Displacement-based methods

EDCE: Civil and Environmental Engineering
CIVIL 706 - Advanced Earthquake Engineering
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• Assumptions
• Reinforced concrete: chord rotation
• Capacity curve
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• Unreinforced masonry
• Example: masonry shear walls
Link to force-based methods

• Application of behavior factor $q$ specified in the codes SIA 262-266

• For existing buildings, non ductile behavior ($q = 1.5$ or $q = 2.0$ for reinforced concrete)

• Displacement-based method: more realistic approach of real seismic behavior

• Allows to justify an higher value of $q$
Displacement-based Method (DbM)

• Applicable to deformable structures

• Brittle failure modes excluded:
  - shear failure
  - bending with concrete failure before steel yielding
  - bending with steel failure without sufficient plastic deformation
  - failure of overlapping zones or anchorage zones
Displacement-based Method (DbM)

- More realistic approach of real behavior

![Diagram showing force vs. displacement with small and large changes at different points.](image-url)
Method included in SIA 2018

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ETH
Reinforced concrete: chord rotation

shear wall  column  beam

\[
\begin{align*}
V & \quad \theta \\
M & \quad V \\
M & \quad V
\end{align*}
\]
Reinforced concrete: chord rotation

- Angle formed by the tg to element axis at $M_{\text{max}}$ location and the chord joining this point to the zero moment point

- Represents the element solicitation (removal of rigid displacements)

- Verification by the comparison (demand-capacity) of chord rotations at the element level
Chord rotation $\theta$: definitions

- shear wall
- column
- beam

- rigid body rotation
- $\theta$ = chord
- $\theta$ = tangent to element axis
- $\theta$ = zero moment point

$V$, $M$, $V_R$, $M_L$, $M_R$
Yielding chord rotation $\theta_y$

\[
\Delta_y = \frac{F_y L_v^3}{3EI} = \frac{M_y L_v^2}{EI} \cdot \frac{3}{3} = \phi_y \frac{L_v}{3} \cdot L_v = \theta_y L_v
\]
Ultimate chord rotation $\theta_u$

\[ L_{pl} = a_{st} \left( 0.08 \cdot L_v + 0.022 \cdot f_s \cdot d_{bl} \right) \]

$h_{pl} = \text{plastic zone}$

$h_{sp} = \text{strain penetration}$

$L_{pl} = \text{plastic hinge}$
Ultimate chord rotation $\theta_u$

$$\Delta_u = \theta_u L_v = \theta_y L_v + (\phi_u - \phi_y) L_{pl} \cdot (L_v - 0.5 \cdot L_{pl})$$
Non-linear force-displacement relationship

\[ \Delta = f(\phi_y), \quad F_y = f(M_y), \quad \Delta_u = f(\phi_y, \phi_u), \quad F_u = f(M_u), \]

\[ \Delta_y, F_y, \Delta_u, F_u \]
Bending moment-curvature relationship

\[ \phi_y = \phi'_y \frac{M_n}{M'_y} \]
Bending moment-curvature relationship

- shear wall

\[ N = -1099 \text{ kN} \]
Bending moment-curvature relationship

- First yield: $s = \varepsilon_y, \varepsilon_c = 0.002$
- Nominal strength: $s = 0.015, \varepsilon_c = 0.004$
- Ultimate: $s = s_{\text{max}}, \varepsilon_c = \varepsilon_{c_{\text{max}}}$
Bending moment-curvature relationship

![Graph showing the bending moment-curvature relationship with 1st approximation and modified approximation marks.](image)

- Bending moment $[\text{MNm}]$
- Curvature $[10^{-3} \text{ m}^{-1}]$
- $M_n$, $M'_y$, $\phi_y$, $\phi'_y$, $\phi_u$
Bending moment-curvature relationship

- nominal yield curvature $\phi_y$

6.2.9 Pour des éléments de construction courants, la courbure nominale de plastification $\phi_y$ peut être estimée comme suit:

Colonne ronde: $$\phi_y = 2.25 \cdot \frac{\varepsilon_{sk}}{D}$$ (11)

Colonne ou poutre rectangulaire: $$\phi_y = 2.1 \cdot \frac{\varepsilon_{sk}}{l_s}$$ resp. $$\phi_y = 2.1 \cdot \frac{\varepsilon_{sk}}{h_b}$$ (12)

Paroi porteuse rectangulaire: $$\phi_y = 2.0 \cdot \frac{\varepsilon_{sk}}{l_w}$$ (13)

$\varepsilon_{sk}$ valeur caractéristique de l’allongement de plastification de l’acier d’armature passive

$D$ diamètre de la colonne

$l_s$ longueur de la colonne dans la direction de sollicitation

$h_b$ hauteur d’une poutre de section rectangulaire

$l_w$ longueur de la paroi porteuse
Bending moment-curvature relationship

- Material mechanical properties

Figure 4  Diagramme contrainte-déformation spécifique idéalising pour l’évaluation des constructions en béton selon la méthode basée sur les déformations
Material mechanical properties

a) concrete BH300, SIA 168/68
1) $f_{\text{cd}}$ acc. to SIA 262
2) $f_{\text{ck}}$ acc. to SIA 2018
3) $f_{\text{ck}}(t)$ acc. to SIA 2018

b) steel IIIa, SIA 168/68
1) $f_{\text{sd}}$ acc. to SIA 262
2) $f_{\text{sk}}$ approx.
3) $f_{\text{sk}}$ type
Material mechanical properties

- **Characteristic values**

- **Partial factor for deformation capacity**
  \[ \gamma_D = 1.3 \]

- **Ultimate concrete deformation**
  \[ \varepsilon_{cmax} = \varepsilon_{cu} = 0.004 \text{ (in general)} \]

- **Peak steel strain**
  \[ \varepsilon_{smax} = \alpha \varepsilon_{su} \]
  - in general: \( \alpha = 1.0 \)
  - if \( M_u < 2M_{cr} \): \( \alpha = 0.5 \)
  - special cases: \( \alpha = ? \)
Application of DbM

• Non-linear element behavior
• Equivalent SDOF
• Force-displacement relationship of structure
• Capacity curve
• Seismic displacement demand
• Shear strength verification
Example: reinforced concrete

Main properties

- 5 stories
- Building end of 60’
- 1 x direction « wall »
- 1 x direction « frame »
- Zone 3b, CS C, CO I
Example RC: dimensions and structure

- \( m_5 = 213t \)
- \( m_4 = 380t \)
- \( m_3 = 380t \)
- \( m_2 = 380t \)

- Total mass: \( m_{\text{tot}} = 1733t \)
Example RC: element dimensions

shear wall cross-section

beam at mid span

beam at connection

column type A

column type B
Example RC: shear walls direction

\[ F_d \]

\[ \text{w} \]

\[ \text{walls} \]

\[ m = 1163 \text{t} \]

\[ h = 11.95 \text{m} \]

\[ k^{*} = \frac{m*}{F_d} \]

\[ k_{\text{MDOF}} = \sum k_{\text{walls}} \]

\[ k^{*} \]

\[ h^{*} = 11.95 \text{m} \]
Example RC: shear walls direction

• Force-displacement relationship

![Diagram showing force-displacement relationship for shear walls and a building. The diagram includes points indicating the yield force \( F_y \) and the ultimate force \( F_u \) for both shear walls and the building. The horizontal displacement is denoted as \( w/\Gamma \) with values 0.039 m and 0.119 m for shear walls and the building respectively.](image-url)
Example RC: shear walls direction

- ADRS spectrum with capacity curve

![Graph showing elastic design spectrum and capacity curve with labels and annotations](image-url)
Example RC: frame direction

• Force-displacement rel.: \( \sum \) frames A, B and C

• Torsion neglected
Example RC: frame direction

• Modelisation
Example RC: frame direction

- Column element type A

![Diagram showing a column element with forces and moments labeled.](image)

- Plastic hinge at $L_{pl}/2$
- Bending moment $M$,
- Axial force $N$,
- Shear force $V$,
- Curvature and bending moment graphs with plastic rotation:
  - Curvature: $[10^{-3} \text{ m}^{-1}]$,
  - Bending moment: $[\text{KNm}]$,
  - Plastic rotation: $[10^{-3}]$.
Example RC: frame direction

- Beam element

![Beam element diagram]

- Rotule plastique $M/\theta$
  (rigide-plastique avec adoucissement)

![Graphs showing moment vs. rotation and curvature]

- Type 2
- Type 8
- Type 6
- Type 4
Example RC: frame direction

- Force-displacement relationship

![Graph showing force-displacement relationship for a building comprised of 2 frames A and 1 frame B. The graph displays global horizontal force vs. horizontal displacement. Key points include:

  - Building response:
    - Total building force $F_y = 2032$ kN
    - Total building displacement $w_y = 0.098$ m
    - Onset of plastic hinge softening

  - Frame A:
    - Force $w_{u} = 0.182$ m

  - Frame B:
    - Force $w_{u} = 0.182$ m

  - Onset of plastic hinge softening

  - Diagram scale:
    - Horizontal displacement $W_d$ in [m]
    - Global horizontal force $F_d$ in [kN]
Example RC: frame direction

- Ultimate deformation of frame B

\[ W_d \]

\[ F_d \]
Example RC: frame direction

- ADRS spectrum with capacity curve

![Graph showing ADRS spectrum with capacity curve.](image)
Unreinforced masonry

Dörflingen douane

Vernier

Yverdon-Les-Bains

Oberriet
DbM: unreinforced masonry

- Assumptions (FEMA 356)

plastic deformations concentrated in some elements (piers)

\[ h_{\text{eff}} = \text{The effective height of the component under consideration} \]
\[ \Delta_{\text{eff}} = \text{The differential displacement between the top and bottom of the component} \]
DbM: unreinforced masonry

- Limitation: out-of-plane failure excluded
Masonry: out-of-plane failure

• Control with h/t ratio (FEMA 356)

<table>
<thead>
<tr>
<th>Wall Types</th>
<th>$S_{X1} \leq 0.24g$</th>
<th>$0.24g &lt; S_{X1} \leq 0.37g$</th>
<th>$S_{X1} &gt; 0.37g$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Walls of one-story buildings</td>
<td>20</td>
<td>16</td>
<td>13</td>
</tr>
<tr>
<td>First-story wall of multistory building</td>
<td>20</td>
<td>18</td>
<td>15</td>
</tr>
<tr>
<td>Walls in top story of multistory building</td>
<td>14</td>
<td>14</td>
<td>9</td>
</tr>
<tr>
<td>All other walls</td>
<td>20</td>
<td>16</td>
<td>13</td>
</tr>
</tbody>
</table>
Masonry: deformation capacity

- Compression depending (tests EPFZ)
Masonry: deformation capacity

- Influence of compression

![Graph showing the influence of compression on lateral force and drift for different masonry structures.](image)
Masonry: deformation capacity

- In function of normal force (K. Lang, 2002) based on test results

\[
\delta_u = 0.8 - 0.25 \cdot \sigma_n
\]
Strength of unreinforced masonry

- In function of the inclination of $\sigma_2$
Lateral strength of URM elements

- Determination with stress fields

**Principle:**

superposition of a vertical field and an inclined field
Lateral strength of URM elements

- Determination with stress fields
Lateral strength of URM elements

• Determination with stress fields

– équilibre:

\[ N_v + N_n = N \]
\[ N_v \cdot e_{1v} + N_n \cdot e_{1n} = M_1 \]
\[ N_v \cdot e_{2v} + N_n \cdot e_{2n} = M_2 = M_1 + V \cdot h \]

– limitation des sollicitations:

\[ f_\alpha = \frac{N_v}{l_{2v} \cdot t \cdot (\cos \alpha)^2} \leq f_{my} \]
\[ l_{2v} = l_w - 2 \cdot e_{2v} \]

avec

\[ f_v = \frac{N_n}{l_{2n} \cdot t} \leq (f_{mx} - f_{my}) \]
\[ l_{2n} = l_w - 2 \cdot e_{2n} \]

\[ \tan \alpha \leq \tan \varphi \]

– résistance:

\[ V = N_v \cdot \tan \alpha \]
Failure modes

• Rocking
Failure modes

• Shear
Lateral strength of URM elements

- Simplified formulae (FEMA, EC8)

rocking (EC8):

\[ V_{Rd,R} = \frac{l_w \cdot N_{xd}}{2 \cdot h_0} \left(1 - 1.15 \cdot \frac{N_{xd}}{l_w \cdot t_w \cdot f_{xd}} \right) \]

shear (EC8):

\[ V_{Rd,S} = f_{vd} \cdot t \cdot l_c \]

with

\[ f_{vd} = f_{vd0} + 0.4 \cdot N_{xd} \cdot \frac{l_c}{t} \]

\[ f_{vd} \leq 0.065 \cdot f_b \]
Masonry: deformation capacity

• According to EC8

rocking:

\[ \delta_u = 0.8 \cdot \frac{h_0}{l_w} \quad [\%] \]

shear:

\[ \delta_u = 0.4 \quad [\%] \]
Unreinforced masonry buildings

- Swiss typical building (Yverdon)
Example: unreinforced masonry

- Simplified typology
Example: unreinforced masonry

- Lateral strength in transversal direction

<table>
<thead>
<tr>
<th></th>
<th>(N_d) [kN]</th>
<th>without coupling effect (V_{Rd}) [kN]</th>
<th>with total coupling effect (V_{Rd}) [kN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>(l_w = 2.0) m</td>
<td>180</td>
<td>22</td>
<td>96</td>
</tr>
<tr>
<td>(l_w = 5.0) m</td>
<td>360</td>
<td>117</td>
<td>216</td>
</tr>
<tr>
<td>(l_w = 6.0) m</td>
<td>1520</td>
<td>367</td>
<td>624</td>
</tr>
<tr>
<td>total</td>
<td></td>
<td>690</td>
<td>1440</td>
</tr>
</tbody>
</table>
Example: unreinforced masonry

- Displacement determination (Lang 2002)
Example: unreinforced masonry

- Capacity curve in transversal direction

![Capacity curve in y direction](image)
Unreinforced masonry buildings

- Swiss typical building ("in line" building)

RC rigid floors, diaphragm effect
Very regular, no torsion
3 shear walls, long. direction
Zone Z1, $a_{gd} = 0.6 \text{ m/s}^2$
Soil E, $S = 1.40$
$H_{\text{storey}} = 2.85 \text{ m}$
$M_{\text{storey}} = 470 \text{ t}$
$V_{M,\text{base}} = 880 \text{ kN}$
Top, $w_y = 20 \text{ mm}$
Top, $w_u = 35 \text{ mm}$
Unreinforced masonry buildings

- Swiss typical building ("in line" building)
Unreinforced masonry buildings

- Swiss typical building (“in line” building)
Masonry: lot of uncertainties

• Real element deformation capacity?

• Effective coupling effect?

• Effective element stiffness (crack pattern)?

• Research efforts needed
Real behavior?

• Coupling effect (frame effect)

Flexible floors (without coupling effect):

\[ h_0 \approx \frac{2}{3} \cdot h_{tot} \]

Very stiff lintels (total coupling effect):

\[ h_0 \approx \frac{1}{2} \cdot h_{st} \]