IPESE
Industrial Process and Energy Systems Engineering

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Uncertainty: Sensitivity Analysis & Robust Optimization

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Introduction

"It is difficult to make predictions, especially about the future"
Danish proverb

Introduction

Sources:
[a] Y. N. Harari, Homo Deus, 2015.

The energy transition

Average US citizen vs. stone-age hunter: 60x more energy consumption[a]

 = 

81.1% world primary energy supply in the year 2014[3]

Main source of anthropogenic GHG emissions[4]

IEA projections to 2050[5]:
+60% energy demand (vs. 2011)
+70% GHG emissions (vs. 2011)

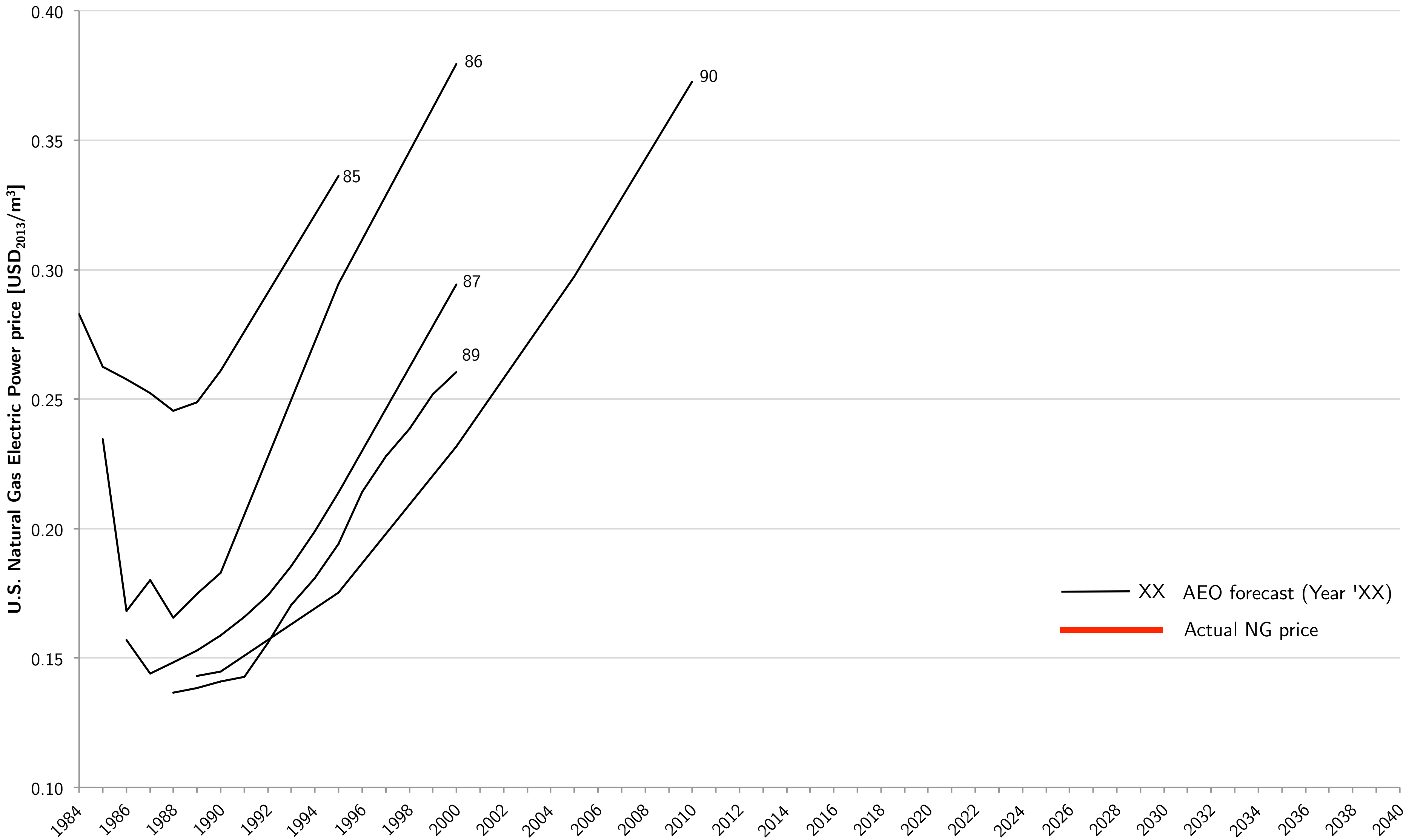
To target the 2°C ΔT limit CO_2 emissions need to be halved by 2050[6]

Strategic Energy Planning
Scale: urban/national/industrial
Time horizon: 20-50 years

Common approach:
Long-term deterministic optimization models based on forecasts

Introduction

Energy forecasting: learning from the past



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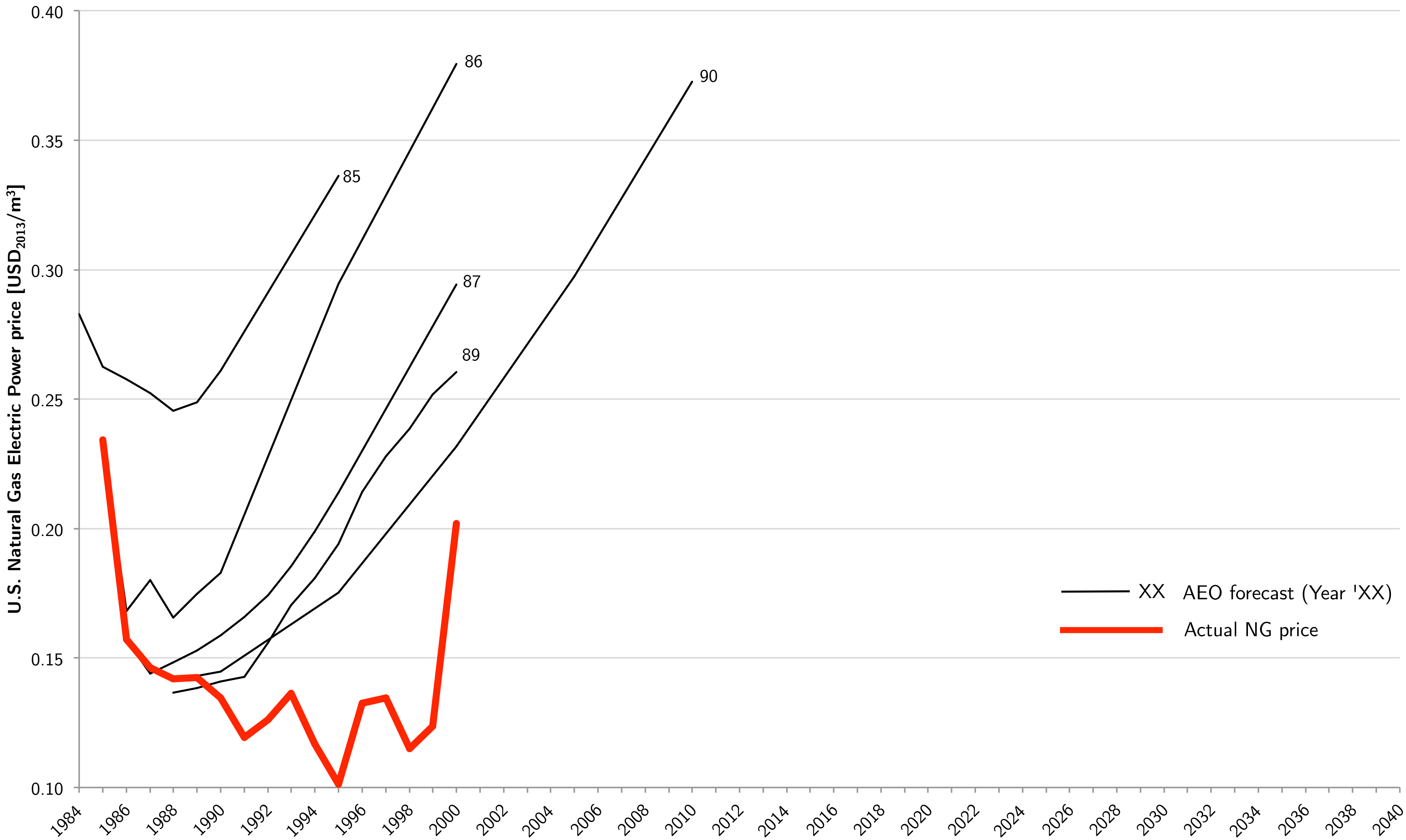

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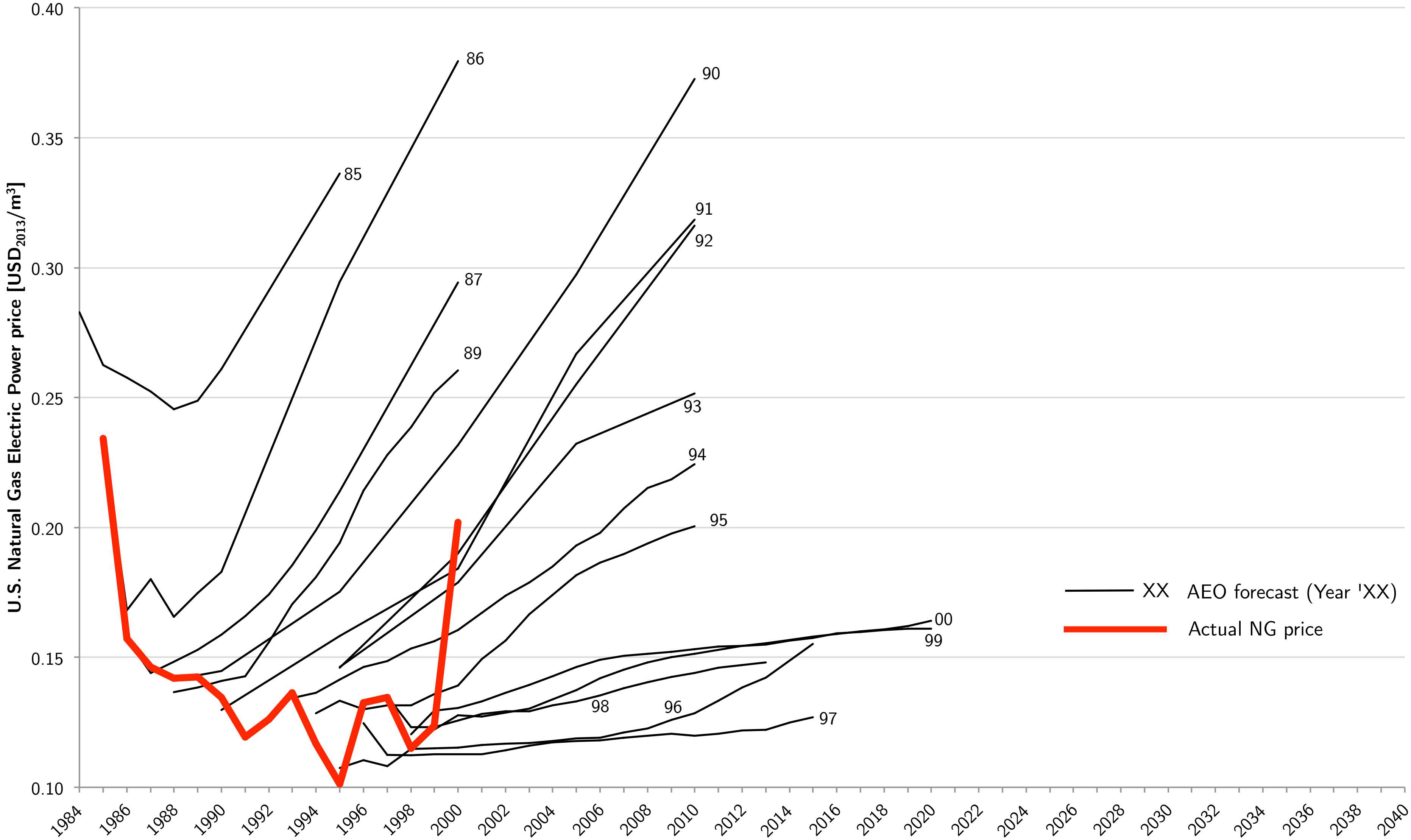

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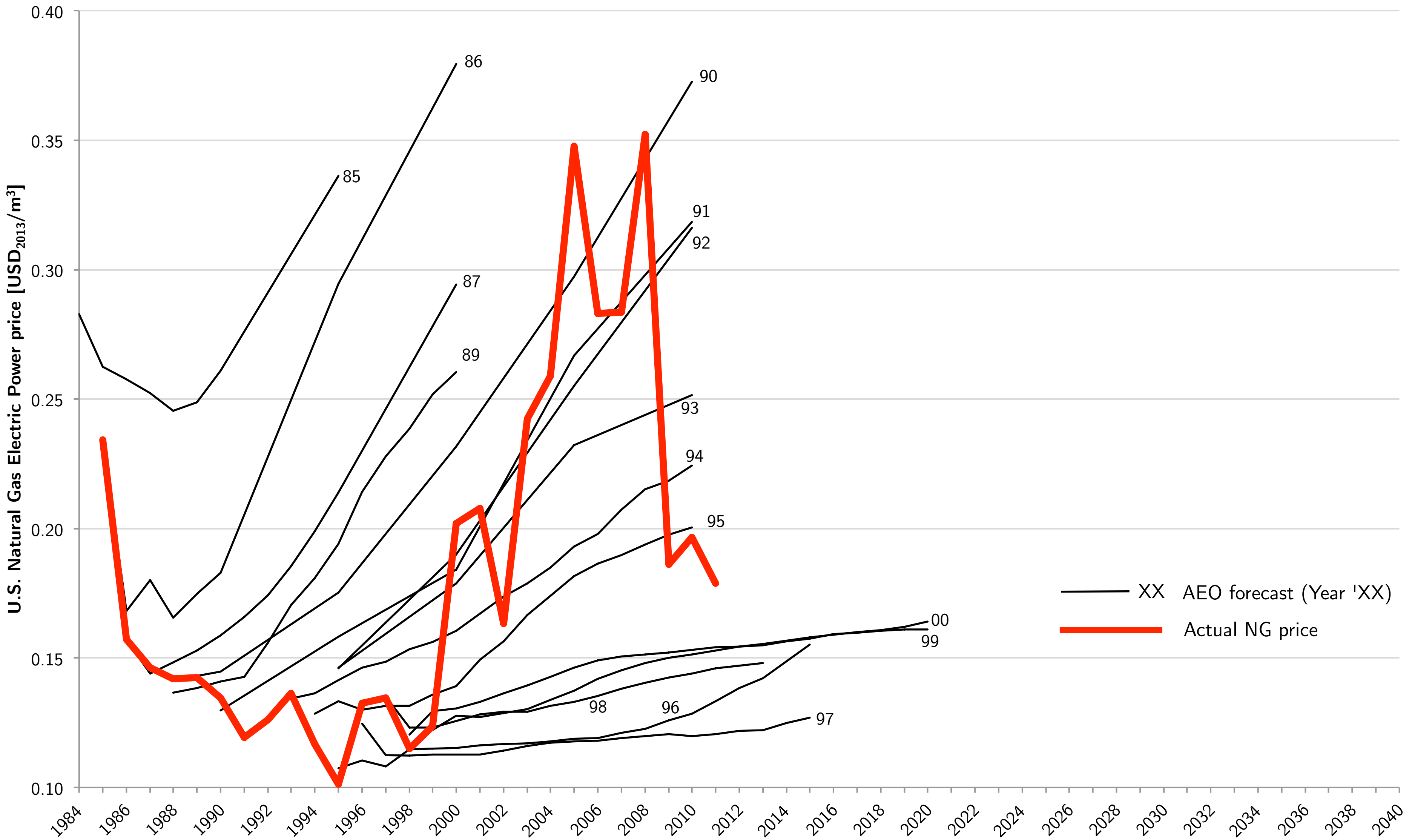

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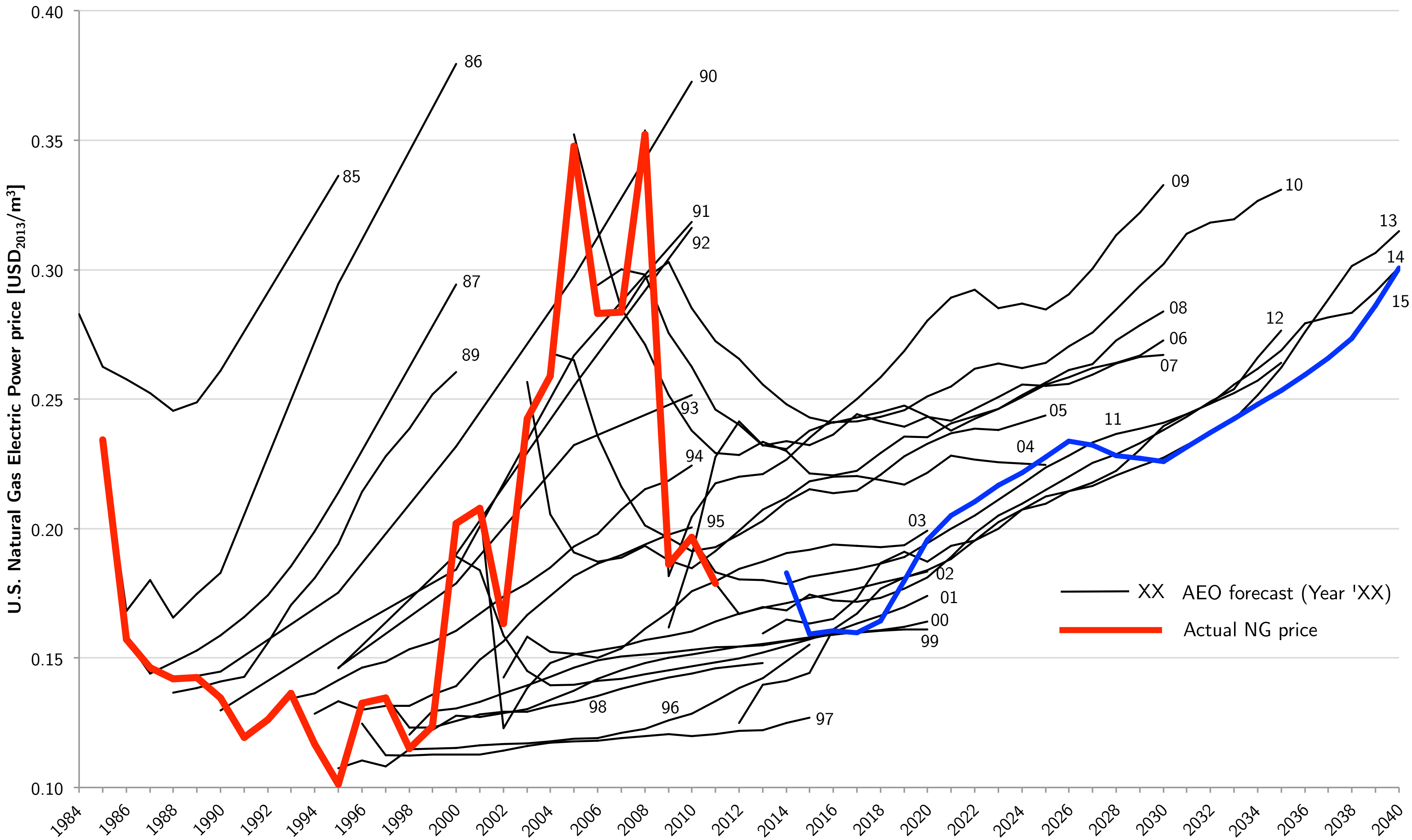

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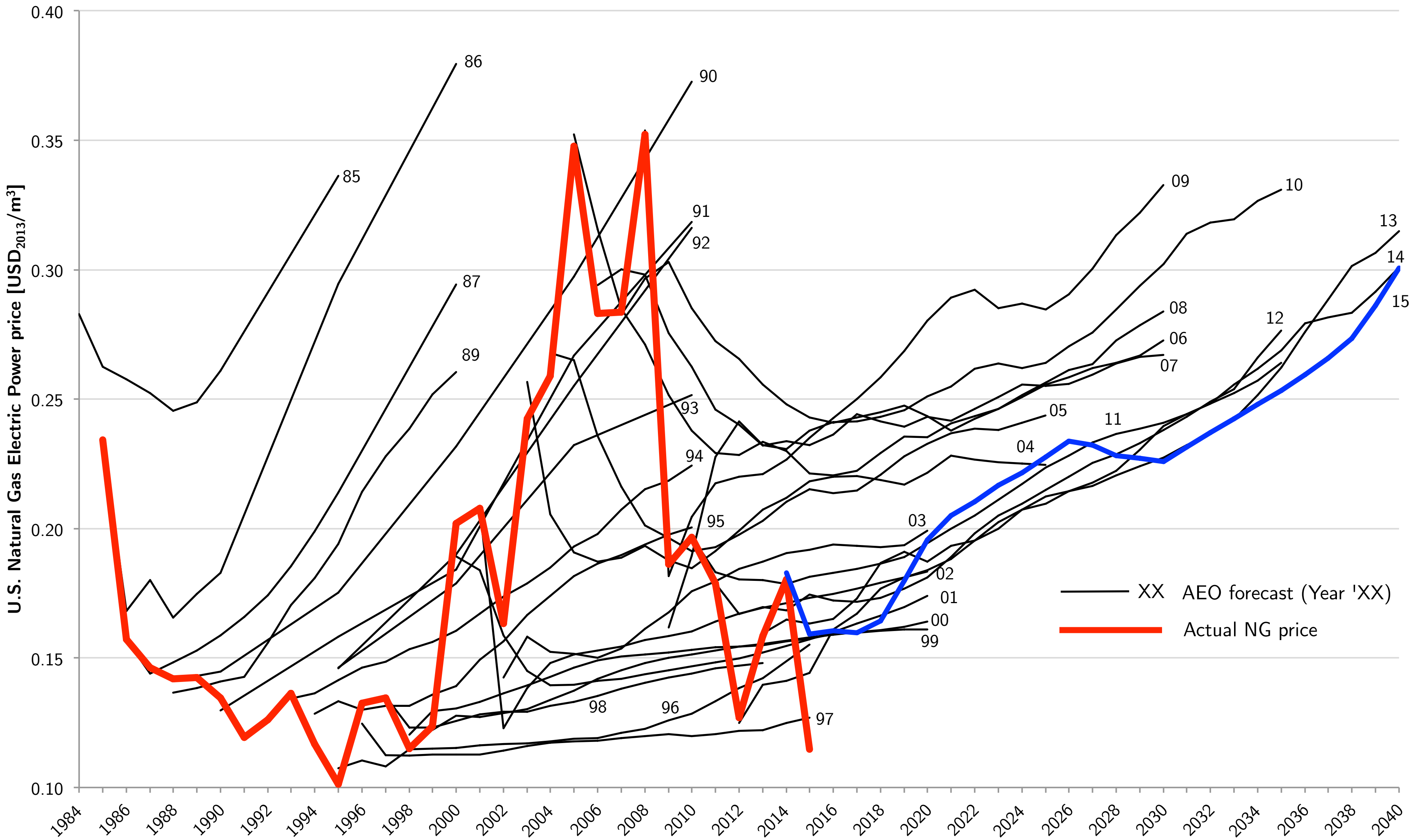


Introduction

Energy forecasting: learning from the past

Long-term, strategic planning for urban and national energy systems

Long time horizons (20-50 years)

Based on forecasting

Low penetration of renewables and/or new (efficient) technologies

Overcapacity[29] if forecasts are wrong

Furthermore:
- energy models are "nonvalidatable", i.e. doomed to inaccuracy[16,17]
- backcasting: models have missed pivotal events[13,14]

Need for accounting of uncertainty in long-term energy modeling[30-33]

Craig et al.[28]: *"Long-run forecasting methods for energy [...] will likely fall prey to the inherent unpredictability of pivotal events"*

Introduction

Gaps

Still low penetration of uncertainty in the energy field[34,35]. Why? Grossmann et al.[36]:

I. Energy models
   - Computationally expensive[27][39]
   - Complex formulation (as originally deterministic)
   - Sector-specific (often electricity)[40-42]

II. Quantification of input uncertainties
   - Scarce quantity and quality of available data[47]
   - Difficulty of defining probability distributions[27,48]
   - Focus on few *a priori* selected parameters and scarce documentation

III. Methods to incorporate uncertainties in energy models
   - Sensitivity analysis → seldom used, few parameters
   - Optimization under uncertainty → computational burden

Global sensitivity analysis

Published as:
S. Moret, V. Codina Gironès, M. Bierlaire and F. Maréchal. *Characterization of input uncertainties in strategic energy planning models*. Applied energy, 2017.

Global sensitivity analysis

Literature review & contributions

	Method(s)[a]	Uncertain Parameters	Application & Model type[b]	Output(s) of interest
Schulz et al. [107]	LSA (scenarios)	fuel prices, inv. cost, subsidies	wood-to-SNG (LP)	FEC, strategy
Kattan and Ruble [108]	LSA	fuel prices	comparison of boilers for residential heating	energy price
Siler-Evans et al. [109]	LSA	cost (fuels and inv), interest rate, efficiencies, lifetime	distributed CHP	NPV
Kim et al. [50]	LSA	feedstock and by-product prices	biomass-to-fuel (LP)	Obj. (cost)
Koltsaklis et al. [41]	LSA (scenarios)	cost (NG, emissions), inv. cost, elec. demand	national power system planning (MILP)	Obj. (cost), other
Pantaleo et al. [110]	LSA (scenarios)	demand, fuel prices, climate, infrastructure	biomass integration in urban systems (MILP)	various
Beckers et al. [111]	LSA	costs, interest rate, lifetime, geothermal resource	geothermal energy	levelized cost
Fazlollahi [112]	EFAST	cost (fuels and inv.), interest rate, emissions	urban energy system (MILP+GA)	Objs. (cost, GWP, efficiency)
Pernet [113]	LSA, EE, VB	heat and exergy demand, efficiencies	urban energy system (MILP)	Obj. (exergy), Tech size
Pye et al. [47]	SP, SRC	inv. cost, build rates, resource avail. and prices	national energy model (LP)	Obj. (cost), emissions
Han et al. [114]	LSA	feedstock price, yields, solvent:biomass ratio	ethanol production	energy price
Lythcke-Jørgensen et al. [86]	EE, UA	fuel price, inv. cost, emissions	multi-generation system design (MILP+GA)	Objs. (NPV, GWP)
Mian [115]	EE, MC	cost (fuels and inv.), efficiencies, interest rate, temperatures	hydrothermal gasification plant design (MINLP)	Obj. (cost)

[a] Abbreviations: local sensitivity analysis (LSA), uncertainty analysis (UA), elementary effect (EE), variance-based (VB), scatterplots (SP), standardized regression coefficients (SRC), Monte Carlo (MC)

[b] Optimization model types: linear programming (LP), mixed-integer linear programming (MILP), mixed-integer non linear programming (MINLP), genetic algorithm (GA). If the model type is not indicated, the model is not based on optimization.

Global sensitivity analysis

Literature review & contributions

Sensitivity analysis studies "*how uncertainty in the output of a model can be apportioned to different sources of uncertainty in the model input*" [54]

In the literature:

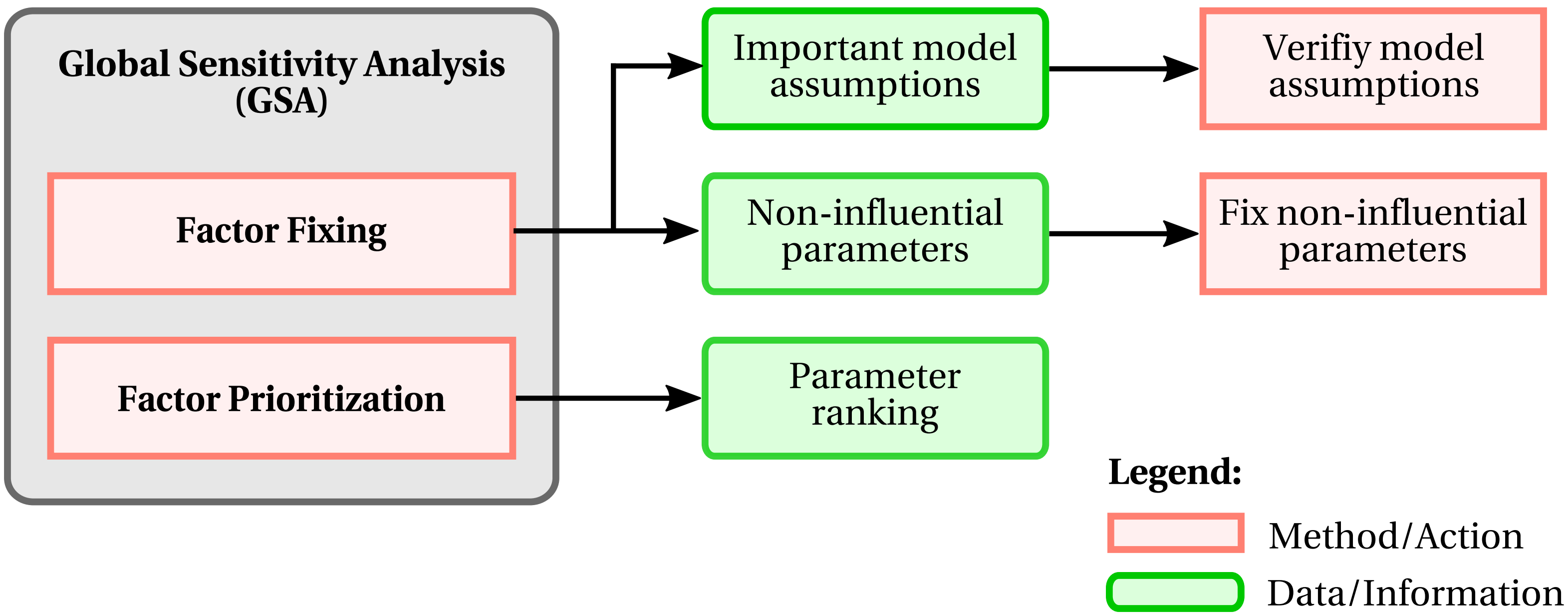
- Variety of methods, but mostly local sensitivity analysis (LSA)
- In general, few applications[34]
- Focus on a few uncertain parameters selected *a priori* [55]

- Large number of input parameters → two-stage global sensitivity analysis method
- Systematic consideration of *all* parameters in the analysis

Global sensitivity analysis

Two-stage GSA

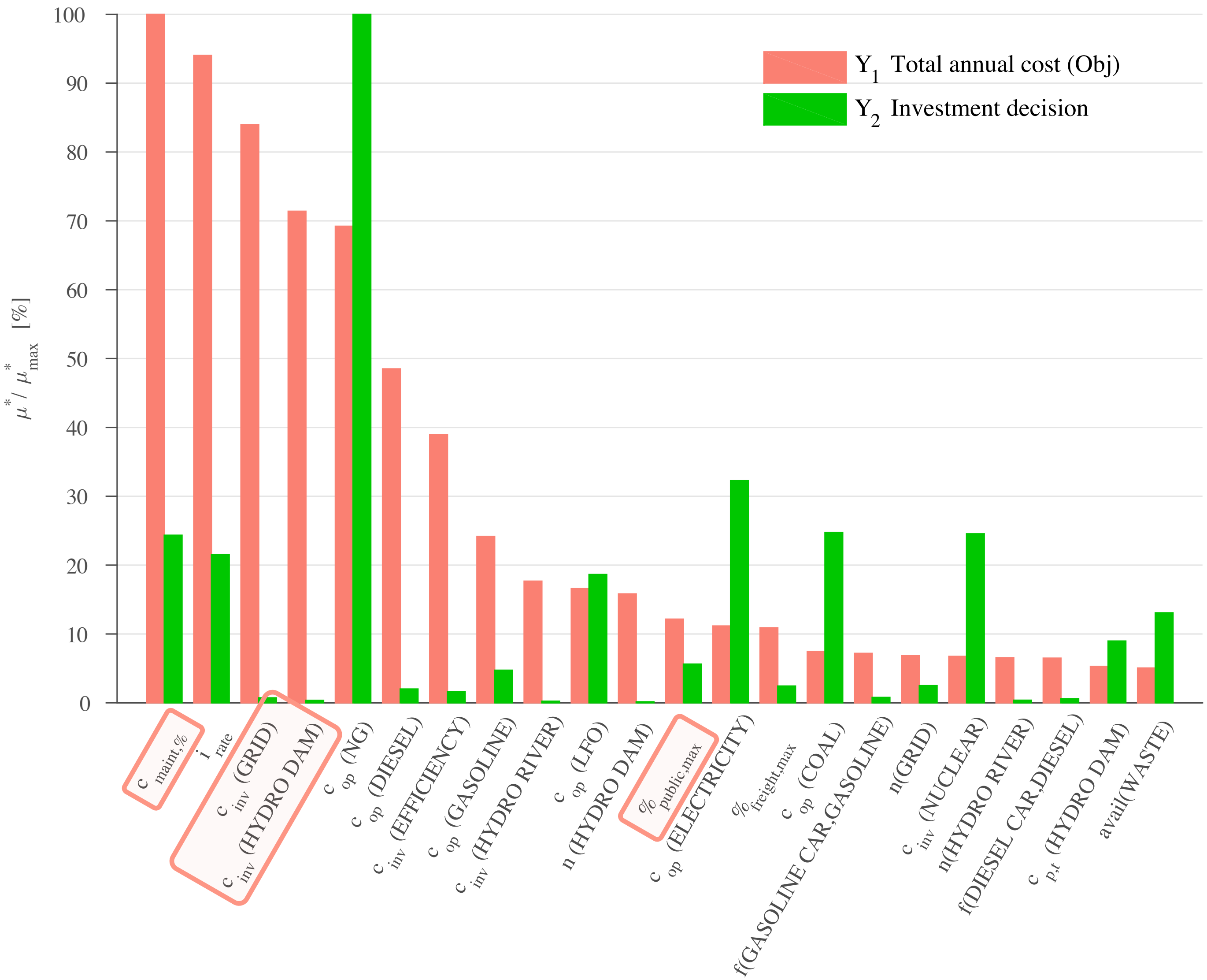
The method was first proposed by Campolongo et al. in the late 1990s; here, it is updated to state-of-the-art GSA methods.
To date, never applied to energy planning problems.



Sensitivity is linked to the calculation of sensitivity indices

Global sensitivity analysis

Case study: I. Factor fixing



370 parameters
22 > 5% of max
53 > 1% of max

- Screening is effective!
- Importance of assumptions → all parameters
- Importance of choosing the output of interest

Global sensitivity analysis

Case study: II. Factor prioritization

Factor prioritization is performed on the first 10 parameters in the factor fixing step

Rank	Parameter	S_i	S_{T_i}
1	i_{rate}	4.830E-01	4.906E-01
2	c_{op}(NG)	4.412E-01	4.586E-01
3	c_{op}(LFO)	1.900E-02	1.914E-02
4	c_{op}(ELECTRICITY)	1.400E-02	3.398E-02
5	c_{inv}(NUCLEAR)	5.732E-03	1.871E-02
6	c_{op}(COAL)	5.487E-03	1.272E-02
7	$c_{p,t}$(HYDRO DAM)	2.320E-03	2.137E-03
8	$avail$(WASTE)	1.501E-03	2.026E-03
9	c_{inv}(DHN)	1.158E-03	1.310E-03
10	$c_{p,t}$(HYDRO RIVER)	6.922E-04	1.744E-03

From the theory[54], $S_{T_i} >= S_i$. $S_{T_i} - S_i$ is a measure of how strongly the i-th parameter is involved in interactions with other inputs.

Robust optimization

Partly published as:
S. Moret, M. Bierlaire and F. Maréchal. *Robust optimization for strategic energy planning*. Informatica, 2016.

Robust optimization

Sources:
- Refs. [123], [36], [57-59], [124]

Literature review & contributions

Sensitivity analysis:
Change parameter values in deterministic model

Optimization under uncertainty:
Uncertainty integrated in optimization model formulation

Stochastic programming

Optimizes the expected value of the objective

- Scenario tree to model uncertainty
- Assumption is that PDFs are known

Limitations:

- Difficult to define PDFs[27][124]
- Quickly leads to intractable models sizes[59]

Robust optimization

Worst-case realizations of uncertainty

- First proposed by Soyster[60] in 1970s: all parameters at worst case → over-conservative solutions

Main developments:

- Ben-Tal and Nemirovski[61] → non-linear
- Bertsimas and Sim[62] → linear

Robust optimization

Literature review & contributions

- Increasing interest in the last 20 years[143]
- Fields: inventory and logistics, finance, revenue management, queuing networks, machine learning, energy systems and the public good[146]
- The linear approach by Bertsimas and Sim[62] is the most diffused
- Uncertain parameters: demand and prices
- Typical application: electricity sector[127][132][134][136][138]

In general:

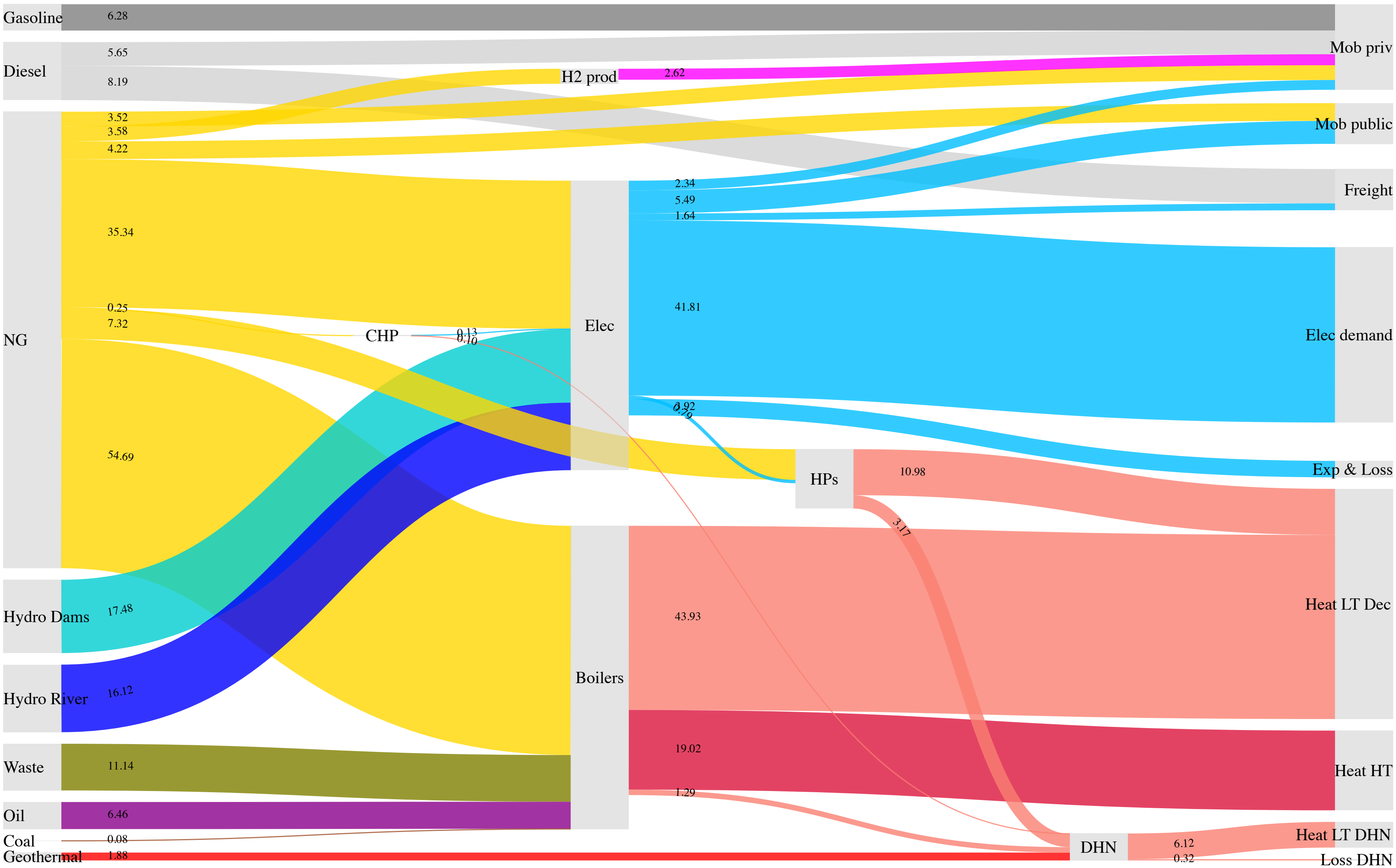
- Still rather limited applications
- Specific parameters and applications → limited by complex model formulations

Bridge gap between OR methods and energy systems applications

- First application of Bertsimas and Sim[62] to a strategic energy planning problem
- Novel developments to consider multiplied uncertain parameters
- Integration of Babonneau et al.[153] to consider uncertainty in the constraints
- General RO framework: all parameters in objective function and other constraints
- Real application: Swiss energy strategy

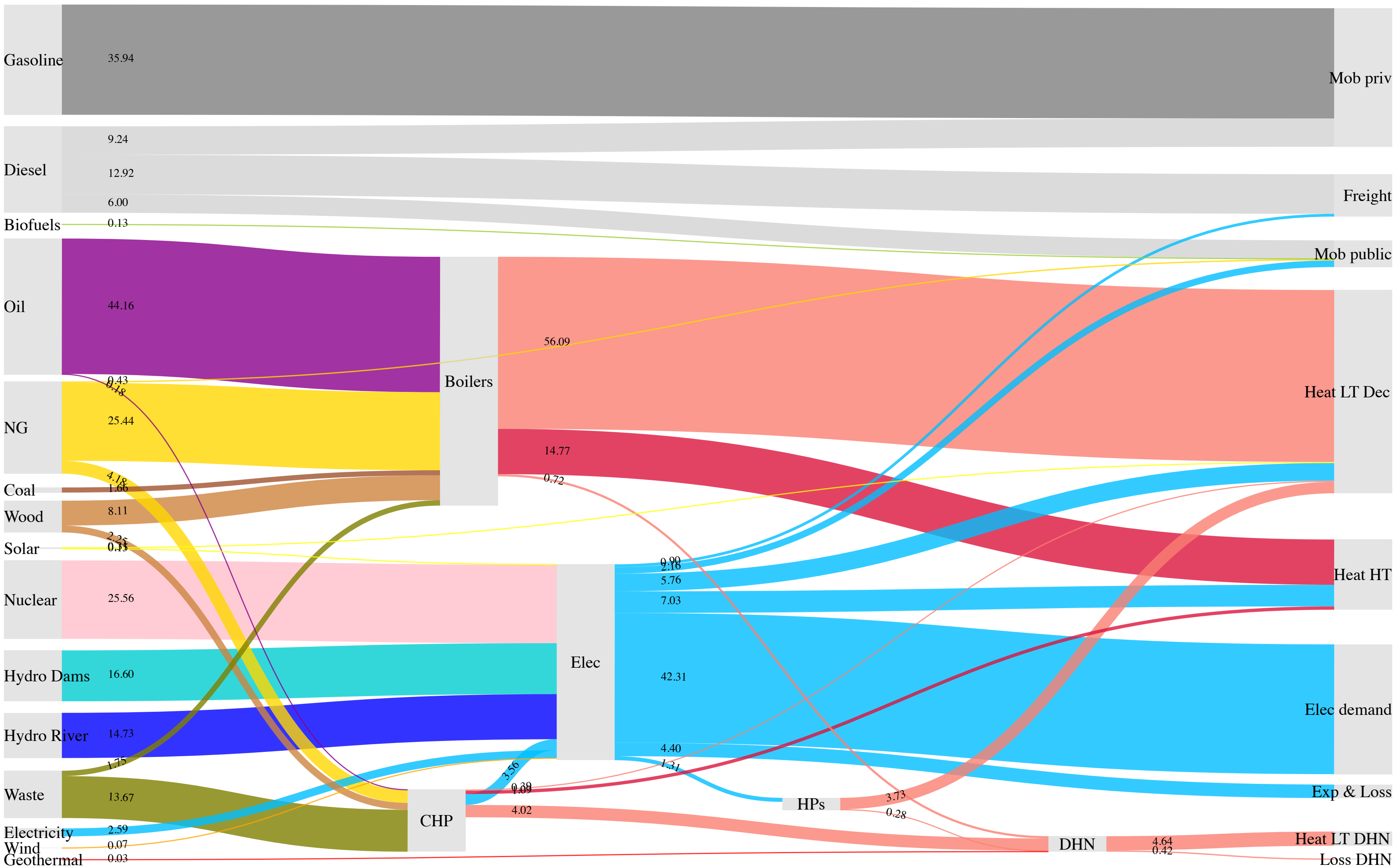
Robust optimization

Switzerland in 2035: deterministic cost optimal solution



Robust optimization

Switzerland in 2011



Robust optimization

The robust approach

$$\min \sum_j c_j \mathbf{x}_j$$ ← Objective function (e.g. minimizing total cost)

$$\text{s.t.} \sum_j a_{ij}\mathbf{x}_j \le b_i \quad \forall i$$ ← Constraints (e.g. energy balance)

$$l_j \le \mathbf{x}_j \le u_j \quad \forall j$$

Soyster: Protection against all uncertain parameters at worst case. Very conservative
Bertsimas & Sim: "*nature will be restricted in its behavior, in that only a subset of the coefficients will change in order to adversely affect the solution*"

Without loss of generality, uncertainty is considered for the coefficients a_{ij}

$$a_{ij} \in [a_{ij} - \delta_{a,ij}, a_{ij} + \delta_{a,ij}], j \in J_i$$

The "protection parameter" controls the number of uncertain parameters at worst case:

$$\Gamma_i = [0, |J_i|]$$

→ $\Gamma_i = 0$ Deterministic MILP, no parameter at worst case

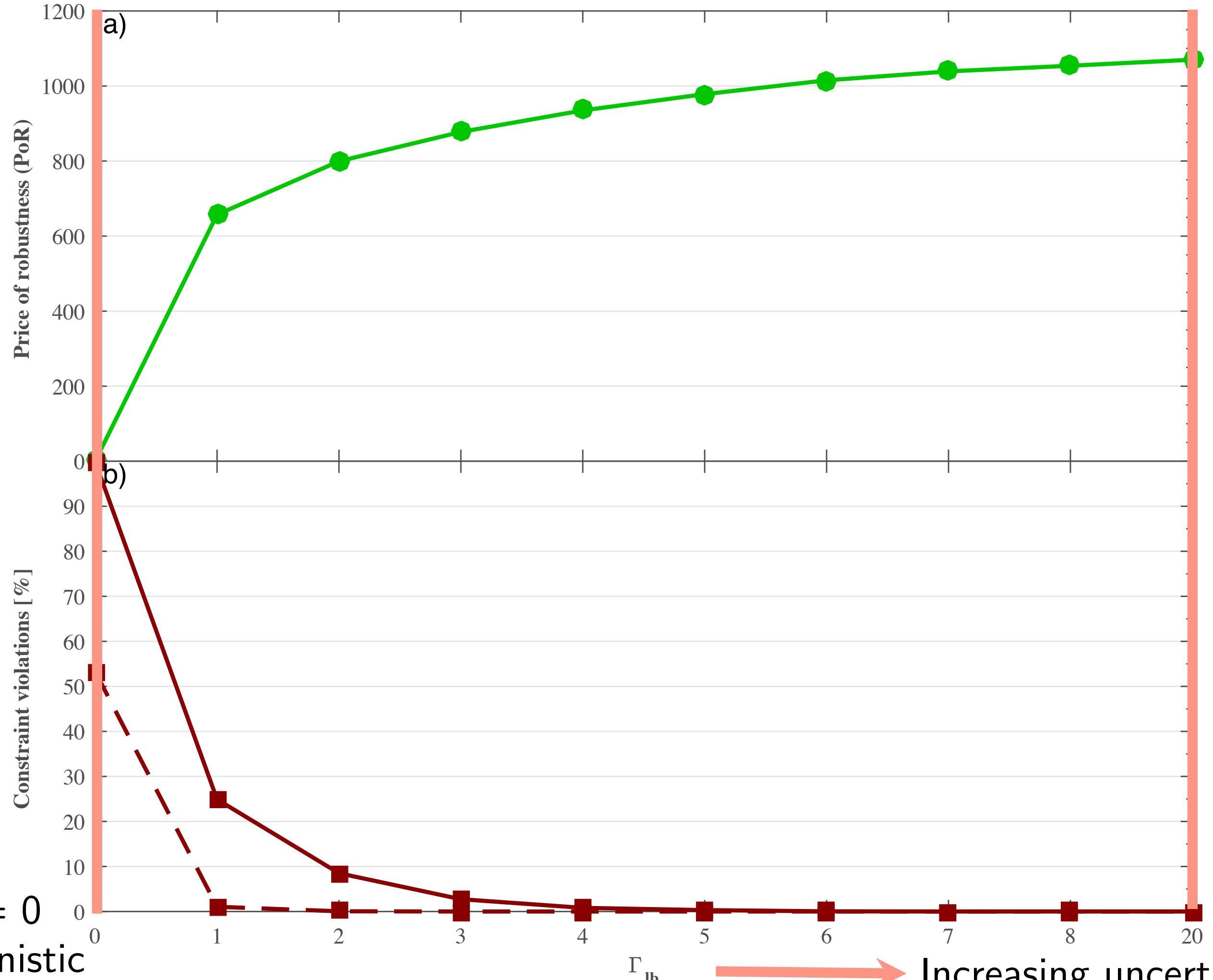
→ $\Gamma_i = |J_i|$ All parameter at worst case (Soyster)

Robust optimization

The robust approach

Cost increases!

The system is more reliable (less risk of not meeting demand)



$\Gamma = |J|$
Soyster's solution → All params at worst case

$\Gamma_{obj} = 0$
Deterministic solution

→ Increasing uncertainty

Robust optimization

Why can't we directly apply this method?

Why is it difficult to apply this approach to energy models?

$$\mathbf{C_{tot}} = \sum_{j \in TECH} \mathbf{C_{inv}}(j) + \sum_{j \in TECH} \mathbf{C_{maint}}(j) + \sum_{i \in RES} \sum_{t \in T} \mathbf{C_{op}}(j,t) =$$

$$= \sum_{j \in TECH} \frac{i_{rate}(i_{rate}+1)^{n(j)}}{(i_{rate}+1)^{n(j)}-1} c_{inv}(j)\mathbf{F}(j) + \sum_{j \in TECH} c_{maint}(j)\mathbf{F}(j) + \sum_{i \in RES} \sum_{t \in T} c_{op}(i,t)\mathbf{F_t}(i,t)\, t_{op}(t)$$

Two main difficulties:

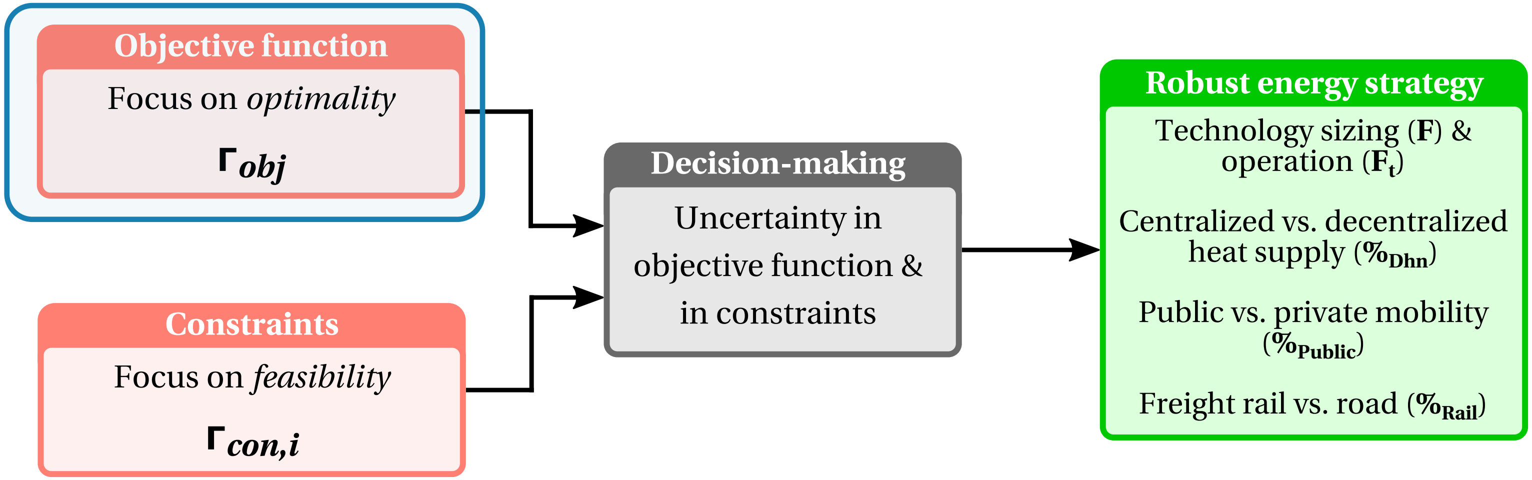
1. Objective function: cannot account for the uncertainty of multiplied parameters
2. Constraints: Difficult to account for uncertainty in the constraints

Both issues are addressed to provide a robust optimization framework

Robust optimization

Robust optimization framework

First, robust formulations are separately derived for the objective function and for the other constraints



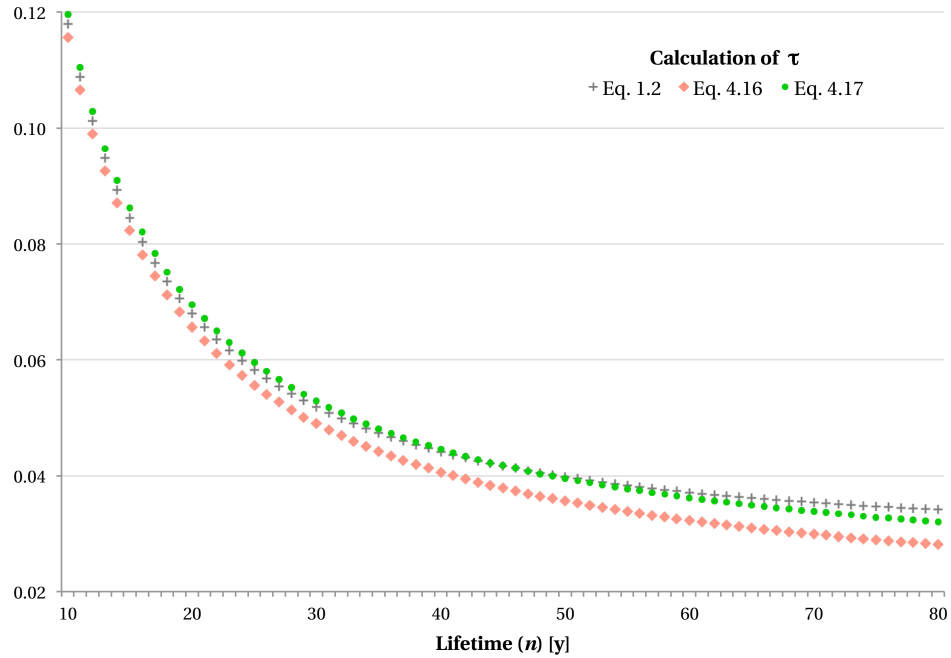
Robust optimization

Uncertainty in the objective function

$$\sum_{j \in TECH} \frac{i_{rate}(i_{rate}+1)^{n(j)}}{(i_{rate}+1)^{n(j)}-1} c_{inv}(j)\mathbf{F}(j)$$

A novel robust formulation is demonstrated in two steps:

1. $$\mathbf{Obj_1} = \sum_{j \in TECH} \frac{i_{rate}(i_{rate}+1)^{n(j)}}{(i_{rate}+1)^{n(j)}-1} c_{inv}(j)\mathbf{F}(j) \approx \sum_{j \in TECH} \left(\alpha \frac{i_{rate}}{2} + \frac{1}{n(j)}\right) c_{inv}(j)\mathbf{F}(j)$$



Robust optimization

Uncertainty in the objective function

$$\sum_{j \in TECH} \frac{i_{rate}(i_{rate}+1)^{n(j)}}{(i_{rate}+1)^{n(j)}-1} c_{inv}(j)\mathbf{F}(j)$$

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1. $\mathbf{Obj_1} = \sum_{j \in TECH} \frac{i_{rate}(i_{rate}+1)^{n(j)}}{(i_{rate}+1)^{n(j)}-1} c_{inv}(j)\mathbf{F}(j) \approx \sum_{j \in TECH} (\alpha \frac{i_{rate}}{2} + \frac{1}{n(j)}) c_{inv}(j)\mathbf{F}(j)$

2. $\min \sum_j (a_j + a'_j) c_j \mathbf{x}_j$

$$
\begin{aligned}
\min\ & \sum_j (a_j + a'_j) c_j \mathbf{x}_j + \mathbf{z_u}\Gamma_u + \mathbf{z_{u'}}\Gamma_{u'} + \mathbf{z_v}\Gamma_v + \sum_j (\mathbf{p}_j + \mathbf{p}'_j + \mathbf{q}_j) \\
\text{s.t.}\ & \mathbf{z_u} - \boldsymbol{\eta}_j + \mathbf{p}_j \geq \delta_{a,j} c_j \mathbf{x}_j && \forall j \\
& \mathbf{z_{u'}} - \boldsymbol{\eta}'_j + \mathbf{p}'_j \geq \delta_{a',j} c_j \mathbf{x}_j && \forall j \\
& \mathbf{z_v} - \boldsymbol{\pi}_j - \boldsymbol{\pi}'_j + \mathbf{q}_j \geq \delta_{c,j} (a_j + a'_j) \mathbf{x}_j && \forall j \\
& \boldsymbol{\eta}_j + \boldsymbol{\pi}_j \geq \delta_{a,j} \delta_{c,j} \mathbf{x}_j && \forall j \\
& \boldsymbol{\eta}'_j + \boldsymbol{\pi}'_j \geq \delta_{a',j} \delta_{c,j} \mathbf{x}_j && \forall j \\
& \mathbf{x}_j, \mathbf{p}_j, \mathbf{p}'_j, \mathbf{q}_j, \boldsymbol{\pi}_j, \boldsymbol{\pi}'_j, \boldsymbol{\eta}_j, \boldsymbol{\eta}'_j, \mathbf{z_u}, \mathbf{z_{u'}}, \mathbf{z_v} \in \mathbb{R}^+
\end{aligned}
$$

Robust optimization

Uncertainty in the objective function

$$\begin{aligned}
\min \quad & \sum_{j \in TECH} \left(\tau_r + \tau_n(j)\right) c_{inv}(j) \mathbf{F}(j) + \sum_{j \in TECH} c_{maint}(j) \mathbf{F}(j) + \sum_{i \in RES} \sum_{t \in T} c_{op}(i,t) \mathbf{F_t}(i,t) t_{op}(t) \\
& + \mathbf{z_r} \Gamma_r + \mathbf{z_n} \Gamma_n + \mathbf{z_{inv}} \Gamma_{inv} + \mathbf{z_0} \Gamma_0 + \sum_{j \in TECH} \left(\mathbf{p_r}(j) + \mathbf{p_n}(j) + \mathbf{p_{inv}}(j) + \mathbf{p_{maint}}(j)\right) + \sum_{i \in RES} \sum_{t \in T} \mathbf{p_{op}}(j)(j,t) \\
\text{s.t.} \quad & \mathbf{z_r} + \sum_{j \in TECH} \left(-\boldsymbol{\eta}_\mathbf{r}(j) + \mathbf{p_r}(j)\right) \geq \delta_{\tau_r} \sum_{j \in TECH} \left(c_{inv}(j) \mathbf{F}(j)\right) \\
& \mathbf{z_n} - \boldsymbol{\eta}_\mathbf{n}(j) + \mathbf{p_n}(j) \geq \delta_{\tau_n}(j) c_{inv}(j) \mathbf{F}(j) && \forall j \in TECH \\
& \mathbf{z_{inv}} - \boldsymbol{\pi}_\mathbf{inv}(j) - \boldsymbol{\pi}'_\mathbf{inv}(j) + \mathbf{p_{inv}}(j) \geq \delta_{inv}(j) \left(\tau_r + \tau_n(j)\right) \mathbf{F}(j) && \forall j \in TECH \\
& \boldsymbol{\eta}_\mathbf{r}(j) + \boldsymbol{\pi}_\mathbf{inv}(j) \geq \delta_{\tau_r} \delta_{inv}(j) \mathbf{F}(j) && \forall j \in TECH \\
& \boldsymbol{\eta}_\mathbf{n}(j) + \boldsymbol{\pi}'_\mathbf{inv}(j) \geq \delta_{\tau_n}(j) \delta_{inv}(j) \mathbf{F}(j) && \forall j \in TECH \\
& \mathbf{z_0} + \mathbf{p_{maint}}(j) \geq \delta_{maint}(j) \mathbf{y_{maint}}(j) && \forall j \in TECH \\
& \mathbf{F}(j) \leq \mathbf{y_{maint}}(j) && \forall j \in TECH \\
& \mathbf{z_0} + \sum_{t \in T} \mathbf{p_{op}}(i,t) \geq \delta_{op}(i) \sum_{t \in T} \mathbf{y_{op}}(i,t) && \forall i \in RES \\
& \mathbf{F_t}(i,t) t_{op}(t) \leq \mathbf{y_{op}}(i,t) && \forall i \in RES, \forall t \in T \\
& \mathbf{z_r}, \mathbf{z_n}, \mathbf{z_{inv}}, \mathbf{z_0}, \mathbf{p_r}, \mathbf{p_n}, \mathbf{p_{inv}}, \mathbf{p_{maint}}, \mathbf{p_{op}}, \boldsymbol{\eta}_\mathbf{r}, \boldsymbol{\eta}_\mathbf{n}, \boldsymbol{\pi}_\mathbf{inv}, \boldsymbol{\pi}'_\mathbf{inv}, \mathbf{y_{maint}}, \mathbf{y_{op}} \in \mathbb{R}^+
\end{aligned}$$

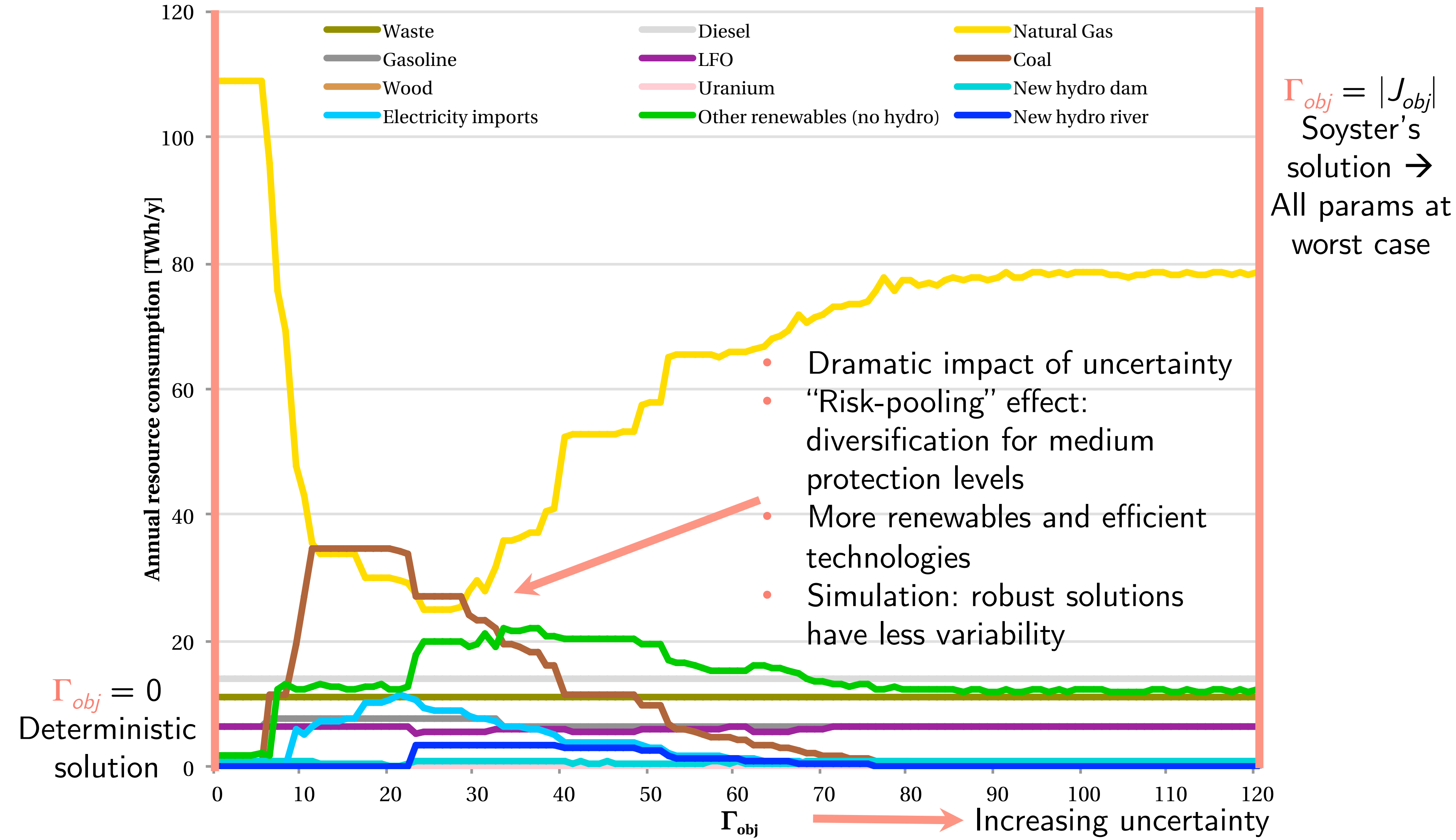
Four control parameters. But, in the case of the studied problem, it can be imposed that:

$$\mathbf{z_{obj}} \Gamma_{obj} = \mathbf{z_r} \Gamma_r + \mathbf{z_n} \Gamma_n + \mathbf{z_{inv}} \Gamma_{inv} + \mathbf{z_0} \Gamma_0$$

Γ_{obj} controls the uncertainty of 160 parameters!

Robust optimization

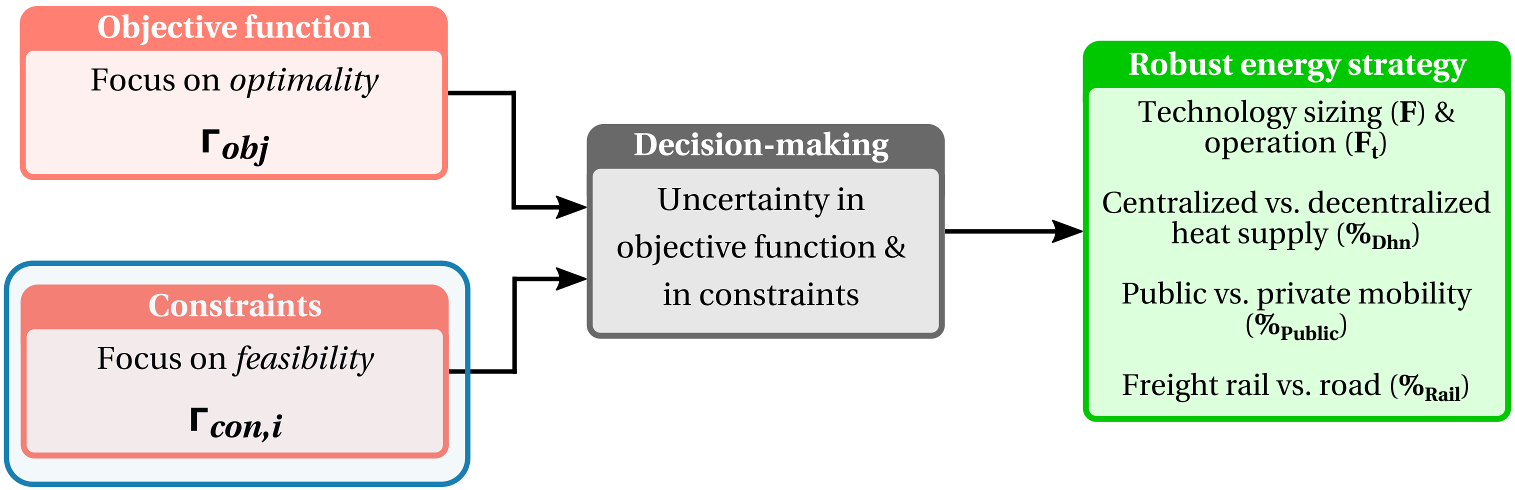
Uncertainty in the objective function



- Dramatic impact of uncertainty
- "Risk-pooling" effect: diversification for medium protection levels
- More renewables and efficient technologies
- Simulation: robust solutions have less variability

Robust optimization

Robust optimization framework



Robust optimization

Uncertainty in the constraints

Uncertainty in the constraints is seldom addressed. Why?

1. Same parameters appearing in multiple constraints[131]
2. Most constraints contain very few (even only one) uncertain parameter[157], e.g. a model has N constraints, each of them with one uncertain parameter → N control parameters $\Gamma_i \in [0;1]$ → combinatorial problem

How to address these issues?

1. Model formulation (modeling for uncertainty)
2. Formulation + idea by Babonneau et al. [153]:

$$\mathbf{ACT}(i) - AF(i) \times CAP(i) \leq 0 \qquad \forall i = 1, \ldots, N$$

Energy carried through "channel" i

Availability of the channel (uncertain)

Channel capacity

Robust optimization

Uncertainty in the constraints: the robust formulation by Babonneau et al.[153]

Babonneau et al.[153]: "*we are interested in protecting the total energy supply [...], not that of each channel separately*"

The idea is adding a redundant constraint summing over the constraint indices and to "robustify" this new constraint → instead of N $\Gamma_i \in [0;1]$, this gives one $\Gamma \in [0;N]$. The problem becomes tractable!

$$\sum_{i=1}^{N} \left(\mathbf{ACT}(i) - AF(i) \times CAP(i)\right) \leq 0$$

$$\mathbf{ACT}(i) - AF(i) \times CAP(i) \leq 0 \qquad \forall i = 1, \ldots, N$$

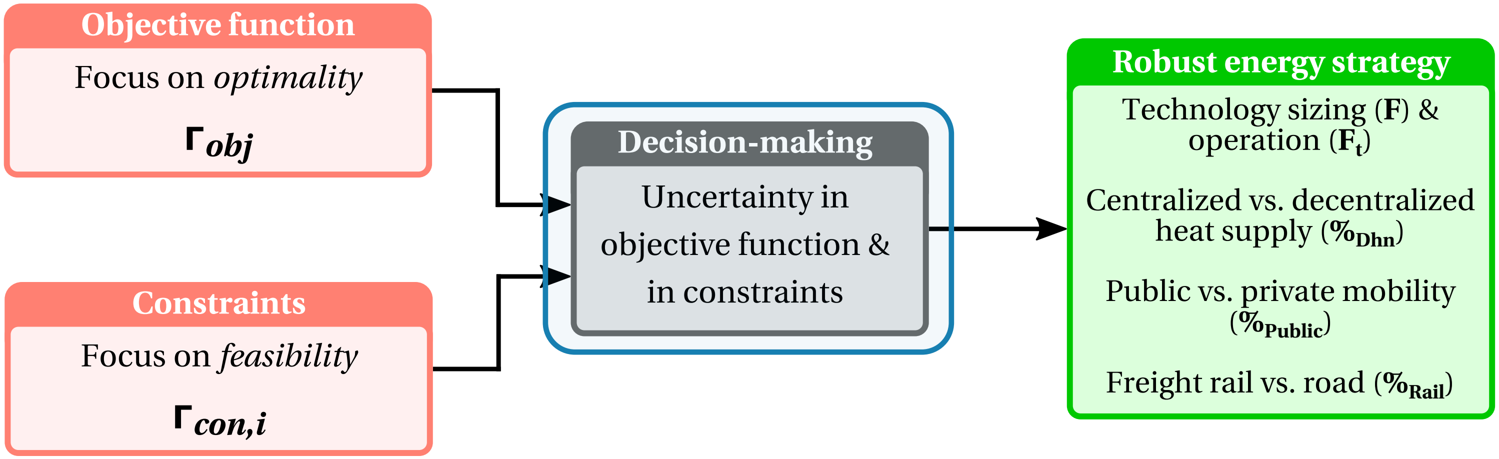
Energy carried through "channel" i

Availability of the channel (uncertain)

Channel capacity

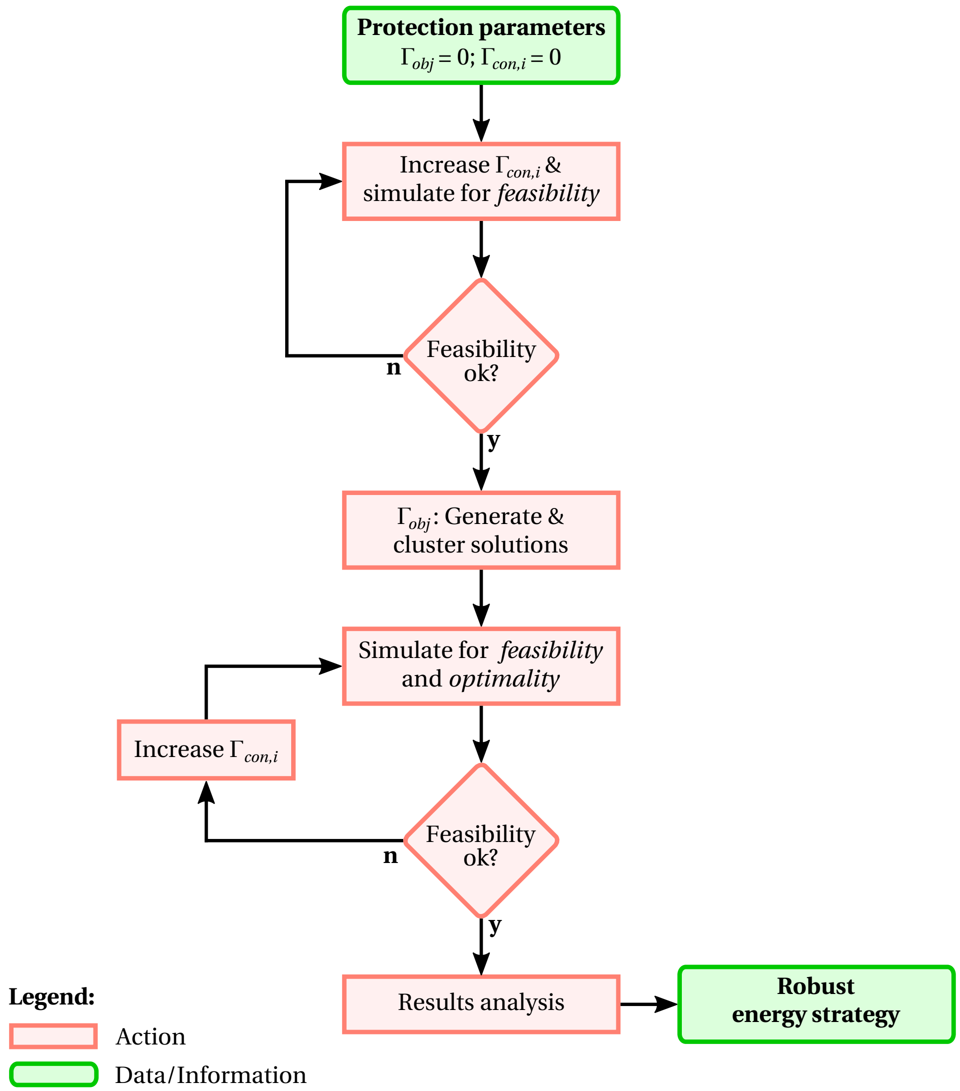
Robust optimization

Robust optimization framework



Robust optimization

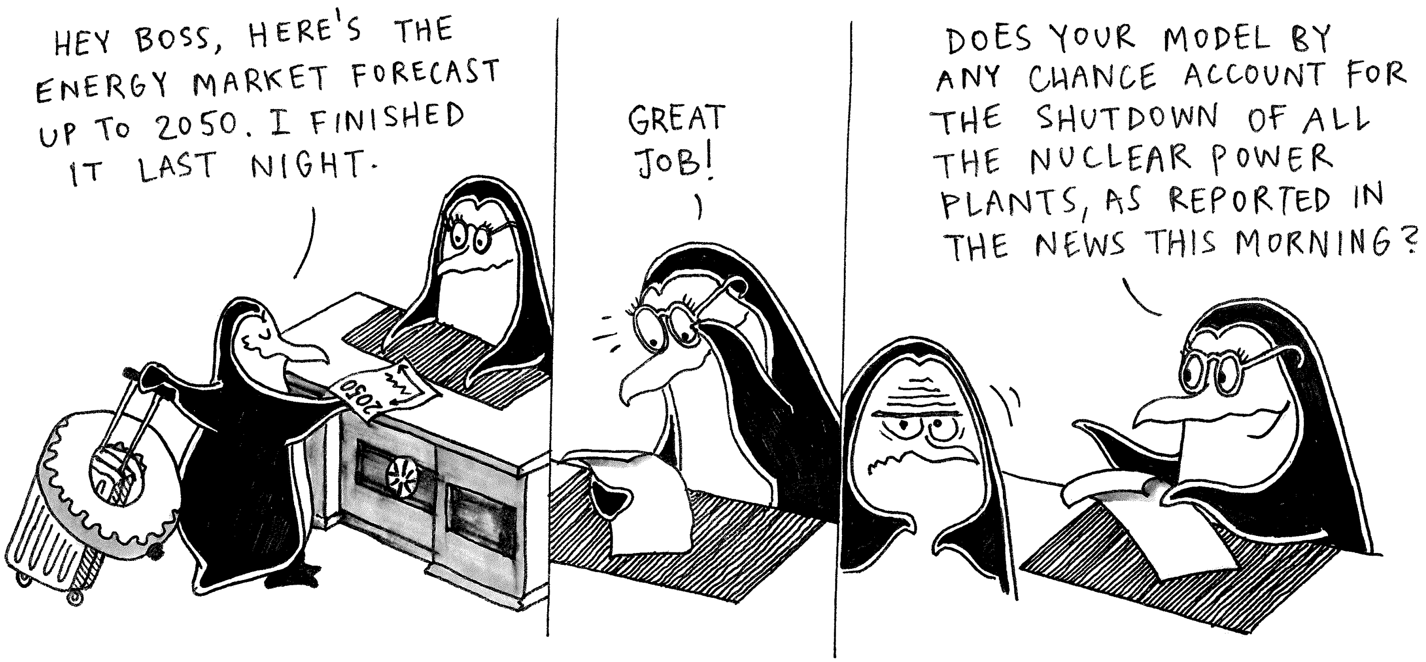
Decision-making method



First feasibility, then optimality

Solutions which are both feasible and cost-effective + limited computational burden

Thank you! Questions?

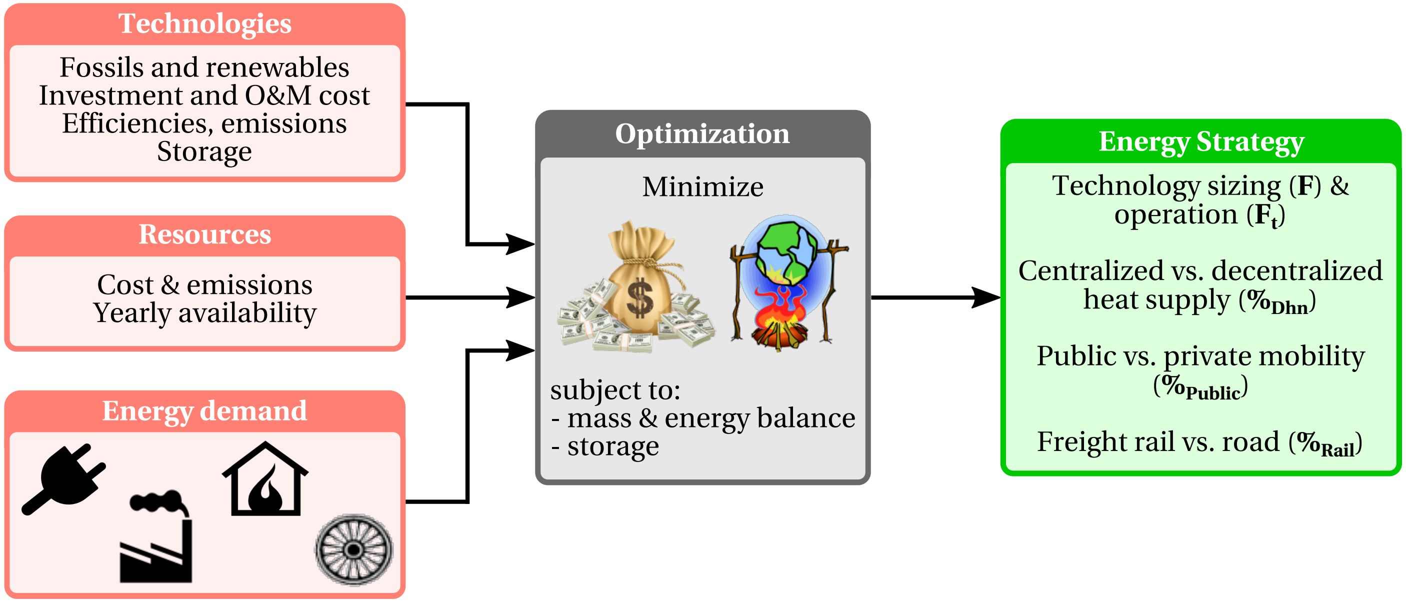


Appendix

Modeling for uncertainty

Contributions

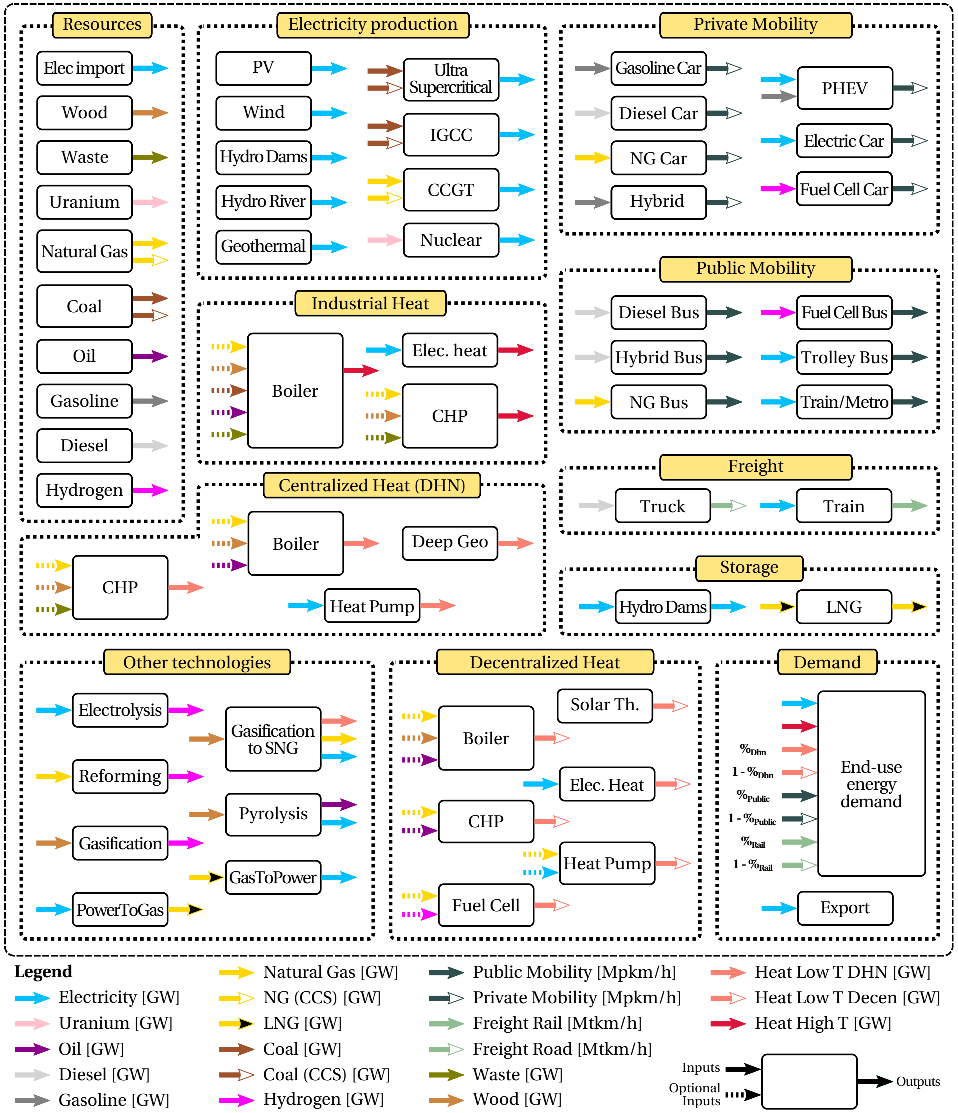
Novel MILP modeling framework for large-scale energy systems



- Energy-based model
- "Snapshot" model: optimization of the energy system in a future target year
- Simplified yet complete energy system: inclusion of heating and mobility
- Multiperiod formulation: seasonality of demand and energy storage
- Concise structure and low computational time → uncertainty applications

Modeling for uncertainty

Case study: the Swiss energy system

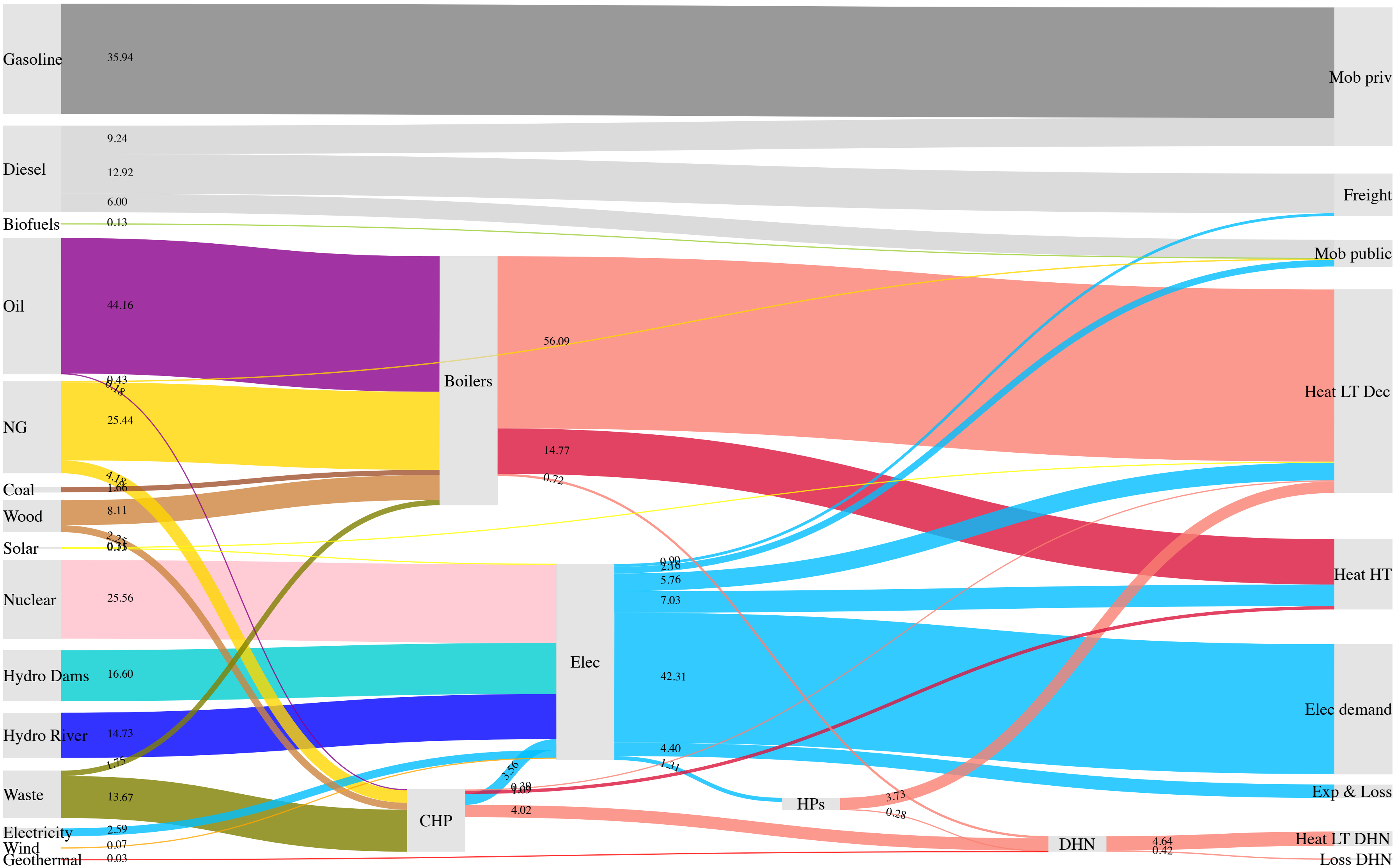


- 20-year time horizon
- Monthly resolution
- Additional constraints for CH
- Model complexity:
  - 1633 decision variables
  - 118 binaries
  - 56 integers
  - Solved in 0.25"

Modeling for uncertainty

Sources:
- Refs. [78-83]

Model validation: Switzerland in 2011



Modeling for uncertainty

Sources:
- Refs. [78-83]

Model validation: Switzerland in 2011

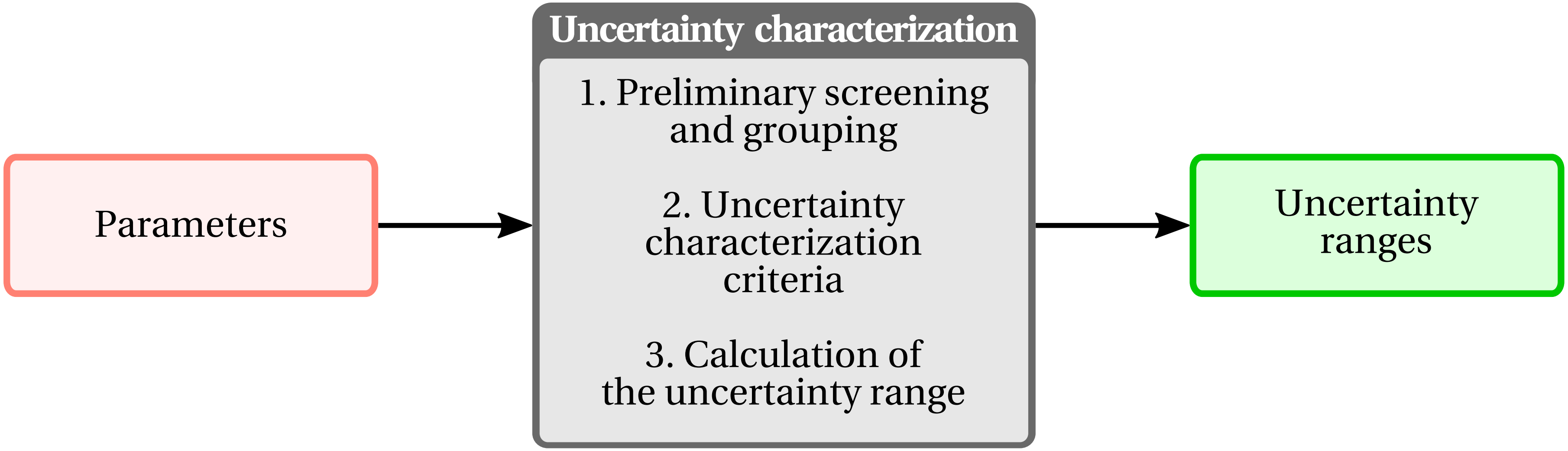
			Actual 2011	MILP	Δ	Units
Primary Energy Consumption		Gasoline	35.94	37.36	1.42	TWh
		Diesel	28.16	26.16	-2.00	TWh
		NG	30.05	28.40	-1.65	TWh
		Elec. imports	2.59	2.76	0.17	TWh
		Coal	1.66	1.43	-0.23	TWh
		Solar	0.46	0.48	0.02	TWh
		Geothermal	0.03	0.02	-0.01	TWh
		Waste	15.41	10.65	-4.76	TWh
		Oil	44.34	46.20	1.86	TWh
		Wood	10.36	9.32	-1.04	TWh
		Total	169.0	162.8	-6.21	TWh
Technologies Output		Boilers	71.59	72.53	0.94	TWh
		CHP	9.06	8.58	-0.48	TWh
		HPs	4.02	4.23	0.21	TWh
GHG emissions (fuels)			47.51[a]	46.92	-0.59	$MtCO_2$-eq.
Installed Technologies	HPs	Installed units	191.8	160.6	-31.2	kUnits
		Total	2.87	1.66	-1.21	GW_{th}
	CHP[b]	Installed units	41	51	10	Units
		Total	0.96	1.02	0.06	GW_{th}

Reasonable trade-off between time and accuracy, compatible with the level of detail of available data.

[a] Total GHG emissions following the Kyoto protocol [84], removing the direct non-energy related emissions from industrial processes.
[b] Large CHP installation (> 1 MW). 2011 Data for HPs and CHP in [78]

Uncertainty characterization

The method



Global sensitivity analysis

First stage: factor fixing

Goal: identifying non-influential parameters, i.e. parameters that can be fixed anywhere in their range of variation without significantly affecting the output of interest.

$$Y = h(\theta_1, \theta_2, \ldots, \theta_k)$$

The total effect sensitivity index of the *i*-th parameter is defined as:

$$S_{T_i} = \frac{\mathbb{E}_{\theta_{\sim i}}(V_{\theta_i}(Y|\theta_{\sim i}))}{V(Y)}$$

Average of V(Y) if only θ_i is varying

If $S_{T_i} = 0$, then θ_i is non-influential. But, S_{T_i} is expensive to calculate!

Elementary effect (Morris) method[119][120][51]:

- One-at-a-time GSA method
- Discrete sampling: *r* "trajectories", at every step only one of the *k* input varies of $\pm\Delta$
- Elementary Effect of the *i*-th input:

$$EE_{ij}^m = \frac{\delta Y_j}{\delta \theta_i} \frac{\sigma_{\theta_i}}{\sigma_{\theta_j}}$$

$$\mu_{ij}^* = \frac{1}{r} \sum_{m=1}^{r} |EE_{ij}^m|$$

Good proxy for $\mathbf{S_{T_i}}$

$r\,(k+1)$ model runs ($r = 15 \div 100$)

Global sensitivity analysis

Second stage: factor prioritization

- Goal: ranking influential parameters (<20) emerging from the first phase
- Variance-based methods[121]
- The first-order effect sensitivity index of the *i*-th parameter is defined as:

$$S_i = \frac{V_{\theta_i}(\mathbb{E}_{\theta_{\sim i}}(Y|\theta_i))}{V(Y)}$$

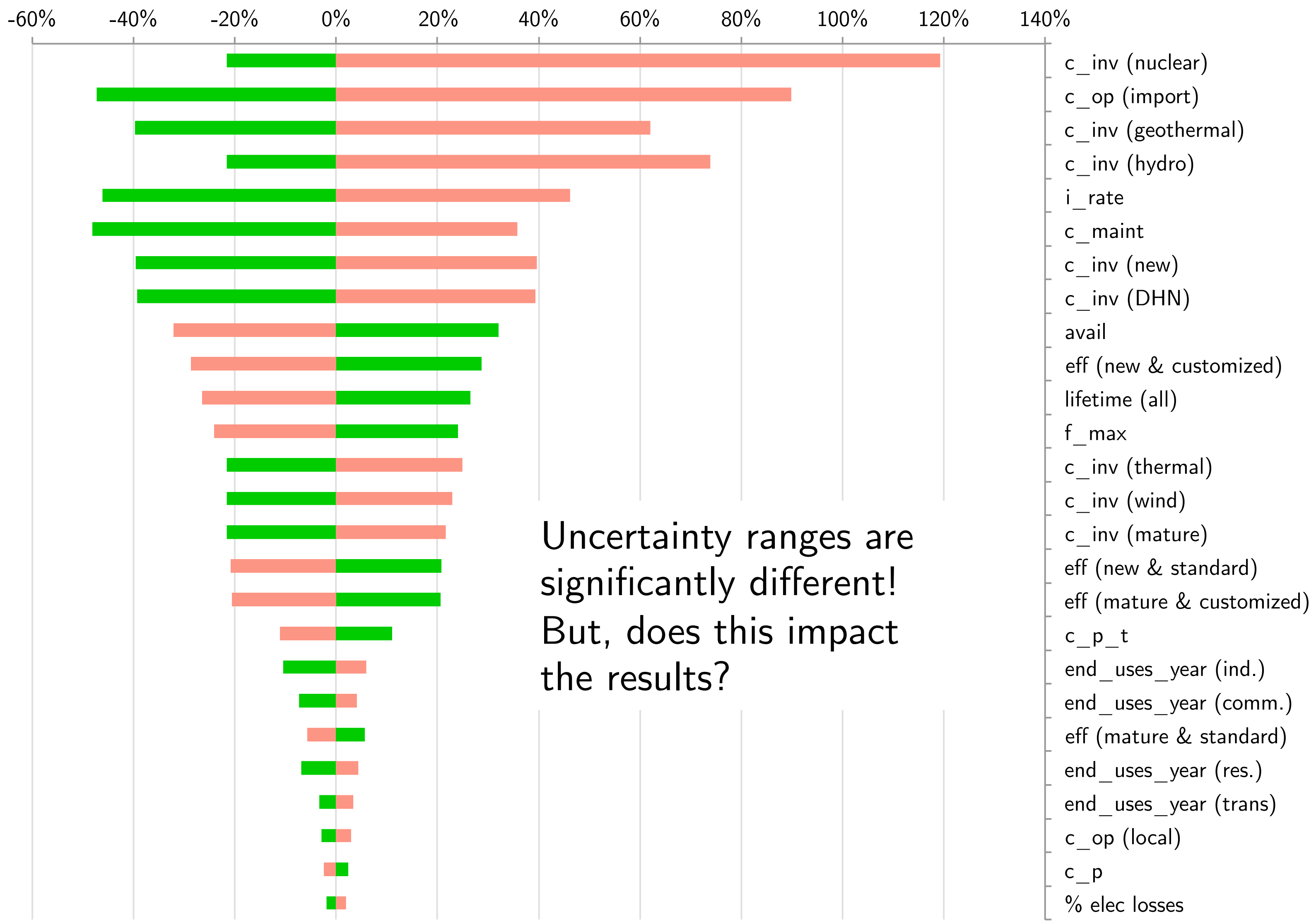
Reduction of V(Y) if fixing only θ_i

- S_i in [0;1]. If $S_i \rightarrow 1$, then θ_i is very influential
- $n(k+2)$ model runs, with n = x100-x1000

The calculation of both the first-order (S) and total effect (S_T) sensitivity indices by variance-based methods offers a good, synthetic characterization of the sensitivity pattern of a model.

Uncertainty characterization

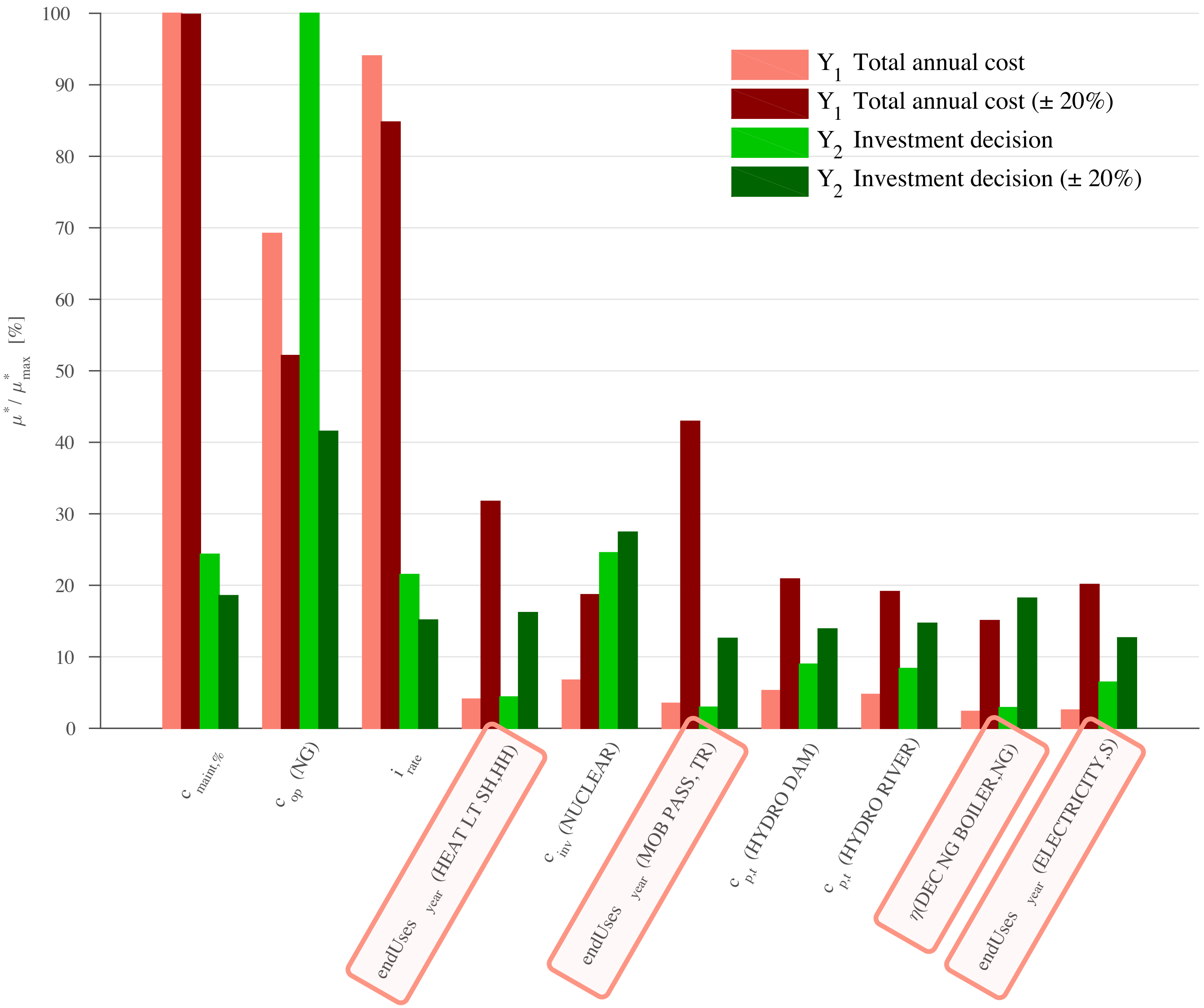
Case study: III. Calculation of the range



Uncertainty ranges are significantly different! But, does this impact the results?

Global sensitivity analysis

Does uncertainty characterization matter in energy planning?



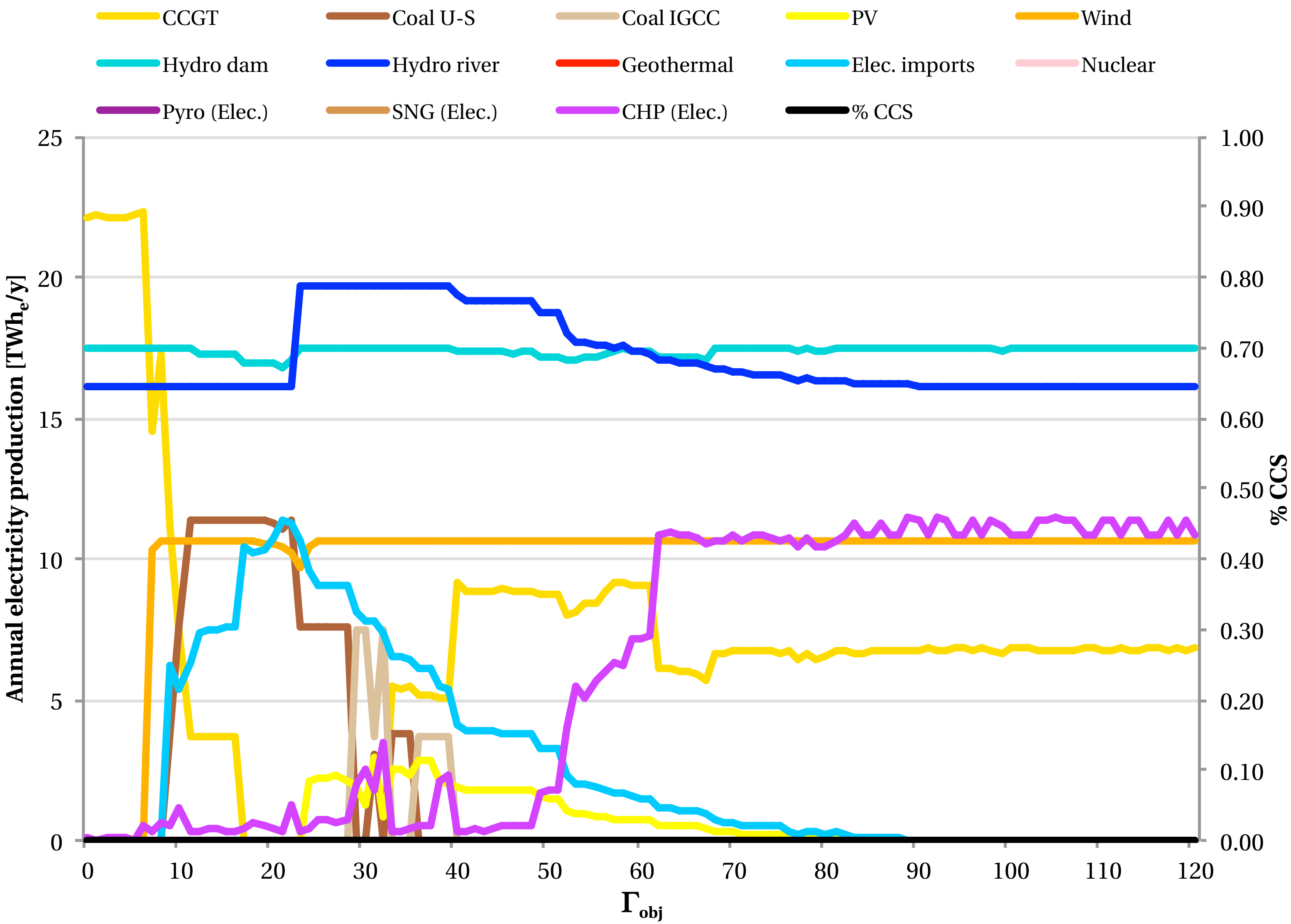
Obtained ranges vs. ±20% uncertainty for all parameters

Results:

- Non-influential → influential
- Influential → non-influential

Robust optimization

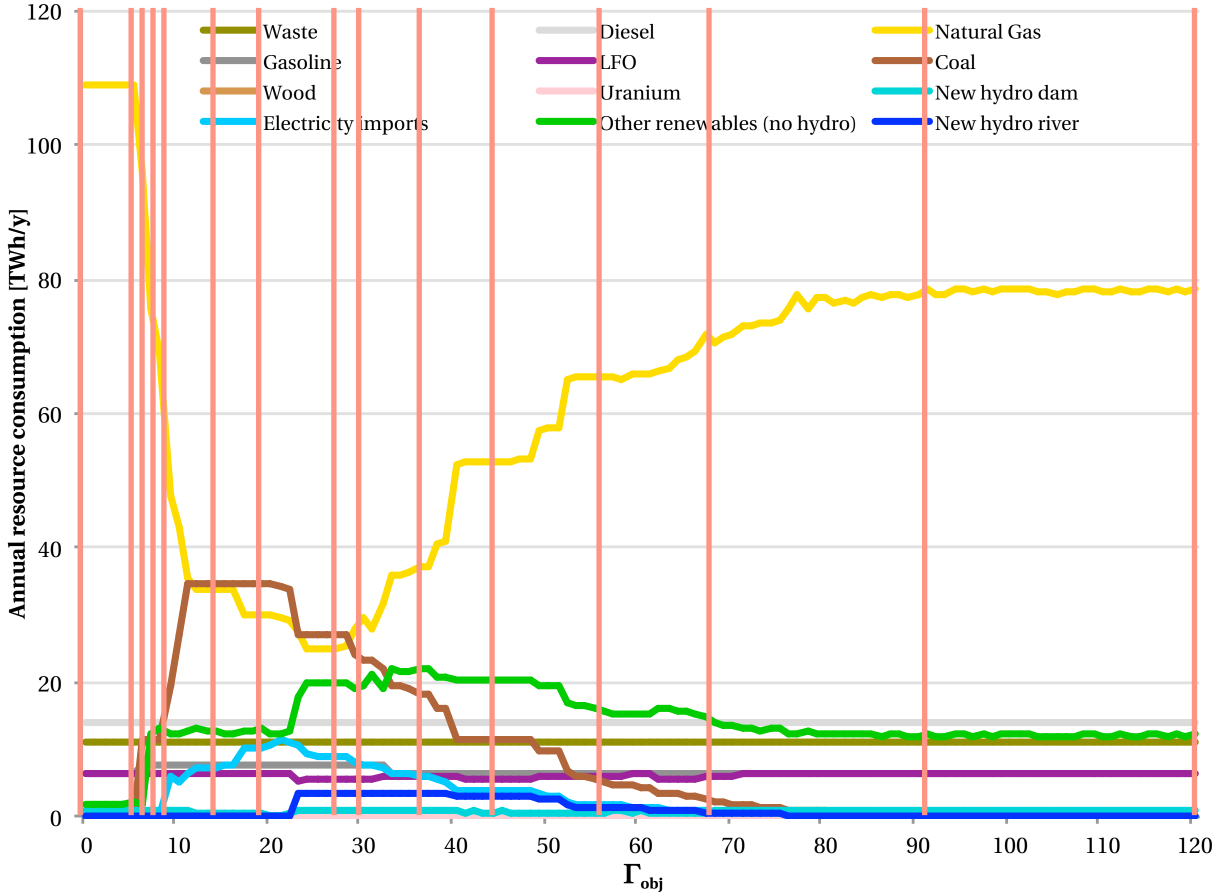
Uncertainty in the objective: results



Robust optimization

Uncertainty in the objective: evaluation of the robust solutions

Considering cost uncertainty in the objective corresponds to generating solutions in different scenarios → cannot compare robust objective values → simulation studies[124]



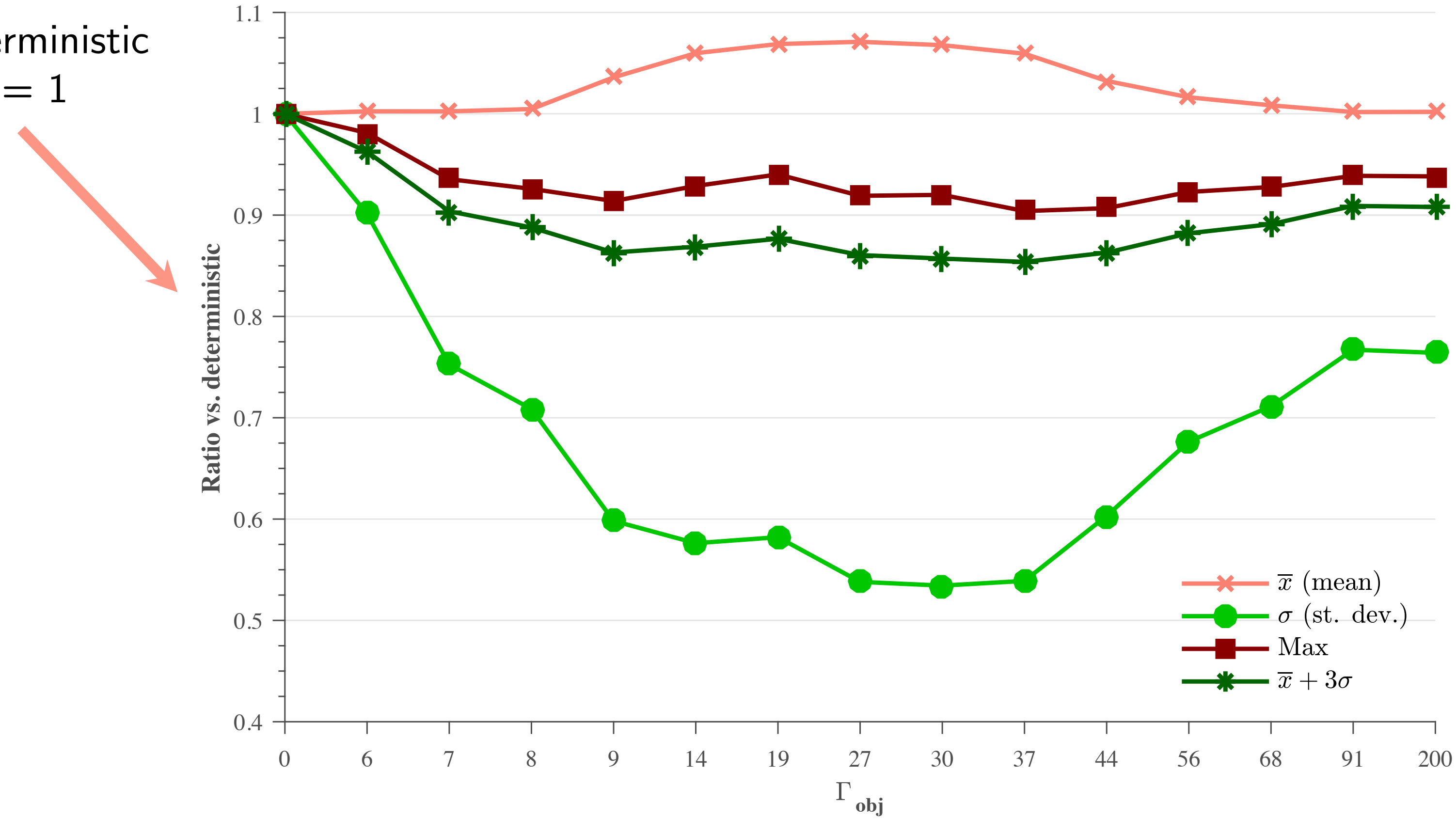
First, select 15 representative solutions (k-medoids[156] clustering)

Then, simulation study: Fix all decision variables (investment & operation), simulate 10000 times on all cost parameters (entire range)

Robust optimization

Uncertainty in the objective: evaluation of the robust solutions

Deterministic
= 1



- Test 1: robust → lower maximum cost, higher average, lower standard deviation
- Medium protection levels → more stability and protection against worst-case

Robust optimization

Uncertainty in the constraints

GSA results:

Rank	Category	$\overline{\mu^*_{Y_1}}$	$\overline{\mu^*_{Y_2}}$
1	$c_{maint,\%}$	4.749E-01	3.535E+00
2	i_{rate}	4.466E-01	3.124E+00
3	c_{op}	1.063E-01	3.421E+00
4	c_{inv}	2.162E-02	4.622E-01
5	$avail$	1.200E-02	1.185E+00
6	$c_{p,t}$	9.887E-03	7.040E-01
7	$endUses_{year}$	7.834E-03	4.820E-01
8	n	3.581E-03	2.802E-01
9	η	2.428E-03	3.540E-01
10	$Other$	1.894E-03	2.984E-01
11	$\%_{loss}$	6.157E-04	3.681E-01
12	f_{max}	4.953E-04	5.440E-01
13	c_p	5.601E-05	1.584E-01

Robust optimization works "constraint-wise". The objective function is a "special" constraint. The difference with the other constraints is that cost uncertainty does not affect feasibility, e.g. the risk of not meeting demand.

Robust optimization

Uncertainty in the constraints

Uncertainty in the constraints is seldom addressed. Why?

1. Same parameters appearing in multiple constraints[131]
2. Most constraints contain very few (even only one) uncertain parameter[157], e.g. a model has N constraints, each of them with one uncertain parameter → N control parameters $\Gamma_i \in [0;1]$ → combinatorial problem

How to address these issues?

1. Model formulation (modeling for uncertainty)
2. Formulation + idea by Babonneau et al. [153]:

$$\mathbf{ACT}(i) - AF(i) \times CAP(i) \leq 0 \qquad \forall i = 1, \ldots, N$$

Energy carried through "channel" i

Availability of the channel (uncertain)

Channel capacity

Robust optimization

Uncertainty in the constraints: the robust formulation by Babonneau et al.[153]

Babonneau et al.[153]: "*we are interested in protecting the total energy supply [...], not that of each channel separately*"

The idea is adding a redundant constraint summing over the constraint indices and to "robustify" this new constraint → instead of $N\ \Gamma_i \in [0;1]$, this gives one $\Gamma \in [0;N]$. The problem becomes tractable!

$$\sum_{i=1}^{N} \big(\mathbf{ACT}(i) - AF(i) \times CAP(i)\big) \leq 0$$

$$\mathbf{ACT}(i) - AF(i) \times CAP(i) \leq 0 \qquad \forall i = 1, \ldots, N$$

Energy carried through "channel" *i*

Availability of the channel (uncertain)

Channel capacity

Robust optimization

Uncertainty in the constraints: application to the case study

How can the method be applied to the case study?

$$\sum_{t \in T} \mathbf{F_t}(i,t)\, t_{op}(t) \leq avail(i) \qquad \forall i \in RES$$ 2 parameters

$$\mathbf{F_t}(j,t) \leq \mathbf{F}(j)\, c_{p,t}(j,t) \qquad \forall j \in TECH, \forall t \in T$$ 60 uncertain parameters

$$\sum_{i \in RES \cup TECH \setminus STO} f(i,l)\mathbf{F_t}(i,t) + \sum_{j \in STO} (\mathbf{Sto_{out}}(j,l,t) - \mathbf{Sto_{in}}(j,l,t)) - \mathbf{EndUses}(l,t) = 0 \qquad \forall l \in L, \forall t \in T$$

52 + 15 = 67 uncertain parameters

Summing over the indices means that the summed parameters can share the same uncertainty budget. Need of case by case evaluation!

Robust optimization

Uncertainty in the constraints: application to the case study

How can the method be applied to the case study?

$$\sum_{t \in T} \mathbf{F_t}(i,t)\, t_{op}(t) \leq avail(i) \qquad \forall i \in RES$$ → Not possible/meaningful

$$\mathbf{F_t}(j,t) \leq \mathbf{F}(j)\, c_{p,t}(j,t) \qquad \forall j \in TECH, \forall t \in T$$ → Sum over T → $\Gamma_{c_{p,t}}$

$$\sum_{i \in RES \cup TECH \setminus STO} f(i,l)\mathbf{F_t}(i,t) + \sum_{j \in STO} (\mathbf{Sto_{out}}(j,l,t) - \mathbf{Sto_{in}}(j,l,t)) - \mathbf{EndUses}(l,t) = 0 \qquad \forall l \in L, \forall t \in T$$

→ Transform into inequality and sum over L → Γ_{lb}

- Application of the method not always possible/meaningful
- Need to carefully evaluate over which sets summation can be performed

When it is possible..

- Aggregation of constraints allow tractability → few control parameters
- Importance of having a concise deterministic model formulation (compact definition of sets and constraints)

Robust optimization

Uncertainty in the constraints: evaluation of the robust solutions

Constraints → feasibility

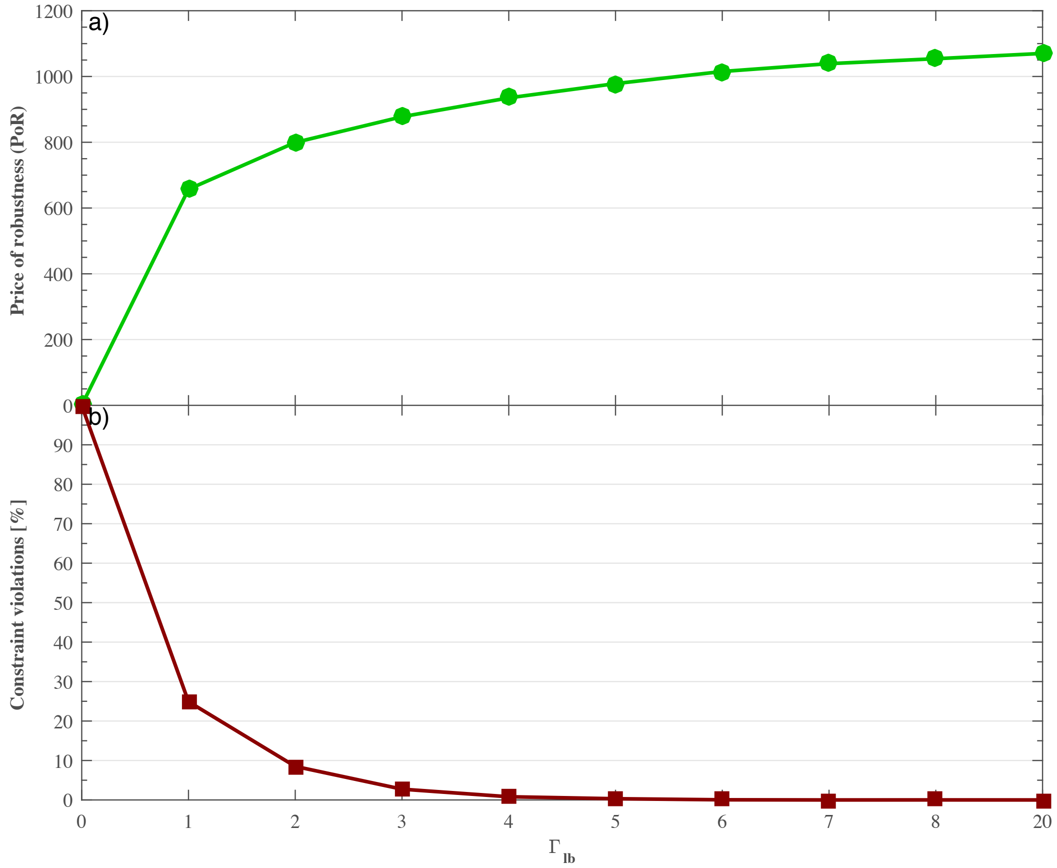
Price of Robustness (PoR)[62]: by increasing protection against worst case, constraint violations (e.g. risk of not meeting demand) are reduced at the price of a higher objective value → simulations with no uncertainty in the objective ($\Gamma_{obj} = 0$)

Focus of Γ_{lb} → Two sets of simulations:

1. First test (to verify theory): fix all decision variables (investment & operation), simulate on both efficiency and demand (entire range)
2. Second test: Fix only investment decision (free resources), simulate on both efficiency and demand (entire range)

Robust optimization

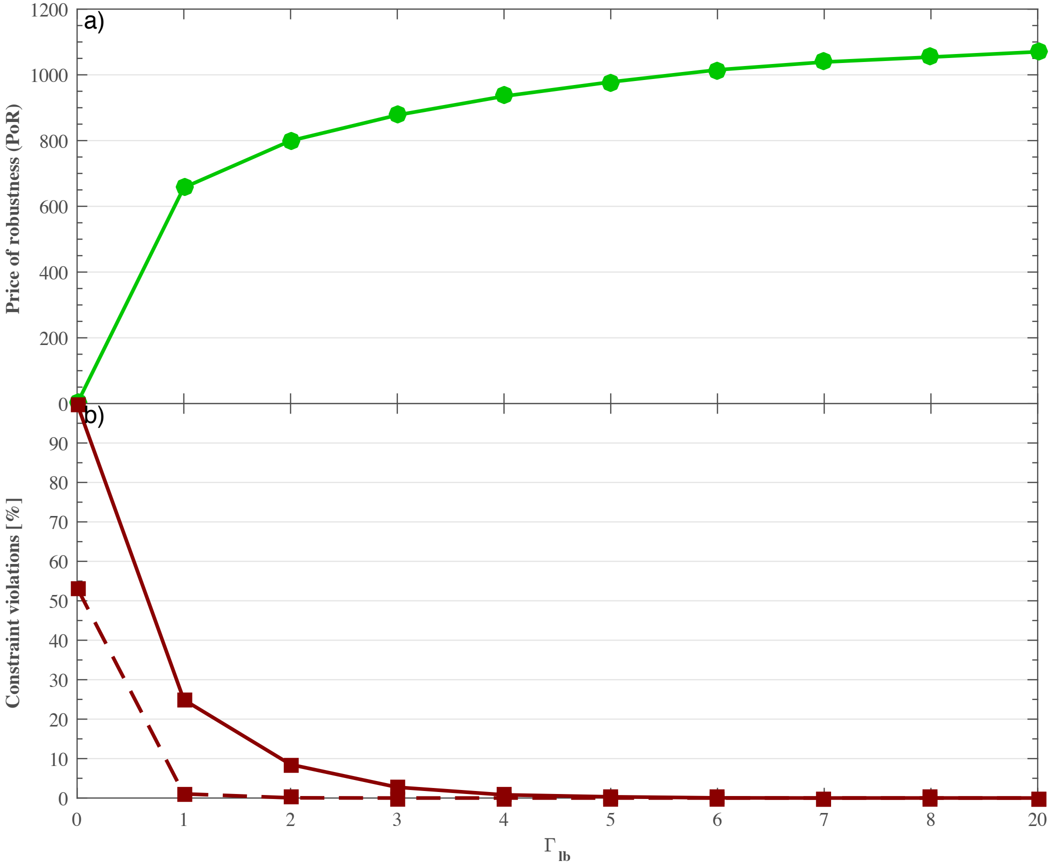
Uncertainty in the constraints: evaluation of the robust solutions



- Test 1: in line with theory → no need of full protection

Robust optimization

Uncertainty in the constraints: evaluation of the robust solutions



- Test 1: in line with theory → no need of full protection
- Test 2: in real situations, infeasibility disappears at very low protection levels
- Overall, uncertainty in the constraints, which is often overlooked, can be very impacting.

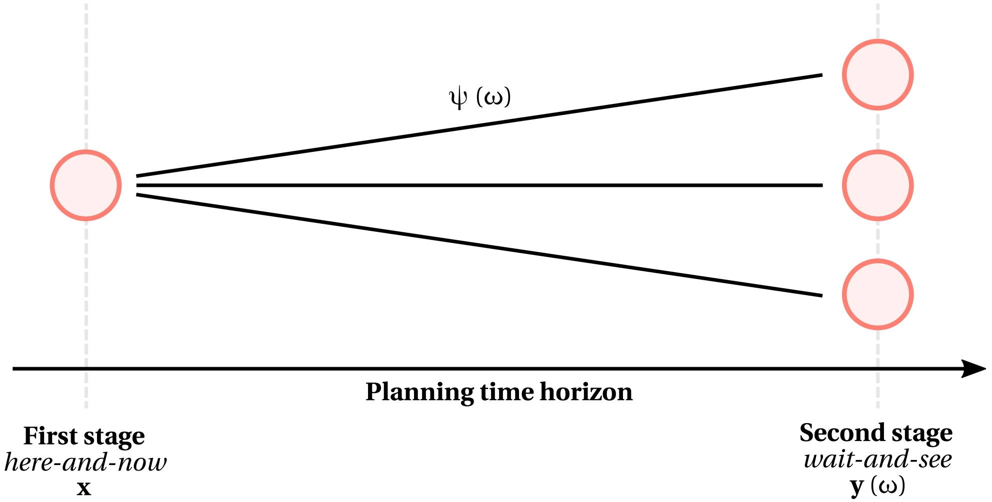
Robust optimization

Sources:
- Ref. [57]

Decision-making: comparison with stochastic programming

"Traditional" approach, it multi-stage nature makes it *"appropriate for long-term [...] planning [...], since it does not fix all the decisions at the initial point of the planning horizon as it allows recourse decisions in future times to adapt in response to how the uncertainties are revealed"* [36]

$$\begin{aligned} \min \quad & c^T \mathbf{x} + \mathbb{E}_{\psi}[\min q(\omega)^T \mathbf{y}(\omega)] \\ \text{s.t.} \quad & \mathcal{A}\mathbf{x} = b \\ & \mathcal{T}(\omega)\mathbf{x} + \mathcal{W}\mathbf{y}(\omega) = \beta(\omega) \\ & \mathbf{x}, \mathbf{y}(\omega) \in \mathbb{R}^+ \end{aligned}$$



Stochastic version of the model using DET2STO[162]:

- First-stage (investment) vs. second-stage decision variables (operation)
- 3 values for the parameters (low-medium-high) → 3^{θ} scenarios
- Solving time: $\theta = 5$ → >1h; $\theta = 6$ → >3d; LP: $\theta >= 8$ → >8GB RAM
- Chosen $\theta = 7$ parameters (c_{op} and i_{rate}) → 2.4 million variables LP (3h)

Robust optimization

Decision-making: comparison with stochastic programming

Optimal investment strategy

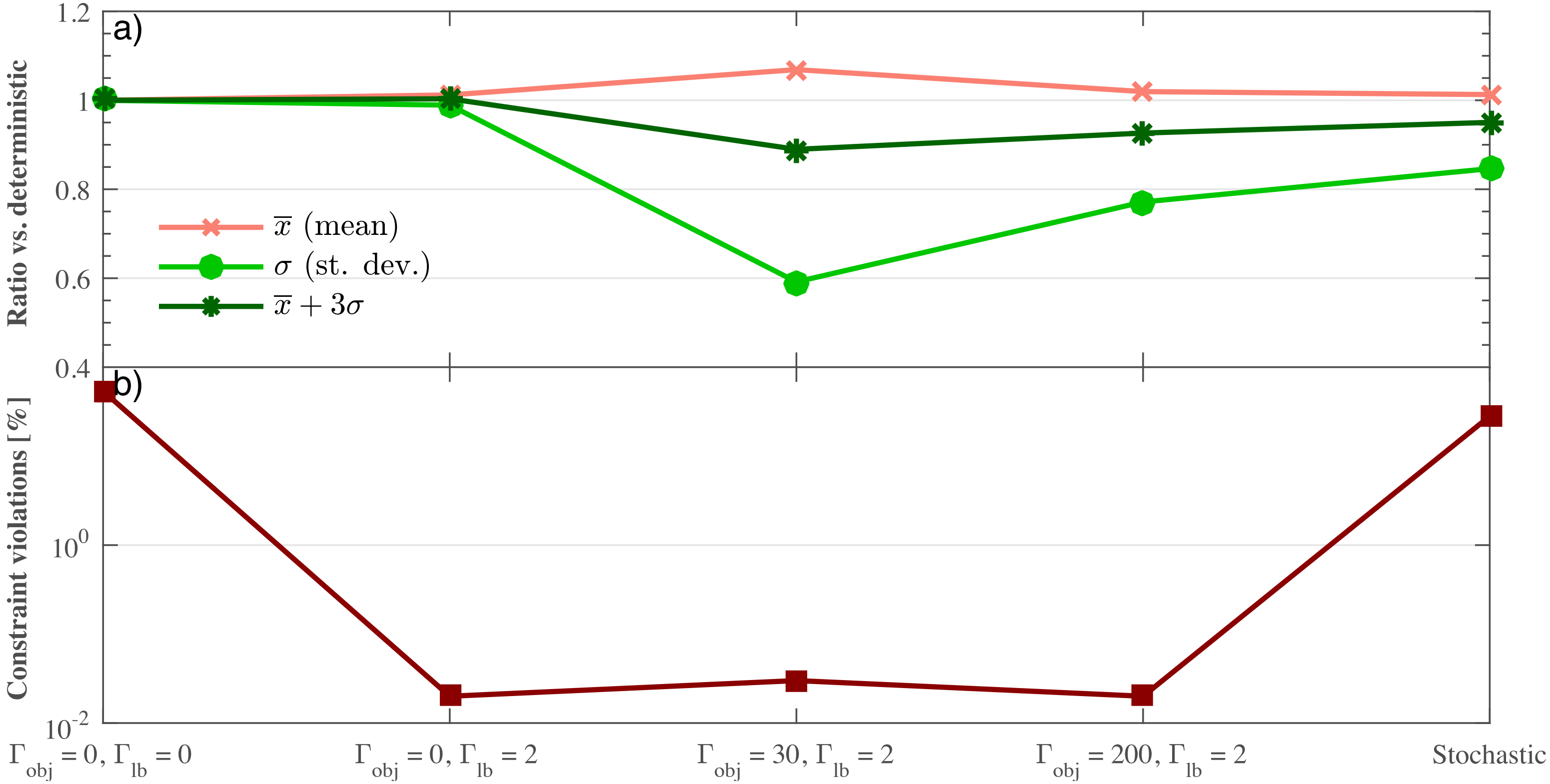
	Technology	Installed size	Units
Electricity Production	**CCGT**	0.52	[GW_e]
	Coal	0	[GW_e]
	PV	0	[GW_e]
	Wind	5.30	[GW_e]
	New Dam	0.44	[GW_e]
	New River	0	[GW_e]
Heat Production	**Boilers**	22.0	[GW_{th}]
	CHP	1.31	[GW_{th}]
	Elec. HPs	2.18	[GW_{th}]
	Solar Th.	0	[GW_{th}]
	Deep Geo	0.77	[GW_{th}]
	$\%_{Dhn}$	0.3	[-]

Simulation study to compare stochastic vs robust:

- All uncertain parameters → check feasibility and optimality
- Only investment decisions are fixed, operation is left free

Robust optimization

Decision-making: comparison with stochastic programming



- Stochastic vs deterministic: +1.3% mean, -15% stdev, high constraint violations
- Robust: higher average, much lower stdev, much lower against constraint violations
- Robust can be a good alternative to stochastic for big problems