N2 method and Rocking behaviour
NL Time History Analysis check

EDCE: Civil and Environmental Engineering
CIVIL 706 - Advanced Earthquake Engineering
Content

• Equal displacement rule
• Target displacement, EC 8 part 1, Annex B
• Approximate graphical assessment
• N2 displacement demand prediction accuracy
• Rocking (S-model and Griffith model)
• Fast assessment for cantilever out-of-plane
• Amplification with floor response spectrum
Equal displacement rule

- Favorable effect, behaviour factor $q$
- Displacement ductility = behaviour factor value
- Avoid brittle failures (shear failures)

Empirical rule: equal displacements

\[ F_y = \frac{F_{el}}{q} \]

\[ x_e = \frac{x_{el}}{q} \]

\[ x_p = x_{el} \]
Equal displacement rule, check NLTHA

• 164 recorded earthquakes, methodology
Equal displacement rule, check NLTHA

- 164 recorded earthquakes, results
EC 8 P1, Annex B: Target displacement

- Long period, equal displacement rule
EC 8 P1, Annex B: Target displacement

- Short period, relative larger displacement

\[
q_u = \frac{S_e(T^*) \cdot m^*}{F_y^*}
\]

\[
d_t^* = \frac{d_{et}^*}{q_u} \left(1 + \left(\frac{q_u - 1}{T_T^*}\right)\right) \geq d_{et}^*
\]
Determination of target displacement

- Alternative procedure used in USA, NZ, etc.

Performance point

\[ S_d = 35 \text{ mm} \]
EC 8 P1, Annex B : Target displacement

- Short period, used assumption \((T_{\text{lim}} = T_C)\)

\[
d_t^* = \frac{d_{et}^*}{q_u} \left(1 + (q_u - 1) \frac{T_C}{T^*}\right) \geq d_{et}^*
\]

\[
q_u = \frac{S_e(T^*) \cdot m^*}{F_y^*}
\]
EC 8 P1, Annex B: Target displacement

- Short period, relative larger displacement

![Graph showing target displacement vs period with plateau region between T_B and T_C for different values of q_u.](image)
EC 8 P1, Annex B: Target displacement

- Short period, graphical approximation

\( S_e \)

\( T < T_C \)

\( e \)

\( e/3 \)
EC 8 P1, Annex B : Target displacement

• Short period, graphical approximation
## Parameters for EC8 (SIA 261) soil classes

<table>
<thead>
<tr>
<th>classe</th>
<th>description</th>
<th>S</th>
<th>$T_B$[s]</th>
<th>$T_C$[s]</th>
<th>$T_D$[s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>massifs rocheux</td>
<td>1,00</td>
<td>0,15</td>
<td>0,4</td>
<td>2,0</td>
</tr>
<tr>
<td>B</td>
<td>gravier ou sable très compact</td>
<td>1,20</td>
<td>0,15</td>
<td>0,5</td>
<td>2,0</td>
</tr>
<tr>
<td>C</td>
<td>compact ou semi-compact</td>
<td>1,15</td>
<td>0,20</td>
<td>0,6</td>
<td>2,0</td>
</tr>
<tr>
<td>D</td>
<td>lâche à semi-compact</td>
<td>1,35</td>
<td>0,20</td>
<td>0,8</td>
<td>2,0</td>
</tr>
<tr>
<td>E</td>
<td>couche C ou D sur A ou B</td>
<td>1,40</td>
<td>0,15</td>
<td>0,5</td>
<td>2,0</td>
</tr>
<tr>
<td>F</td>
<td>sensible, organique</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
Elastic response spectra in EC 8 (SIA 261)

The diagram shows the elastic response spectra for different soil classes according to EC 8 (SIA 261). The x-axis represents the period T [s], and the y-axis represents the ratio $S_e / a_{gd}$. The classes of soil are A, B, C, D, and E, each represented by a different line color. The graph illustrates how the spectral response varies with period for each soil class.
Displacement demand prediction accuracy

• Sets of 12 recorded TH, Abrahamson matching

SIA 261 soil class A, before modification

SIA 261 soil class A, after modification

SIA 261 soil class B, before modification

SIA 261 soil class B, after modification

SIA 261 soil class C, before modification

SIA 261 soil class C, after modification
Displacement demand prediction accuracy

12 modified ESMD-earthquakes for soil class A, $R=2$

Takeda model ($\alpha=0.4, \beta=0.0$), $R=3$

strength reduction factor $R=4$

strength reduction factor $R=5$
Displacement demand prediction accuracy

- 12 modified ESMD-earthquakes for soil class B, $R=2$
- Takeda model ($\alpha=0.4, \beta=0.0$), $R=3$
- Strength reduction factor $R=4$
- Strength reduction factor $R=5$

CIVIL 706 - N2, Rocking, check NLTHA
EDCE-EPFL-ENAC-SGC 2016
Displacement demand prediction accuracy

- 12 modified ESMD-earthquakes for soil class C, R=2
- Takeda model (alpha=0.4, beta=0.0), R=3
- Strength reduction factor R=4
- Strength reduction factor R=5
Displacement demand prediction accuracy

12 modified ESMD-earthquakes for soil class D, R=2

Takeda model (alpha=0.4, beta=0.0), R=3

Strength reduction factor R=4

Strength reduction factor R=5

CIVIL 706 – N2, Rocking, check NLTHA
EDCE-EPFL-ENAC-SGC 2016
Site effect

Earthquake =
source * propagation * site

Site effect due to the reflexions of seismic waves in the "soft » soil deposits
Microzonation : H/V measurements
Microzonation: SASW measurements
Microzonation : Yverdon
Microzonation: Yverdon, response spectra

![Graph showing response spectra for Yverdon, S1, S2, S3, S4]
Microzonation: Yverdon, response spectra

- ADRS format
Microzonation: Yverdon, response spectra

- Zone S1: are usual procedures valid?
Microzonation: Yverdon, response spectra

- Zone S1: are usual procedures valid?
Microzonation: Yverdon, response spectra

- Zone S1: are usual procedures valid?

![Graphs showing response spectra for different reduction factors (R=2, R=3, R=4, R=5)]
Microzonalation : Yverdon, response spectra

• Zone S1 : are usual procedures valid ?
Microzonation: Yverdon, response spectra

- Zone S1: are usual procedures valid?
Conclusions – N2 method

• Equal displacement rule from $T_{\text{lim}} = T_c$

• Target displacement in Part 1, Annex B

• Approximate graphical assessment

• Accuracy ($R=3 \, ✓$, $R<3 \, ✓$, $R>3 \, ✓$)

• Procedure also valid for special response spectra from microzonation studies
Rocking behaviour, check NLTHA

- 164 recorded earthquakes, methodology

164 recorded earthquakes

4 hysteretic models

13 f₀
9 R

9 R

12 MDOF

4 R

Non-linear dynamic response

Non-linear behavior

Non-linear dynamic response

Statistical analysis

Acceleration
time

Displacement
time

Frequency
time
Rocking behaviour, check NLTHA

• Hysteretic models

- S-model
- Flag-model
- Elastoplastic (EP-model)
- Modified Takeda-model

[Diagrams showing force vs. displacement for different hysteretic models]
Rocking behaviour, check NLTHA

- 164 recorded earthquakes, results

**S-model**

**modified Takeda-model**

- $\text{displacement ductility demand}$
- $\text{initial frequency [Hz]}$

- $R = 4$
- $R = 3.5$
- $R = 3$
- $R = 2.5$
- $R = 2$
Rocking behaviour, check NLTHA

• 164 recorded earthquakes, results

![Graphs showing rocking behaviour](image-url)
Rocking behaviour, check NLTHA

• Slightly modified $R - \mu_\Delta$ relationships
Masonry out-of-plane behaviour

- Griffith: rigid body behaviour

\[
\Delta_e = \frac{2}{3} t
\]

Inertia force distribution

\[
F_0 = \frac{Mg}{2} + \frac{F_0}{2} - Mgt/2h
\]

(a) Parapet Wall at incipient Rocking and Point of Instability

(b) Simply-Supported Wall at Incipient Rocking and Point of Instability

\[
\Delta_e = 0
\]

\[
R' = \frac{F_0}{2} + Mgt/2h
\]

Griffith model: rigid body behaviour

- Force – displacement relationships

\[ F_0 = 4(1 + \psi)M_e g t/h \]

\[ F_0 = 4M_e g t/h \]

\[ \Psi = \text{overburden weight}/(Mg/2) \]

(a) Parapet Wall

(b) Simply Supported Wall
Griffith model: rigid body behaviour

- Tri-linear hysteretic model

Table II. Empirically derived trilinear $F-\Delta$ defining displacements.

<table>
<thead>
<tr>
<th>State of degradation at cracked joint</th>
<th>$\Delta_1/\Delta_f$</th>
<th>$\Delta_2/\Delta_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>New</td>
<td>6%</td>
<td>28%</td>
</tr>
<tr>
<td>Moderate</td>
<td>13%</td>
<td>40%</td>
</tr>
<tr>
<td>Severe</td>
<td>20%</td>
<td>50%</td>
</tr>
</tbody>
</table>

Note: Only the positive displacement range is shown.
**Out-of-Plane: rigid body behaviour**

**Several stability conditions**

**Static conditions:** rocking onset

- **Italian code**
  
  \[
  a_0^* = \frac{PGA}{2} \leq \alpha \cdot g
  \]

**Overturning:** Betbeder-Matibet

\[
V \approx \frac{a_{gd}}{10} \leq \frac{1}{2} \cdot \alpha \cdot \sqrt{g \cdot r}
\]

\[
\Rightarrow a_{gd} \leq \frac{5 \cdot t_w}{h_w} \cdot \sqrt{g \cdot r}
\]
Case study, definitions

Cantilever non bearing wall

**Dimensions:**
- \( h_w = 3 \text{ m (3rd storey)} \)
- \( t_w = 0.1 \text{ m - 0.4 m} \)

**Seismic:**
- Z2
- CO I
- soil CSC

**Building:**
- \( T_1 = 0.3 \text{ s} \)
- \( T_1 = 1.2 \text{ s} \)
- \( T_1 = 1.0 \text{ s} \)
- \( T_1 = 1.67 \text{ s} \)

\[ K_{s-eff} = K_0 (\Delta_2 - \Delta_f) / \Delta_2 \]

Note: Only the positive displacement range is shown.
Griffith model: rigid body behaviour

- Sets of 12 recorded TH, methodology
Abrahamson matching for 12 TH – EC8/SIA

SIA 261 soil class A, before modification

SIA 261 soil class A, after modification

SIA 261 soil class B, before modification

SIA 261 soil class B, after modification

SIA 261 soil class C, before modification

SIA 261 soil class C, after modification

Sa [m/s²] vs. period [s] for SIA 261 soil classes before and after modification.
Non linear time history analysis

• Examples
Non linear time history analysis

• Determination using incremental analysis
Non linear time history analysis

- Ultimate increasing value: results assessment
Non linear time history analysis

- Aggregate results for different wall thicknesses

<table>
<thead>
<tr>
<th>$t_w$ [m]</th>
<th>$t_w / H_w$ [-]</th>
<th>$\alpha g$ [m/s$^2$]</th>
<th>$2 \alpha g$ [m/s$^2$]</th>
<th>NL (mean 12 TH &gt; $t_w/2$) $\zeta = 1% - 3% - 5%$ [m/s$^2$]</th>
<th>NL (mean 12 TH &gt; $t_w$) $\zeta = 1% - 3% - 5%$ [m/s$^2$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>0.033</td>
<td>0.3</td>
<td>0.7</td>
<td>0.29 – 0.38 – 0.43</td>
<td>0.35 – 0.51 – 0.61</td>
</tr>
<tr>
<td>0.12</td>
<td>0.040</td>
<td>0.4</td>
<td>0.8</td>
<td>0.35 – 0.46 – 0.52</td>
<td>0.40 – 0.61 – 0.73</td>
</tr>
<tr>
<td>0.15</td>
<td>0.050</td>
<td>0.5</td>
<td>1.0</td>
<td>0.44 – 0.57 – 0.65</td>
<td>0.50 – 0.73 – 0.91</td>
</tr>
<tr>
<td>0.18</td>
<td>0.060</td>
<td>0.6</td>
<td>1.2</td>
<td>0.53 – 0.68 – 0.78</td>
<td>0.60 – 0.93 – 1.09</td>
</tr>
<tr>
<td>0.20</td>
<td>0.067</td>
<td>0.7</td>
<td>1.3</td>
<td>0.58 – 0.76 – 0.86</td>
<td>0.67 – 1.02 – 1.21</td>
</tr>
<tr>
<td>0.25</td>
<td>0.083</td>
<td>0.8</td>
<td>1.6</td>
<td>0.73 – 0.94 – 1.08</td>
<td>0.81 – 1.27 – 1.51</td>
</tr>
<tr>
<td>0.30</td>
<td>0.100</td>
<td>1.0</td>
<td>2.0</td>
<td>0.87 – 1.13 – 1.29</td>
<td>1.00 – 1.46 – 1.81</td>
</tr>
<tr>
<td>0.40</td>
<td>0.133</td>
<td>1.3</td>
<td>2.6</td>
<td>1.16 – 1.51 – 1.72</td>
<td>1.33 – 1.95 – 2.42</td>
</tr>
</tbody>
</table>
Amplification with floor response spectra

- Methodology
Amplification with floor response spectra

- Limit case: stiff building

Armasuisse, out-of-plane benchmark B&H, limit stiff building, fp=3.3333Hz

\[
\frac{S_e(T_1)}{\sqrt{\left(1 - \frac{T_s}{T_1}\right)^2 + 0.02 \cdot \frac{T_s}{T_1}}}
\]

- elastic design spectrum
- average floor spectra, z=5%
- amplification according to Griffith
Amplification with floor response spectra

- Limit case: flexible building

\[
S_e(T_i) \sqrt{\left(1 - \frac{T_s}{T_i}\right)^2 + 0.02 \cdot \frac{T_s}{T_i}}
\]

Armasuisse, out-of-plane benchmark B&H, limit flexible building, \( f_p = 0.83333 \text{Hz} \)
## Amplification with floor response spectra

### $T_1 = 1.2\, s$

<table>
<thead>
<tr>
<th>$t_w$ [m]</th>
<th>$t_w / H_w$ [-]</th>
<th>$\alpha, g$ [m/s²]</th>
<th>$\alpha, g/1.8$ [m/s²]</th>
<th>$\alpha, g/2.3$ [m/s²]</th>
<th>NL (moyenne 12 TH &gt; $t_w/2$) [m/s²]</th>
<th>NL (moyenne 12 TH &gt; $t_w$) [m/s²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>0.033</td>
<td>0.3</td>
<td>0.17</td>
<td>0.13</td>
<td>0.12 – 0.16 – 0.19</td>
<td>0.19 – 0.28 – 0.31</td>
</tr>
<tr>
<td>0.12</td>
<td>0.040</td>
<td>0.4</td>
<td>0.22</td>
<td>0.17</td>
<td>0.15 – 0.19 – 0.23</td>
<td>0.23 – 0.34 – 0.37</td>
</tr>
<tr>
<td>0.15</td>
<td>0.050</td>
<td>0.5</td>
<td>0.28</td>
<td>0.22</td>
<td>0.18 – 0.24 – 0.29</td>
<td>0.29 – 0.41 – 0.46</td>
</tr>
<tr>
<td>0.18</td>
<td>0.060</td>
<td>0.6</td>
<td>0.33</td>
<td>0.26</td>
<td>0.22 – 0.28 – 0.34</td>
<td>0.23 – 0.49 – 0.56</td>
</tr>
<tr>
<td>0.20</td>
<td>0.067</td>
<td>0.7</td>
<td>0.39</td>
<td>0.30</td>
<td>0.24 – 0.31 – 0.38</td>
<td>0.38 – 0.56 – 0.62</td>
</tr>
<tr>
<td>0.25</td>
<td>0.083</td>
<td>0.8</td>
<td>0.44</td>
<td>0.35</td>
<td>0.30 – 0.39 – 0.47</td>
<td>0.48 – 0.68 – 0.77</td>
</tr>
<tr>
<td>0.30</td>
<td>0.100</td>
<td>1.0</td>
<td>0.56</td>
<td>0.43</td>
<td>0.36 – 0.47 – 0.57</td>
<td>0.57 – 0.82 – 0.92</td>
</tr>
<tr>
<td>0.40</td>
<td>0.133</td>
<td>1.3</td>
<td>0.72</td>
<td>0.56</td>
<td>0.48 – 0.62 – 0.76</td>
<td>0.76 – 1.01 – 1.23</td>
</tr>
</tbody>
</table>

### $T_1 = 1.67\, s$

<table>
<thead>
<tr>
<th>$t_w$ [m]</th>
<th>$t_w / H_w$ [-]</th>
<th>$\alpha, g$ [m/s²]</th>
<th>$\alpha, g/2.8$ [m/s²]</th>
<th>$\alpha, g/3.8$ [m/s²]</th>
<th>NL (moyenne 12 TH &gt; $t_w/2$) [m/s²]</th>
<th>NL (moyenne 12 TH &gt; $t_w$) [m/s²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>0.033</td>
<td>0.3</td>
<td>0.11</td>
<td>0.08</td>
<td>0.08 – 0.10 – 0.12</td>
<td>0.00 – 0.00 – 0.00</td>
</tr>
<tr>
<td>0.12</td>
<td>0.040</td>
<td>0.4</td>
<td>0.14</td>
<td>0.11</td>
<td>0.09 – 0.12 – 0.14</td>
<td>0.00 – 0.00 – 0.00</td>
</tr>
<tr>
<td>0.15</td>
<td>0.050</td>
<td>0.5</td>
<td>0.18</td>
<td>0.13</td>
<td>0.11 – 0.15 – 0.18</td>
<td>0.00 – 0.00 – 0.00</td>
</tr>
<tr>
<td>0.18</td>
<td>0.067</td>
<td>0.6</td>
<td>0.21</td>
<td>0.16</td>
<td>0.14 – 0.18 – 0.22</td>
<td>0.00 – 0.00 – 0.00</td>
</tr>
<tr>
<td>0.20</td>
<td>0.083</td>
<td>0.7</td>
<td>0.25</td>
<td>0.18</td>
<td>0.15 – 0.20 – 0.24</td>
<td>0.00 – 0.00 – 0.00</td>
</tr>
<tr>
<td>0.25</td>
<td>0.100</td>
<td>1.0</td>
<td>0.36</td>
<td>0.26</td>
<td>0.19 – 0.25 – 0.30</td>
<td>0.00 – 0.00 – 0.00</td>
</tr>
<tr>
<td>0.30</td>
<td>0.133</td>
<td>1.3</td>
<td>0.46</td>
<td>0.34</td>
<td>0.30 – 0.40 – 0.48</td>
<td>0.00 – 0.00 – 0.00</td>
</tr>
</tbody>
</table>
Amplification with floor response spectra
Conclusions – Rocking (S and Griffith)

• Slight modifications of $R - \mu_\Delta$ relationships for S-model and flag-model

• Onset rocking condition on the safe side for cantilever walls with rigid body behaviour

• Approach also valid for walls in upper stories by taking into account adequate floor response spectra