupancy of the optical cavity is given by $|\alpha|^2$, in the shifted one its steady state is simply the vacuum $|0\rangle_p$. Thus, we obtain

$$\dot\mu = -i[\omega_m a_m^\dagger a_m, \mu] + \tfrac{1}{2}\{\gamma_m[n(\omega_m)+1] + A_-\} \times (2a_m\mu a_m^\dagger - a_m^\dagger a_m\mu - \mu a_m^\dagger a_m) + \tfrac{1}{2}[\gamma_m n(\omega_m) + A_+] \times (2a_m^\dagger\mu a_m - a_m a_m^\dagger\mu - \mu a_m a_m^\dagger). \quad (3)$$

In the first term we have redefined ω_m to include the light-induced shift of the mechanical frequency. The second and third terms correspond, respectively, to cooling and heating induced by the coupling to the thermal bath (contributions $\propto \gamma_m$) and by inelastic laser light scattering processes [cf. Fig. 1(b)] with rates

$$A_\mp = \eta^2 \frac{4\Omega^2}{4\tau^2\Delta_L^2 + 1}\frac{\omega_m^2\tau^3}{4\tau^2(\Delta_L \pm \omega_m)^2 + 1}. \quad (4)$$

In the shifted representation it is simple to understand these cooling and heating processes in terms of perturbation theory in the small parameters $\eta|\alpha|$ and η [cf. Eq. (2)]. To lowest order in η only the states $|0\rangle_p$ and $|1\rangle_p$ participate yielding the same results as for an equivalent dissipative two level system [cf. Fig. 1(b)] [18]. This scenario is thus similar to the laser cooling of a trapped ion in the Lamb-Dicke regime [3,4], or of a nanomechanical resonator coupled to an "artificial atom" [20] or an ion [21]. An important caveat in this analogy is that there is no external driving for $\eta = 0$. Furthermore, though the parameter η^2 will play a role reminiscent of the Lamb-Dicke parameter—determining, for example, the relative spectral weight of the sidebands—the efficiency of the cooling process will depend solely on $\eta^2|\alpha|^2$, and Eq. (4) will remain valid for arbitrary Ω provided η^2 is sufficiently small. This absence of a "direct" driving amplitude also implies that the cubic term in Hamiltonian (2) does not contribute to the master equation (3) as it only generates terms that are higher order in η^2. Thus to the lowest order there is no "diffusive channel" and the theory is equivalent to a quadratic Liouvillian. Furthermore, Eq. (4) coincides with its counterpart in a classical treatment of the canonical variables as used in Ref. [11].

Henceforth we focus on the regime $\Delta_L < 0$ for which there is a net laser cooling rate $\Gamma = A_- - A_+ > 0$. In this regime Eq. (3) has a well-defined steady state that transformed back to the physical representation yields a shifted thermal state. The corresponding average occupancy, to which the system converges on the time scale $1/(\Gamma + \gamma_m)$, is given by $\langle a_m^\dagger a_m\rangle_{\rm SS} = n_f + |\beta|^2$ with $n_f = [\gamma_m n(\omega_m) + A_+]/(\gamma_m + \Gamma)$ and $|\beta|^2 = \eta^2\Omega^4\tau^4/(4\tau^2\Delta_L^2 + 1)^2$. However, the *final temperature* is determined by n_f as the other term corresponds to a coherent shift.

As we start from thermal equilibrium, initially the number of phonons is given by $n_i = n(\omega_m)$. Thus, from the expression for n_f it is clear that for appreciable cooling (i.e., $n_f \ll n_i$) we need $\Gamma \gg \gamma_m$. In this regime

$$n_f = \left[\frac{\gamma_m}{\Gamma} n_i + \tilde{n}_f\right]\left[1 + \mathcal{O}\left(\frac{\gamma_m}{\Gamma}\right)\right], \tag{5}$$

with

$$\tilde{n}_f \equiv \frac{A_+}{\Gamma} = \frac{4\tau^2(\Delta_L + \omega_m)^2 + 1}{16\tau^2\omega_m(-\Delta_L)}. \tag{6}$$

Inspection of Eq. (5) shows that two regimes can be distinguished. In the first regime heating is dominated by the intrinsic dissipation of the mechanical resonance and the final average occupancy is proportional to the initial one. This behavior has been demonstrated experimentally [8,9,11]. On the other hand for sufficiently weak intrinsic dissipation or high laser power, the heating is dominated by the scattering of laser light. In this second regime the final occupancy is given by $\tilde{n}_f$ (cf. Fig. 2). Thus the optimal value of n_f solely depends on the product $\omega_m\tau$ and is found by minimizing with respect to the normalized detuning $\tilde{\delta} \equiv \tau\Delta_L$. This yields the *fundamental limit*

$$\tilde{n}_{\mathrm{TL}} = \min\{\tilde{n}_f(\tilde{\delta})\} = \tfrac{1}{2}\left(\sqrt{1 + 1/4\omega_m^2\tau^2} - 1\right), \tag{7}$$

for $\tilde{\delta}_{\mathrm{opt}} = -\sqrt{1 + 4\omega_m^2\tau^2}/2$. The regime $\omega_m\tau \ll 1$ is in essence the *adiabatic limit* [15], since the cavity dynamics is much faster than the mechanical motion. Some recent experiments fall into this parameter range [8]. Expanding Eq. (7) we obtain $\tilde{n}_{\mathrm{TL}} \approx 1/4\omega_m\tau \gg 1$ precluding ground state cooling. The resulting final temperature is of order $\hbar/k_B\tau$ in complete analogy with the Doppler limit of the laser cooling of trapped atoms [4].

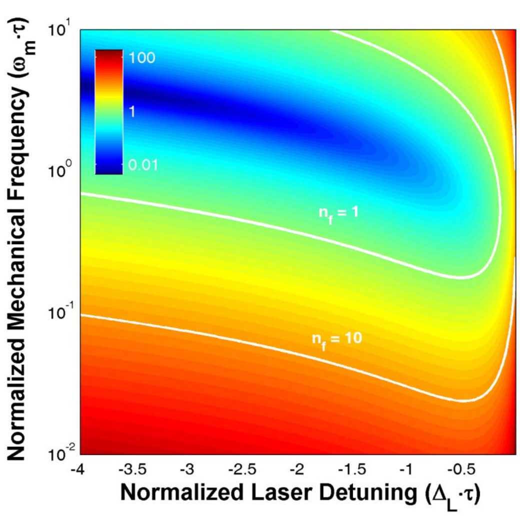

FIG. 2 (color online). Final (steady state) average phonon number $\tilde{n}_f$ as a function of normalized laser detuning ($\Delta_L\tau$) and normalized mechanical angular frequency ($\omega_m\tau$). The contour lines indicate the values $\tilde{n}_f = 1$ and 10, respectively.

We turn now to the regime where retardation effects become significant (i.e., $\omega_m \gtrsim 1/\tau$). This has indeed been observed in recent experimental work pertaining to both amplification [14] and cooling [9,11] of a mechanical oscillator mode. In this regime the optical cavity field cannot respond instantaneously to the mechanical motion and the asymmetry in the Stokes and anti-Stokes scattering rates becomes more pronounced leading into the analog of the "resolved sideband" limit of the laser cooling of trapped atoms. More precisely for $4\omega_m^2\tau^2 \gg 1$, Eq. (7) yields $\tilde{n}_{\mathrm{TL}} \approx 1/16\omega_m^2\tau^2 \ll 1$ implying that in this limit one can achieve *ground state cooling*. A sound benchmark to evaluate the cooling performance is whether average occupancies below unity can be attained. Equation (7) leads to the following criterion: $\tilde{n}_{\mathrm{TL}} < 1 \Leftrightarrow \omega_m\tau > 1/\sqrt{32}$, and Eq. (6) implies that $n_f < 1$ is only possible for $|\Delta_L + 3\omega_m| \le \sqrt{8\omega_m^2 - 1/4\tau^2}$.

Finally, we consider the impact of the intrinsic dissipation on the optimal value of n_f in the regime $\omega_m\tau > 1/\sqrt{32}$. The situation is reminiscent of the "atomic" laser cooling of nanoresonators [20,21] where the finite Q_m also plays a crucial role. However, in the present context, the analysis is simpler and it can be proved that the optimal detuning is still given by $\tilde{\delta}_{\mathrm{opt}}$. Hence, the only relevant issue is the upper bound on P required by the wide time scale separation underpinning our adiabatic treatment of the cooling and heating processes, which in the regime of interest for ground state cooling ($n_f \lesssim 1$) reduces to $A_- \ll 1/\tau$. Thus, our treatment provides an upper bound for the optimal n_f when the finite Q_m is considered. As an illustration we consider the parameters of Ref. [11] (i.e., $\Delta_L\tau > 0.5$, $\omega_m/2\pi = 60$ MHz, $\tau = 3$ ns). For a reservoir temperature of 4 K we have $n_i \approx 1.4 \times 10^3$. If we consider the improvements in the mechanical Q values of toroid microcavities due to vacuum operation ($Q_m = 3 \times 10^4$) a cooling rate of 2.8 MHz is then required to reach $n_i\gamma_m/\Gamma < 1$ [cf. Eq. (5)].

The cooling process gives rise to photons which have frequencies that differ from the pump laser (ω_L). Thus it can be studied in an experiment by measuring the spectrum of the scattered light. As depicted in Fig. 1(a), we consider a one-sided cavity and the relevant observable is the output power. The input-output formalism implies that in the physical representation its spectrum $S(\omega)$ is given by the Fourier transform of $e^{i\omega_L\tau}\langle[\sqrt{1/\tau_{\mathrm{ex}}}a_p^\dagger(t+\tau) + a_{\mathrm{in}}^\dagger(t+\tau)] \times [\sqrt{1/\tau_{\mathrm{ex}}}a_p(t) + a_{\mathrm{in}}(t)]\rangle_{\mathrm{SS}}$. In the shifted representation $a_p(t) \to a_p(t) + \alpha$ and the classical input just adds a c number to the cavity steady state amplitude. Along the lines of our derivation of Eq. (3), $a_p(t)$, $a_p^\dagger(t)$ are treated as environment operators to be reduced to the system operators $a_m(t)$, $a_m^\dagger(t)$ by integrating out the corresponding Heisenberg equations of motion. A straightforward calculation based on perturbation theory and the theory of quantum Markov processes [4,16] then yields

$$S(\omega) \approx \frac{\tau}{\tau_{ex}}\left\{\frac{P}{\hbar\omega_L}\left[\frac{\tau_{ex}}{\tau} - \frac{1-\frac{\tau}{\tau_{ex}}}{\tilde{\delta}^2+\frac{1}{4}}\right]\delta(\omega-\omega_L) + \frac{A_- n_f}{\pi}\frac{\frac{\gamma_{eff}}{2}}{(\omega-\omega_L-\omega_m)^2+\frac{\gamma_{eff}^2}{4}} + \frac{A_+(n_f+1)}{\pi}\frac{\frac{\gamma_{eff}}{2}}{(\omega-\omega_L+\omega_m)^2+\frac{\gamma_{eff}^2}{4}}\right\}; \quad (8)$$

where the relative order of the corrections is given by η^2 for *all* frequencies, we have normalized to number of photons per unit time and unit frequency, and we have defined $\gamma_{eff} \equiv \gamma_m + \Gamma$. As expected there is emission of blueshifted (anti-Stokes) photons associated to cooling and redshifted (Stokes) photons associated to heating. These motional sidebands have a linewidth determined by the effective damping rate γ_{eff} and weights ($N_\mp$) determined by $A_\mp$; namely, $N_- = \frac{\tau}{\tau_{ex}}A_- n_f$ and $N_+ = \frac{\tau}{\tau_{ex}}A_+(n_f+1)$.

The final occupancies can be retrieved by comparing the above spectra [Eq. (8)] for different input powers. The quantity $[N_+(P) + N_-(P)]P_0/[N_+(P_0) + N_-(P_0)]P$ provides an upper bound for the ratio n_f/n_i. Here we have introduced a "reference" low power P_0 for which Γ, $A_+ \ll \gamma_m$ implying $n_f(P_0) \approx n_i$, and assumed that the input power P induces appreciable cooling [i.e., $n_f(P) \ll n_i$]. It is important to note that (given n_i) this upper bound provides an *accurate* direct measurement of the final temperature for $n_f \gg \frac{1}{2}$. On the other hand the worst case scenario occurs for $\omega_m\tau \gg 1$ and $n_f \approx \tilde{n}_f$ where it yields $2n_f$. However, for $n_f \lesssim 1$ an accurate measurement is afforded by the quantity

$$\frac{N_-(P)}{N_+(P)}\frac{N_+(P_0)}{N_-(P_0)}\frac{n_i}{n_i+1} \approx \frac{n_f(P)}{n_f(P)+1} = \frac{\tilde{C}n_i + P\tilde{A}_+}{\tilde{C}(n_i+1)+P\tilde{A}_-}$$

with $\tilde{C} \equiv \hbar\omega_L\tau_{ex}(4\tilde{\delta}^2+1)/4\tau^2\eta^2 Q_m$ and $\tilde{A}_\mp \equiv 4\tau\omega_m/[4(\tilde{\delta} \pm \tau\omega_m)^2+1]$, that provides a clear signature of ground state cooling when it can be achieved. This is in contrast to the case of a laser cooled trapped ion [4] where a well-defined bath associated to the intrinsic dissipation is lacking and detailed balance yields $N_- = N_+$.

In summary, we have derived a quantum theory of cooling using radiation pressure induced dynamical backaction and shown that ground state cooling can be achieved as the optical cavity linewidth becomes smaller than the mechanical frequency, in analogy to atomic sideband cooling. We find that the threshold to attain occupancies below unity is given by $\omega_m\tau > 1/\sqrt{32}$. Furthermore, we have shown how the spectrum of the optical cavity output could be used to measure the final temperatures achieved. Our results are relevant for a wide range of experimental realizations of cavity self-cooling [8,9,11] and could apply to other systems exhibiting dynamical backaction such as an LC circuit with its capacitance modulated by a mechanical oscillator [22].

T. J. K. acknowledges funding via a Max Planck Independent Junior Research Group Grant, a Marie Curie Reintegration Grant (No. CMIRG-CT-2005-031141), a Marie Curie Excellence Grant (No. MEXT-CT-2006-042842), and the Nanosystems Initiative Munich.

Note added.—Recently, we became aware of similar results by Marquardt *et al.* [23].

*zwerger@ph.tum.de
†tjk@mpq.mpg.de

[1] A. Abramovici *et al.*, Science **256**, 325 (1992).
[2] K. C. Schwab and M. L. Roukes, Phys. Today **58**, No. 7, 36 (2005).
[3] D. J. Wineland and W. M. Itano, Phys. Rev. A **20**, 1521 (1979).
[4] S. Stenholm, Rev. Mod. Phys. **58**, 699 (1986); D. Leibfried *et al.*, Rev. Mod. Phys. **75**, 281 (2003).
[5] S. Mancini, D. Vitali, and P. Tombesi, Phys. Rev. Lett. **80**, 688 (1998).
[6] P. F. Cohadon, A. Heidmann, and M. Pinard, Phys. Rev. Lett. **83**, 3174 (1999).
[7] C. Hoehberger-Metzger and K. Karrai, Nature (London) **432**, 1002 (2004).
[8] S. Gigan *et al.*, Nature (London) **444**, 67 (2006).
[9] O. Arcizet *et al.*, Nature (London) **444**, 71 (2006).
[10] D. Kleckner and D. Bouwmeester, Nature (London) **444**, 75 (2006).
[11] A. Schliesser *et al.*, Phys. Rev. Lett. **97**, 243905 (2006).
[12] V. B. Braginsky and F. Khalili, *Quantum Measurement* (Cambridge University Press, Cambridge, U.K., 1992); V. B. Braginsky, S. E. Strigin, and S. P. Vyatchanin, Phys. Lett. A **287**, 331 (2001).
[13] V. B. Braginsky and S. P. Vyatchanin, Phys. Lett. A **293**, 228 (2002); F. Marquardt, J. Harris, and S. Girvin, Phys. Rev. Lett. **96**, 103901 (2006).
[14] T. J. Kippenberg *et al.*, Phys. Rev. Lett. **95**, 033901 (2005).
[15] C. K. Law, Phys. Rev. A **51**, 2537 (1995).
[16] C. W. Gardiner and P. Zoller, *Quantum Noise* (Springer-Verlag, Berlin, 2004).
[17] J. I. Cirac *et al.*, Phys. Rev. A **46**, 2668 (1992).
[18] C. Cohen-Tannoudji, J. Dupont-Roc, and G. Grynberg, *Atom-Photon Interactions: Basic Processes and Applications* (John Wiley & Sons, New York, 1992), Chap. 1.
[19] We adopt an interaction picture for the resonator mode, introduce the projection onto the "relevant part" $\mathcal{P}\rho = |0\rangle\langle 0|_p \mathrm{Tr}_p\{\rho\}$ and the formal parameter ζ such that: $\Delta_L \to \Delta_L\zeta^2$, $\omega_m \to \omega_m\zeta^2$, $\tau \to \tau/\zeta^2$, $\eta \to \eta/\zeta$; and $\gamma_m \to \gamma_m$ in H, $\mathcal{L}'_D$; and finally evaluate the leading order of the expansion of $\mathcal{P}\dot{\rho}$ for $\zeta \to \infty$.
[20] I. Wilson-Rae, P. Zoller, and A. Imamoglu, Phys. Rev. Lett. **92**, 075507 (2004); I. Martin *et al.*, Phys. Rev. B **69**, 125339 (2004).
[21] L. Tian and P. Zoller, Phys. Rev. Lett. **93**, 266403 (2004).
[22] V. B. Braginsky, *Measurement of Weak Forces in Physics Experiments* (University of Chicago Press, Chicago, 1977).
[23] F. Marquardt, J. P. Chen, A. A. Clerk, and S. M. Girvin, following Letter, Phys. Rev. Lett. **99** 093902 (2007).