Tuned Vibration Absorbers for "Lively" Structures

Introduction

Human activities such as walking and jumping can produce dynamic forces with a predominant frequency content around 2 Hz. Coincidentally, 2 Hz is also the fundamental frequency of many structures, especially footbridges. In many cases these structures are lightly damped, a condition described as "lively," and can undergo large vibrations. These vibrations can create a serviceability problem because users can become frightened by the structure's readily perceived motions. Structures that are easily excited are also tempting targets for acts of vandalism. One measure against excessive vibrations is the application of tuned vibration absorbers. They have been known for some time, but have not been widely used, perhaps because civil engineers feel uncomfortable with such machine-like devices. This paper illustrates the theoretical and practical experiences with three structures subjected to human-induced vibrations, whose dynamic behavior has been improved by tuned vibration absorbers. References to examples with wind- and traffic-induced vibrations are given in [1].

Optimal Absorber Parameters

The behavior of the two-degrees-of-freedom system can be visualized considering the dynamic amplification curves in Fig. 2 \(\left(\frac{f_T}{f_H} = 1/100, \zeta_T = 1\%\right)\). The excitation frequency \(f\) is normalized by the eigenfrequency of the structure. Without the absorber, the well-known curve for a single-degree-of-freedom system results. For a structure with an absorber, i.e., for a two-degrees-of-freedom system, the amplification curve typically exhibits two humps, one below the eigenfrequency of the absorber and one above the eigenfrequency of the structure. The optimal absorbing capacity is obtained if both humps have the same height, as shown by the curve for \(f_T = f_{opt}\). For a structure with no damping, the optimal parameters have been determined in [2] as

\[
\begin{align*}
  f_{opt} &= \frac{1}{1 + \frac{m_T}{m_H}} f_H \\
  \zeta_{opt} &= \frac{3 m_T}{m_H} \left(1 + \frac{m_T}{m_H}\right)^3
\end{align*}
\]

Using a dimensionless form, the parameters \(m_{1H} \cdot c_{1H} \cdot k_{1H} \cdot m_T \cdot c_T \cdot k_T \cdot F\) and the two variables \(u_{1H} \cdot u_T\) can be reduced to the five parameters \(\frac{m_T}{m_H}, \frac{f_T}{f_H}, \zeta_T, \zeta_H\) and the two dimensionless variables \(u_T/u_{1H}, u_T/u_{1H}\).

Theoretical Background

Two-Degrees-of-Freedom System

The simplest model for a structure with a tuned vibration absorber is a two-degrees-of-freedom system. It is used here to develop the theoretical background and will be discussed subsequently in connection with the design of tuned vibration absorbers. Fig. 1 shows the general set-up of a two-degrees-of-freedom system.

Summary

In the first part of this paper, the theoretical background and a design procedure for tuned vibration absorbers are presented. In the second part, practical experiences with two footbridges and a diving platform are described. Experiments confirm both the applicability of the proposed design procedure and the effectiveness of the absorbers.

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this surface for the previous example. The minimal amplification is 11.6 at the point $f_1 = 0.99$ and $\xi_T = 6\%$.

The sensitivity of the tuning can be appreciated by considering the contour level for an amplification of 15. To obtain a maximum amplification below that value, the tuning of the frequency has to be in the range $\xi_T = 0.96-1.02$, i.e., within ±3%. On the other hand, the damping value can be in the range of $\xi_T = 3\%-11\%$, which is between half and almost double the optimal value. Clearly the optimal point is relatively insensitive to the damping, but the exact frequency tuning is crucial. By calculating numerically the optimal frequency for different mass ratios and for different structural damping values, the curves given in Fig. 4 have been obtained. The curve for $\xi_T = 0\%$ corresponds to the formula given in Eqs. (1) and (2). For higher structural damping values and high mass ratios, the optimal frequency of the absorber is slightly less than the one given by the formula. However, since tuned vibration absorbers are only effective for lightly damped structures, the formulas in Eqs. (1) and (2) are sufficiently accurate for any practical application.

**Design Procedure**

**Excitation**

The excitation force due to various human activities is described in [1]. It can be expressed as a Fourier series as:

$$F(t) = G + \Delta G_1 \sin(2\pi f_p t)$$

$$+ \Delta G_2 \sin(2 \cdot 2\pi f_p t - \phi_2) + \ldots$$

As a simplified procedure, each harmonic is considered separately. The maximum steady-state response is obtained if the excitation frequency $f = f_p$ equals the frequency of the mode considered. Because the activity rate lies within a certain range, each mode can usually only be excited by a certain harmonic. Depending on the eigenfrequencies of the structure, the first, second or third term is important. The most severe loading is typically due to jumping, with a force amplitude of $\Delta G_1 = 1.8G$. Assuming the weight of the person jumping to be $G = 800$ N, this leads to an amplitude of the first Fourier term of $\Delta G_1 = 1440$ N.

**Equivalent Single-Degree-of-Freedom System**

A first step in designing an absorber is to determine the dynamic parameters of the structure, i.e., the mass and the stiffness of an equivalent single-degree-of-freedom system corresponding to a certain mode of vibration. For practical reasons, the stiffness is usually calculated from the mass and the eigenfrequency. The eigenfrequency can be accurately determined based on experimental data. A reliable value is essential to obtain an optimal frequency tuning. The modal mass can be estimated assuming a mode shape and calculating the corresponding kinetic energy (Rayleigh's quotient). To obtain an equivalent single-degree-of-freedom system with the same displacement as the structure, the assumed mode shape has to be scaled to a value of 1 at the location and in the direction of the absorber. Similar considerations have to be made in a finite element analysis.

**Absorber Parameters**

When designing an absorber, a major design decision is the choice of the mass ratio $m_H/m_T$. The mass ratio directly influences the response of the structure and the relative movement of the absorber mass. The mass ratio can be chosen with the aid of the design curves given in Figs. 5 and 6.

**Notations**

<table>
<thead>
<tr>
<th>Structure parameters</th>
<th>$m_T$</th>
<th>mass</th>
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<tr>
<td>$c_T$</td>
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<tr>
<td>$k_T$</td>
<td>spring constant</td>
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<tr>
<td>$u_T$</td>
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<td>$u_{H0}$</td>
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<td>$f_H$</td>
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<td>$\xi_T$</td>
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**Excitation force**

$$F(t) = \text{force applied to structure}$$

$$G = \text{weight of active person}$$

$$\Delta G_i = \text{force amplitude of } i\text{th harmonic}$$

$$f_p = \text{activity rate}$$

$$\phi_i = \text{phase lag of } i\text{th harmonic}$$

$$t = \text{time}$$

$$f = \text{frequency of excitation}$$
An absorber can only be tuned to one particular frequency, normally, to the frequency of the fundamental mode of the structure. If other modes also have to be damped, more than one absorber is necessary. Typically, if only the fundamental mode is damped by an absorber, higher-order modes dominate the acceleration in a steady-state analysis. However, from a practical point of view, these modes are not important. Although it is possible to excite modes with higher frequencies through the higher Fourier terms of the excitation force, it is generally quite difficult to obtain the steady-state response, which requires maintaining the exact excitation frequency over a long period of time.

**Experimental Considerations**

**Before Installation**

To be able to determine the optimal absorber parameters, the frequency and the damping of the relevant mode of the structure have to be established first. This is most precisely accomplished by testing. A useful test is dropping a sandbag, which excites the structure in a wide frequency band. The eigenfrequencies show up as sharp peaks in the Fourier spectrum.

Tuning the absorber in the laboratory is a valuable, even necessary part of the installation. Frequency and damping can be determined by measuring the free vibration response and interpreting the resulting decay curve. Practically, free vibration is achieved by exciting the absorber manually and then removing the force, or by moving the absorber with a lifting device and then suddenly stopping the absorber’s motion. Forced vibration is only valid if the excitation is well defined. For example, measuring the vibrations during manual excitation does not yield a unique eigenfrequency because of the high damping (typically 10%). One problem which has to be considered is that the damping devices themselves are not purely viscous, and also contribute to the stiffness. This means that the damping and the frequency cannot be adjusted independently. As can be seen from Fig. 3, the damping does not need to be very accurate, whereas exact frequency tuning is crucial.

**After Installation**

Final tuning of the absorber frequency is done after the installation. An accurate method consists in exciting the structure equipped with the absorber by dropping a sandbag. This impulsive force covers a wide frequency range and the Fourier spectrum of the structural response resembles the amplification curve of Fig. 2. If the absorber is optimally tuned, two humps of equal height will appear. The method only works for smaller structures where the imposed energy is high enough.

Other useful tests are comparisons with the absorber locked and unlocked. Locking the absorber can be achieved simply by somehow restraining the movement of its mass. The ratio of the maximum values with the absorber locked and unlocked are a measure for the effectiveness of the absorber. For these comparisons, it is important to apply an identical force in the corresponding experiments. Ideally, a mechanical shaker would be used for the excitation, but from a practical point of view it is much simpler, although not as accurate, to excite the structure by human activities such as jumping and running using a pace-giving metronome.
Practical Examples

Girder Footbridge

The first example is a footbridge with four spans as shown in Fig. 7. It consists of two steel girders and a concrete slab made up of plates of size 2.5 m x 2.5 m, which are supported by neoprene bearings. The cross section is shown in Fig. 8. The bridge showed excessive vibration under normal use primarily due to the low damping of the steel girders. The fundamental frequency was determined experimentally to be 2.46 Hz and the damping ratio to lie in the range of $\zeta_H = 0.2-0.4\%$, depending on the displacement amplitude. The modal mass was determined using Rayleigh's quotient as $m_H = 30500$ kg. The stiffness follows as $k_H = 7280$ kN/m.

The design excitation force was taken to be $\Delta G_1 = 1280$ N, corresponding to one person jumping with the resonance frequency in the center of the main span. Assuming a damping ratio of $\zeta_H = 0.25\%$ leads to a dynamic amplification factor of 200 and a steady-state acceleration amplitude of 8.4 m/s². An acceptable value of acceleration was assumed to be about 0.5 m/s². Thus the dynamic amplification factor had to be reduced from 200 down to 12. Referring to the design curves in Fig. 5, the mass ratio has to be about 0.012. The actual absorber mass chosen was 350 kg, corresponding to a mass ratio of 0.0115. This leads to a theoretical steady-state acceleration of 0.053 g. The optimal absorber parameters are $f_{opt} = 0.0989 \cdot f_H = 2.43$ Hz and $\zeta_{opt} = 6.5\%$. The relative displacement of the absorber mass, according to the design curves in Fig. 6, is $\pm 15$ mm.

The design of the absorber is shown in Fig. 9. The main problem was to construct an absorber taking into account the limited space available. The two identical absorbers consist each of a mass of 175 kg supported by four springs and of two damping devices. The damping device is a rod submerged in a viscous fluid. The frequency can be modulated by a variable mass and the damping can be adjusted by varying the submerged length of the rod.

The absorber was first tuned in the laboratory before it was installed on the bridge. It was fine-tuned on site, using the spectrum of the response from a dropped sandbag (Fig. 10). The absorber mass was adjusted until the two humps between 2 and 3 Hz were approximately equal. The design excitation force was taken to be $\Delta G_1 = 1280$ N, corresponding to one person jumping with the resonance frequency in the center of the main span. Assuming a damping ratio of $\zeta_H = 0.25\%$ leads to a dynamic amplification factor of 200 and a steady-state acceleration amplitude of 8.4 m/s². An acceptable value of acceleration was assumed to be about 0.5 m/s². Thus the dynamic amplification factor had to be reduced from 200 down to 12. Referring to the design curves in Fig. 5, the mass ratio has to be about 0.012. The actual absorber mass chosen was 350 kg, corresponding to a mass ratio of 0.0115. This leads to a theoretical steady-state acceleration of 0.053 g. The optimal absorber parameters are $f_{opt} = 0.0989 \cdot f_H = 2.43$ Hz and $\zeta_{opt} = 6.5\%$. The relative displacement of the absorber mass, according to the design curves in Fig. 6, is $\pm 15$ mm.

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The absorber was first tuned in the laboratory before it was installed on the bridge. It was fine-tuned on site, using the spectrum of the response from a dropped sandbag (Fig. 10). The absorber mass was adjusted until the two humps between 2 and 3 Hz were approximately equal. Accelerations due to one person jumping are shown in Fig.11. The high effectiveness of the absorber is demonstrated by comparing the values with the absorber locked and unlocked.

The maximum values of selected tests with one person and with the absorber locked and unlocked are shown in Table 1. The ratio, which is an indication of the effectiveness of the absorber, is also given. Clearly, with the absorber locked, the maximum values

<table>
<thead>
<tr>
<th>Test</th>
<th>Accelerations (m/s²)</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Locked</td>
<td>Unlocked</td>
<td></td>
</tr>
<tr>
<td>Jumping</td>
<td>9.0</td>
<td>0.48</td>
</tr>
<tr>
<td>Walking</td>
<td>1.2</td>
<td>0.24</td>
</tr>
<tr>
<td>Running</td>
<td>1.7</td>
<td>0.59</td>
</tr>
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</table>

Table 1: Girder footbridge: maximum vertical accelerations
depend on the kind of loading and its duration. The steady-state case is only obtained for jumping. For walking and running, the time needed to cross the bridge is too short to reach steady-state conditions. The values with the absorber unlocked all lie around the acceptable value of 0.5 m/s².

**Cable-Stayed Footbridge**

The second example is a cable-stayed footbridge with a steel girder (Fig. 12). This case is different from the other two in the respect that the installation of an absorber was planned before the bridge was built. Therefore, the first analyses were performed by finite element calculations. The fundamental frequency was determined to be $f_{H} = 1.98$ Hz, the modal stiffness $k_{H} = 3500$ N/mm and the modal mass $m_{H} = 23000$ kg. The damping ratio was assumed to be $\xi_{H} = 0.5\%$. The location for the absorber was chosen because this is where the fundamental mode shape has its largest amplitude. Using an excitation force of $\Delta G_{1} = 1.8 \times 800$ N acting at the location of the absorber yields an acceleration of 6.3 m/s², which is about ten times the acceptable value. With a planned absorber mass of 800 kg this value would decrease to 0.46 m/s². This absorber mass was chosen somewhat larger than absolutely necessary, because space was no problem in this case. The placement of the absorber is shown in Fig. 13.

The design of the absorber is shown in Fig. 14. Because the absorber parameters were based on a finite element analysis rather than on measurements, the absorber was designed to be adjustable for a wide range of frequencies. In addition to variation of the mass, there were also three sets of springs with different stiffnesses available. The actual absorber mass after the frequency tuning was 1000 kg, corresponding to a mass ratio of 0.044, leading to a theoretical maximum acceleration of 0.44 m/s² and a maximum relative movement of the absorber mass of ±14 mm.

After the bridge was built, the eigenfrequencies were determined experimentally by analyzing the free vibrations produced by a dropped sandbag and by a person jumping. The fundamental frequency was $f_{H} = 2.12$ Hz, which is close to the calculated value of 1.98 Hz. Higher modes were also quite close to the calculated values. The damping ratio was about 0.5% for low acceleration amplitudes in the range of 0.1–0.2 m/s². However, for larger amplitudes the damping was much more than anticipated. Up to an amplitude of 1 m/s² it was about 1%, and for higher amplitudes it was even larger. Because the fundamental frequency was close to the predicted value, the different sets of springs were not actually needed. The tuning was performed in the laboratory by varying the mass only.

Various tests with one or two persons jumping or running were performed. Fig. 15 shows the comparison of the acceleration response due to one person jumping, both with the absorber locked and unlocked. The absorber is not as effective as in the first example. One reason is that the locked case used as a reference already has a low value because the structure itself has considerable damping, especially for higher displacement amplitudes. The other reason is that the influence of higher modes is no longer negligible, as can be seen from the time history. Maximum values for one person jumping and running are shown in Table 2. Although the ratios between the locked and the unlocked cases are rather low, the absorber performs well in that it reduces the response from an excessive to an acceptable value.

<table>
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<tr>
<th>Test</th>
<th>Accelerations (m/s²)</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Jumping</td>
<td>Locked 1.1</td>
<td>0.45</td>
</tr>
<tr>
<td></td>
<td>Unlocked 0.45</td>
<td>0.3</td>
</tr>
</tbody>
</table>

*Table 2: Cable-stayed footbridge: maximum vertical accelerations*
The last example is a reinforced concrete diving platform (Fig. 16). It consists of a Y-shaped shaft with two slabs at the 3 m and 5 m levels. The two branches are connected by a tie-beam at the level of the lower slab. The platform could be excited excessively by shaking it horizontally on the railing parallel to the pool. Some cracks in the shaft were observed.

In a first stage, the structure was thoroughly studied, both by finite element calculations and experimental modal analysis. The analysis showed that the main vibrations were in the first mode as depicted in Fig. 16. Measured values were $f_1 = 2.6-2.7$ Hz for the fundamental frequency and $\zeta_1 = 1.5-2.0\%$ for the damping ratio. A special problem in this example was the fact that the movement of the structure at the absorber location is oblique, with a larger component in the horizontal direction, as indicated. An ideal absorber would have to move in the same direction, but such a device is difficult to construct. Instead it was decided to design a horizontally moving absorber. The classical solution of a pendulum was not feasible in this case because the length of the pendulum would only be about 3 cm. The novel design finally used is shown in Fig. 17. It consists of a mass hanging on steel plates acting as springs. It moves like a pendulum but the stiffness is provided by the springs. Besides the mass, the stiffness can also be varied by adjusting the length of the springs. The damping device is again a rod submerged in a viscous fluid. The rod, however, moves horizontally instead of vertically.

The modal mass of the platform was determined by the finite element model with the appropriate scaling of the mode shape as $m_1 = 12000$ kg. For the design of the absorber a reduction factor for the accelerations of about 5 was postulated. Assuming a damping ratio of $\zeta_1 = 1.5\%$, the dynamic amplification has to be reduced from 33 down to 7. According to the design curves in Fig. 5, the corresponding mass ratio is 0.03. The mass actually chosen was 335 kg corresponding to a mass ratio of 0.028.

First testing of the platform showed an amplitude-dependent eigenfrequency which was somewhat higher than the one determined by the experimental modal analysis, because in the meantime the cracks in the shaft had been repaired by epoxy injection. The frequency was 2.9 Hz for acceleration amplitudes of 0.05 m/s$^2$ and 2.7 Hz for amplitudes of 2 m/s$^2$. The absorber was tuned for an expected frequency of 2.8 Hz.

Tests with the absorber locked and unlocked were performed exciting the structure by jumping or by shaking it on the railing. Fig. 18 shows the comparison of the accelerations induced by two persons shaking with the absorber locked and unlocked. Maximum values of accelerations due to shaking by two persons and due to jumping by one person are compared in Table 3. As can be observed, the absorber is very effective. The desired reduction factor of 5 was more or less obtained in the tests shown here and in other tests.

<table>
<thead>
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<th>Accelerations (m/s$^2$)</th>
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<tbody>
<tr>
<td>Jumping</td>
<td>1.3</td>
<td>0.3</td>
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<tr>
<td>Shaking</td>
<td>3.2</td>
<td>0.5</td>
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<td>Table 3: Diving platform: maximum horizontal accelerations</td>
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Conclusions

Vibration absorbers can be designed based on simple dynamics. The two design charts in Figs. 5 and 6, and the formulas from Den Hartog given in Eqs. (1) and (2) are sufficient to determine the absorber’s parameters. Recommended values for the mass ratio are 0.03–0.04 if space for the tuned vibration absorber is not restricted. Lower values, down to 0.01, still lead to effective absorbers for lightly damped structures. This choice, however, leads to large relative movements of the absorber mass.

The exact tuning of the absorber frequency is crucial, whereas the damping need only be adjusted approximately. For long-term operation of tuned vibration absorbers it is important that the device is accessible for maintenance and re-tuning in case the dynamic properties of the structure change.

References
