STATIC NONLINEAR ANALYSIS

Advanced Earthquake Engineering
CIVIL-706

Instructor:
Lorenzo DIANA, PhD
By the end of today’s course...

You will be able to answer:

• What are NSA advantages over other structural analysis methods?
• How to perform an NSA?
• What are the key elements when performing NSA?
Earthquake Engineering Assessment

GROUND MOTION

Corresponding response spectra

Multiple records

Dynamic analysis

Multi-mode pushover analysis (MPA)

Simplified MDOF dynamic analysis

Nonlinear static procedures (NSP’s)

Simplified SDOF dynamic analysis

high RELATIVE UNCERTAINTY low

Source: FEMA 440
# Earthquake Engineering Assessment

<table>
<thead>
<tr>
<th>Structure</th>
<th>Action</th>
<th>Static</th>
<th>Dynamic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td></td>
<td>Linear static analysis</td>
<td>Linear dynamic analysis</td>
</tr>
<tr>
<td>Non-linear</td>
<td></td>
<td>Non linear static analysis</td>
<td>Non linear dynamic analysis</td>
</tr>
</tbody>
</table>
Linear static analysis

- Seismic forces are applied as equivalent static forces
- Regular (plan and vertical) buildings

**NTC 2008:**

\[ T_1 = C_1 \times H^{3/4} \]

- \( T_1 \): period of the structure
- \( C_1 \): factor related to material
- \( H \): height of the structure
Equivalent Force Method

\[ F_i = F_h \times z_i \times W_i / \sum z_j \times W_j \]

\[ F_h = S_d (T_1) \times W \times \lambda / g \]

- **W**: total mass of the structure
- **\( \lambda \)**: factor related to the number of stories
- **\( W_i \) and **\( W_j \)**: mass of i and j
- **\( z_i \) and **\( z_j \)**: height of i and j
Equivalent Force Method

**PERIODO DI RIFERIMENTO DELLA STRUTTURA:**

\[ H = 13.44 \text{ (m)} \quad C_1 = 0.075 \]

\[ T_1 = C_1 \cdot H^{\frac{3}{4}} = 0.526 \]

**CALCOLO DELLA FORZA DA APPLICARE AD OGNI MASSA:**

\[ W = 16438 \text{ kN} \quad \text{Peso complessivo della costruzione;} \]

\[ \lambda = 0.85 \quad \text{Vale 0.85 se la struttura ha almeno 3 orizzontamenti (} T_1 < 2T_c \text{); 1 negli altri casi} \]

\[ F_h = S_{dNTC}(T_1) W \lambda = 2.138 \times 10^3 \text{ kN} \]

\[ F_i = F_h \cdot z_i \cdot W_i / \sum z_j W_j \]

**Graph:**

- **T = 0.526 sec**
- **S_{dNTC}(x) = 0.153**

**Data Points:**

- \( W_1 = 1216.76 \text{ kN} \)
- \( W_2 = 3321.845 \text{ kN} \)
- \( W_3 = 3766.032 \text{ kN} \)
- \( W_4 = 8134 \text{ kN} \)

- \( z_1 = 13.2 \text{ m} \)
- \( z_2 = 10.7 \text{ m} \)
- \( z_3 = 7.5 \text{ m} \)
- \( z_4 = 4.3 \text{ m} \)

- \( F_1 = F_h \cdot W_1 / z_1 = 299.039 \text{ kN} \)
- \( F_2 = F_h \cdot W_2 / z_2 = 661.822 \text{ kN} \)
- \( F_3 = F_h \cdot W_3 / z_3 = 525.925 \text{ kN} \)
- \( F_4 = F_h \cdot W_4 / z_4 = 631.255 \text{ kN} \)
Linear dynamic analysis

- Seismic forces are applied as a dynamic action on structures
- Regular (plan and vertical) buildings
- NO mix structures
- \[ M \cdot \ddot{x} + C \cdot \dot{x} + K \cdot x = - M \cdot e_x \cdot \ddot{x}_g \]

- **M** mass matrix
- **C** damping matrix
- **K** stiffness matrix
- **e_x** vector of seismic direction
Linear dynamic analysis

Figure 5.10: La méthode du spectre de réponse permet d'évaluer la réponse maximale d'une structure. La position des modes dans le spectre de réponse détermine les valeurs spectrales associées aux réponses modales maximales.

- Response spectrum method
- Overlapping of effects (SRSS: square root of the sum of the square)
Linear dynamic analysis

E quindi:

Come visto per l’analisi statica lineare si determinano le forze sui diversi elementi strutturali

$S_d$(modo 1)

$T$(modo 1)
Linear dynamic analysis

E quindi:

Come visto per l’analisi statica lineare si determinano le forze sui diversi elementi strutturali

$S_d$(modo 3)

$T$(modo 3)
Linear dynamic analysis

Equindi:

Come visto per l'analisi statica lineare si determinano le forze sui diversi elementi strutturali.

$S_4(\text{modulo 7})$

$T(\text{modulo 7})$
Non linear time history analysis

- **Advantage**: the complexity of the dynamic load is considered + the dynamic behavior of structure
- **Disadvantage**: time consuming
Non linear static procedure

• How does it work? Capacity vs. Demand
Nonlinear static procedure

- Necessary elements?
  - Acceleration-displacement response spectrum (ADRS)
  - Capacity Curve

- Static nonlinear analysis

\[ V (\text{force}) \]

\[ \Delta (\text{disp.}) \]
Nonlinear static procedure

- Developing capacity curves:
  - **Displacement-based** methods
Nonlinear static procedure

• Assumptions
  – Response: maximum displacement
  – Deformation: “most often” following the first mode

• Benefits
  – Displacement-based analysis
  – Nonlinear behavior of the structure

• Drawbacks
  – Simplified approach \((\text{static})\)
  – Damping difficult to represent
Content

• Capacity curve
• Seismic damage
• Acceleration-Displacement Response Spectrum (ADRS)
• Performance Point
• Large-scale vulnerability assessment
Content

• **Capacity curve**
• Seismic damage
• Acceleration-Displacement Response Spectrum (ADRS)
• Performance Point
• Large-scale vulnerability assessment
Capacity Curve

• **Capacity curve** defines the capacity of the structure independent to any seismic demand

![Capacity Curve Diagram]

- **Objectives:**
  - Estimate the maximum horizontal displacement
  - Estimate the (global) ductility of the structure
Capacity Curve

- Pushover
- Simplified analytical method (EC8)
- DBV method
Capacity Curve

• Pushover

Load pattern: distribution of forces should represent the dynamic behavior

– Triangular (if building regular enough)
– Following the first mode
– Other simplified distributions depending on the building
Capacity Curve

• Pushover

Load pattern: importance definition

– Effect of lateral load can change the capacity curve
Capacity Curve

• Pushover

Different model type

Structural model (FEM, AEM, etc.)
Capacity Curve

- **Force/Displacement to spectral coordinates**

![Image of capacity curve diagram]

\[ \Gamma = \text{mass participation factor} \]

\[ m^* = \text{equivalent mass} \]

\[ Sa = \frac{V}{m^*\Gamma} \]

\[ Sd = \frac{\Delta u}{\Gamma} \]
Unreinforced masonry (URM) Example

URM building in Yverdon-les-Bains
Unreinforced masonry (URM) Example

Architectural plan
Unreinforced masonry (URM) Example

Simplification for the modeling
Unreinforced masonry (URM)

Key-parameter: **Frame effect** (coupling of horizontal/vertical elements)

- flexible slab (w/o frame effect)

\[ h_0 = \frac{2}{3} \times h_{total} \]

- Infinitely stiff lintels (total frame effect)

\[ h_0 = \frac{1}{2} \times h_{storey} \]
Unreinforced masonry (URM)

Example

Extreme case studies:

\[ h_0 = \frac{2}{3} \times h_{total} = 6.7 \, m \]

\[ h_0 = \frac{1}{2} \times h_{story} = 1.25 \, m \]
Unreinforced masonry (URM) Example

A) Frame effect not considered

<table>
<thead>
<tr>
<th>Wall</th>
<th>lw=2.0 m</th>
<th>lw=2.4 m</th>
<th>lw=4.0 m</th>
<th>lw=5.0 m</th>
<th>lw=6.0 m</th>
<th>TOT [dir Y]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nxd [kN]</td>
<td>180</td>
<td>160</td>
<td>1200</td>
<td>360</td>
<td>1520</td>
<td>-</td>
</tr>
<tr>
<td>VRd [kN]</td>
<td>21.7</td>
<td>24.6</td>
<td>123.3</td>
<td>113.6</td>
<td>304.1</td>
<td>618</td>
</tr>
<tr>
<td>δu [%]</td>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
<td>-</td>
</tr>
<tr>
<td>Δy [mm]</td>
<td>10.3</td>
<td>7.1</td>
<td>10</td>
<td>5.6</td>
<td>10.5</td>
<td>7.9</td>
</tr>
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<td>27.7</td>
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<td>24.2</td>
<td>27.9</td>
<td>24.2</td>
</tr>
<tr>
<td>k [kN/mm]</td>
<td>2.1</td>
<td>3.5</td>
<td>12.4</td>
<td>20.2</td>
<td>29</td>
<td>77.8</td>
</tr>
</tbody>
</table>
Unreinforced masonry (URM) Example

A) Frame effect not considered

\[ V_{Rd,R} = \frac{l_w \cdot N_{xd}}{2 \cdot h_w} \left( 1 - 1.15 \cdot \frac{N_{xd}}{l_w \cdot t_w \cdot f_{xd}} \right) \]

\[ V_{Rd,S} = \left( 1.5 \cdot \frac{f_{va0}}{0.85 \cdot f_{xd}} + 0.4 \right) \cdot N_{xd} \approx 0.5 \cdot N_{xd} \]
Unreinforced masonry (URM) Example

A) Frame effect not considered

\[ \Delta_y = V_{Rd} \cdot H_{tot} \left( \frac{h_p \cdot (3 \cdot h_0 - h_p)}{6 \cdot E \cdot I_{eff}} + \frac{\kappa}{G \cdot A_{eff}} \right) \]

\[ \Delta_y = \delta_y \cdot H_{tot} \]

\[ \delta_y = \frac{d_y}{h_p} \]

\[ \Delta_u = \Delta_y + (d_u - d_y) \]

\[ \delta_u = \frac{d_u}{h_p} = 0.8\% \]
Unreinforced masonry (URM) Example

A) Frame effect not considered
Unreinforced masonry (URM) Example

A) Frame effect not considered

\[ V_{Rdy} = (4 \times 21.7 + 2 \times 113.6 + 1 \times 304) = 618 \text{ kN} \]
\[ k_E = (4 \times 2.1 + 2 \times 20.2 + 1 \times 29) = 77.8 \text{ kN} / \text{mm} \]
\[ \Delta_y = \frac{618}{77.8} = 7.94 \text{ mm} \]
\[ \Delta_u = 24.2 \text{ mm} \]
Unreinforced masonry (URM) Example

A) Frame effect not considered

<table>
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<tr>
<th>Wall</th>
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<td>21.7</td>
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<td>123.3</td>
<td>113.6</td>
<td>304.1</td>
<td>837.6</td>
</tr>
<tr>
<td>δu [%]</td>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
<td>-</td>
</tr>
<tr>
<td>Δy [mm]</td>
<td>10.3</td>
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<td>10</td>
<td>5.6</td>
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<td>27.5</td>
<td>24.2</td>
<td>27.9</td>
<td>25.3</td>
</tr>
<tr>
<td>k [kN/mm]</td>
<td>2.1</td>
<td>3.5</td>
<td>12.4</td>
<td>20.2</td>
<td>29</td>
<td>98.6</td>
</tr>
</tbody>
</table>
Unreinforced masonry (URM)

Example

A) Frame effect not considered
Unreinforced masonry (URM) Example

A) Frame effect not considered

\[ V_{Rdx} = (4 \times 123.3 + 14 \times 24.6) = 837.6 \text{ kN} \]
\[ k_E = (4 \times 12.4 + 2 \times 14 \times 3.5) = 98.6 \text{ kN / mm} \]
\[ \Delta_y = \frac{837.6}{98.6} = 8.50 \text{ mm} \]
\[ \Delta_u = 25.3 \text{ mm} \]
Unreinforced masonry (URM) Example

B) Frame effect considered

<table>
<thead>
<tr>
<th>Wall</th>
<th>lw=2.0 m</th>
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<td>360</td>
<td>1520</td>
<td>-</td>
</tr>
<tr>
<td>VRd [kN]</td>
<td>57.8</td>
<td>65.6</td>
<td>219.4</td>
<td>180.3</td>
<td>541.2</td>
<td>1133</td>
</tr>
<tr>
<td>δu [%]</td>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
<td>0.4</td>
<td>0.8</td>
<td>-</td>
</tr>
<tr>
<td>Δy [mm]</td>
<td>10.2</td>
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<td>12.0</td>
<td>6.5</td>
<td>14.5</td>
<td>9.8</td>
</tr>
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<td>Δu [mm]</td>
<td>27.6</td>
<td>25.7</td>
<td>29.0</td>
<td>14.9</td>
<td>30.9</td>
<td>14.9</td>
</tr>
<tr>
<td>k [kN/mm]</td>
<td>5.7</td>
<td>8.6</td>
<td>18.3</td>
<td>27.6</td>
<td>37.4</td>
<td>115.4</td>
</tr>
</tbody>
</table>
Unreinforced masonry (URM) Example

B) Frame effect considered

![Graph showing lateral strength vs. top ultimate displacement for different wall lengths.](image)
Unreinforced masonry (URM) Example

B) Frame effect considered

\[
V_{\text{Rdy}} = (4 \times 57.8 + 2 \times 180.3 + 1 \times 541.2) = 1133 \text{ kN}
\]

\[
k_E = (4 \times 5.7 + 2 \times 27.6 + 1 \times 37.4) = 115.4 \text{ kN/mm}
\]

\[
\Delta_y = \frac{1133}{115.4} = 9.81 \text{ mm}
\]

\[
\Delta_u = 14.9 \text{ mm}
\]
Unreinforced masonry (URM) Example

B) Frame effect considered

<table>
<thead>
<tr>
<th>Wall</th>
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<td>-</td>
</tr>
<tr>
<td>VRd [kN]</td>
<td>57.8</td>
<td>65.6</td>
<td>219.4</td>
<td>180.3</td>
<td>541.2</td>
<td>1796</td>
</tr>
<tr>
<td>δu [%]</td>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
<td>0.4</td>
<td>0.8</td>
<td>-</td>
</tr>
<tr>
<td>Δy [mm]</td>
<td>10.2</td>
<td>7.6</td>
<td>12.0</td>
<td>6.5</td>
<td>14.5</td>
<td>9.3</td>
</tr>
<tr>
<td>Δu [mm]</td>
<td>27.6</td>
<td>25.7</td>
<td>29.0</td>
<td>14.9</td>
<td>30.9</td>
<td>25.7</td>
</tr>
<tr>
<td>k [kN/mm]</td>
<td>5.7</td>
<td>8.6</td>
<td>18.3</td>
<td>27.6</td>
<td>37.4</td>
<td>193.6</td>
</tr>
</tbody>
</table>
Unreinforced masonry (URM) Example

B) Frame effect considered

![Diagram showing lateral strength vs. top ultimate displacement for different building lengths](image)
Unreinforced masonry (URM) Example

B) Frame effect considered

\[ V_{Rdy} = (4 \times 219.4 + 14 \times 65.6) = 1796 \text{ kN} \]
\[ k_E = (4 \times 18.3 + 14 \times 8.6) = 193.6 \text{ kN/mm} \]
\[ \Delta_y = \frac{1796}{193.6} = 9.28 \text{ mm} \]
\[ \Delta_u = 25.7 \text{ mm} \]
Unreinforced masonry (URM) Example

Frame not considered

Frame considered

Static nonlinear analysis
Unreinforced masonry (URM) Example

Seismic safety factor obtained

Passing to ADSR format

\[ m^* = \sum m_i \Phi_i \quad \text{(equivalent mass)} \]
\[ (1.0+0.75+0.50+0.25+0.10) \times 250 = 625 \text{ t} = m^* \]

\[ \Gamma = \frac{\sum m_i \Phi_i}{\sum m_i \Phi_i^2} = \frac{m^*}{\sum m_i \Phi_i^2} \]
\[ (1.0+0.75+0.50+0.25+0.10) \times (1.0^2+0.75^2+0.50^2+0.25^2+0.10^2) = 2.5/1.875 = 1.33 = \Gamma \]
Unreinforced masonry (URM) Example

Seismic safety factor obtained

<table>
<thead>
<tr>
<th></th>
<th>dir</th>
<th>$V_{Rd}$ [KN]</th>
<th>$S_{ay}$ [m/s²] = $V_{Rd} / m^* \cdot \Gamma$</th>
<th>$\Delta_u / \Gamma$ [mm]</th>
<th>$d^*$ [mm]</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frame not considered</td>
<td>Y</td>
<td>618</td>
<td>0.74</td>
<td>18.2</td>
<td>14.4</td>
<td>1.26</td>
</tr>
<tr>
<td></td>
<td>X</td>
<td>838</td>
<td>1.01</td>
<td>19</td>
<td>12</td>
<td>1.58</td>
</tr>
<tr>
<td>Frame considered</td>
<td>Y</td>
<td>1133</td>
<td>1.36</td>
<td>11.2</td>
<td>9.9</td>
<td>1.13</td>
</tr>
<tr>
<td></td>
<td>X</td>
<td>1796</td>
<td>2.16</td>
<td>19.3</td>
<td>5.6</td>
<td>3.45</td>
</tr>
</tbody>
</table>
Displacement based vulnerability method (DBV)

- DBV method = development of capacity curve through few geometrical and mechanical parameters. Method developed for masonry and reinforced concrete existing buildings.

MASONRY BUILDINGS

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Figure 2-5. Capacity curve assumed for masonry buildings in terms of stiffness, strength and ultimate displacement capacity (Lagomarsino and Cattari 2013).
Displacement based vulnerability method (DBV)

The equivalence between the multi degree of freedom (MDOF) and the single substitute one (SDOF) is established using the procedure proposed by Fajfar (2000) and assumed as reference also in Eurocode 8-Part 1 (2005), thus by introducing the \( \Gamma \) coefficient and the equivalent mass \( m^* \).

The capacity curve assessment is associated with a certain analysis direction (\( \text{dir} = X,Y \)).
Displacement based vulnerability method (DBV)

- \( a_{y,dir} = \frac{F_{dir}}{m^* \Gamma} \)  
  Yield acceleration

- \( T_{y,dir} = 2\pi \sqrt{\frac{m^*}{k^*_{dir}}} \)  
  Fundamental period

- \( d_4 = \varepsilon \cdot d_{u,s.s.}(dir) + (1 - \varepsilon) \cdot d_{u,unif.}(dir) \)  
  Ultimate displacement capacity
Displacement based vulnerability method (DBV)

\[ a_{y,dir} = \frac{F_{dir}}{m^* \Gamma} \]

Yield acceleration

- \( F_{dir} \): Total base shear capacity
- \( m^* \): Equivalent mass of SDOF
- \( \Gamma \): Coefficient representing the modal participation factor and it requires of assuming a modal shape \( \Phi \)

\[ \Gamma = \frac{\sum m_i \Phi_i}{\sum m_i \Phi_i^2} = \frac{m^*}{\sum m_i \Phi_i^2} \]
Displacement based vulnerability method (DBV)

• $a_{y,\text{dir}} = \frac{F_{\text{dir}}}{m*\Gamma}$

Yield acceleration

$F_{\text{dir}}$  
Total base shear capacity
Directly related to the shear strength offered by resistant walls area at the first floor

$F_{\text{dir}} = A_{1,\text{dir}} \cdot \tau_{u,\text{dir}} \cdot \xi \cdot \zeta_{\text{res}}$

$\tau_{u,\text{dir}}$  
Ultimate shear strength of masonry

$\xi$  
Coefficient aimed to penalize the strength as a function of the main prevailing failure mode expected at scale of masonry piers, it is assumed equal to 1 in the case of prevalence of shear failure mechanisms and 0.8 in the case of compression-bending failure mechanisms

$\zeta_{\text{res}}$  
Corrective factor aimed to consider peculiarities of existing buildings
Displacement based vulnerability method (DBV)

\[ T_{y,dir} = 2\pi \sqrt{\frac{m^*}{k^*_{dir}}} \]

**Fundamental period**

- \( k^*_{dir} \) Stiffness of the SDOF system

\[ k^*_{dir} = \zeta_{rig} \cdot \frac{G}{H^2} \cdot \sum_{i=1}^{N} A_{i,dir} \cdot h_i \]

\( \zeta_{rig} \) Corrective factor taking into account the coupling effect due to spandrels and the flexural contribution
Displacement based vulnerability method (DBV)

Figure 2-6. Global response for three cases of coupled walls: from the case of very weak spandrels (a) to the shear type idealization (c); case (b) represents an intermediate condition (Lagomarsino and Cattari, 2013).
Displacement based vulnerability method (DBV)

\[ \zeta_{res} = \zeta_1 \zeta_2 \zeta_3 \]

- \( \zeta_1 \): It takes into account the non-homogeneous size of masonry piers.
- \( \zeta_2 \): It takes into account the influence of geometric and shape irregularities in the plan configuration.
- \( \zeta_3 \): It takes into account the influence of spandrels stiffness that directly affects the global collapse mechanism.
Displacement based vulnerability method (DBV)

- $\zeta_{res} = \zeta_1 \zeta_2 \zeta_3$

### Table 2-3. Corrective factors ranges for strength evaluation (Lagomarsino and Cattari 2013).

<table>
<thead>
<tr>
<th>Corrective factors</th>
<th>Uniform failure mode</th>
<th>Soft-story failure mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\zeta_1$</td>
<td>from 0.8 to 1</td>
<td></td>
</tr>
<tr>
<td>$\zeta_2$</td>
<td>from 0.75 to 1</td>
<td></td>
</tr>
<tr>
<td>$\zeta_3$</td>
<td>from 0.6 to 1</td>
<td>1</td>
</tr>
</tbody>
</table>
Displacement based vulnerability method (DBV)

- $\zeta_{rig} = \zeta_4 \zeta_5$

$\zeta_4 = \frac{1}{1 + \frac{1.2 E (\frac{h_p}{b_p})^2}{G}}$

coefficient aimed to take into account the influence of the flexural component on the stiffness

$h_p$ and $b_p$ height and width of masonry piers

$\zeta_5$ coefficient related to the characteristics of spandrels
Displacement based vulnerability method (DBV)

• $\zeta_{\text{res}} = \zeta_1 \zeta_2 \zeta_3$

Table 2-4. Corrective factors ranges for stiffness evaluation (Lagomarsino and Cattari 2013).

<table>
<thead>
<tr>
<th>Corrective factors</th>
<th>Uniform failure mode</th>
<th>Soft-story failure mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\zeta_4$</td>
<td>from 0.4 to 0.8</td>
<td></td>
</tr>
<tr>
<td>$\zeta_5$</td>
<td>from 0.7 to 1</td>
<td>1</td>
</tr>
</tbody>
</table>
Displacement based vulnerability method (DBV)

- \[ d_4 = \varepsilon \cdot d_{u,s.s.}(dir) + (1 - \varepsilon) \cdot d_{u,unif.}(dir) \]

Ultimate displacement capacity

\[
d_{u,\text{uniform}}(dir) = \delta_{u,\text{dir}} \frac{Nh}{\Gamma} \]

ultimate displacement of masonry piers according to a prevailing failure mode

\[
d_{u,\text{soft story}}(dir) = \delta_{u,\text{dir}} h + d_{y,\text{dir}} \left(1 - \frac{\Gamma}{N}\right) \]

\[
\delta_{u,\text{dir}} \quad \text{ultimate drift of masonry piers according to a prevailing failure mode in pier for the examined direction (if shear or flexural one)}
\]

\[
d_{y,\text{dir}} \quad \text{yield displacement computed starting from } a_{y,\text{dir}} \text{ and } T_{y,\text{dir}}
\]

\[
\varepsilon \quad \text{Fraction assigned to the soft story global failure mode}
\]
Displacement based vulnerability method (DBV)

- $d_1 = 0.7d_y$

- $d_2 = \rho_2 d_y$

$\rho_2$ coefficient varying as a function of the prevailing global failure mode. It is proposed to assume a value equal to 1.5 in case of soft story failure mode and 2 in case of the uniform one.
Displacement based vulnerability method (DBV)

• The reliability of the new vulnerability models has been verified through a comparison between the capacity curves evaluated by using the mechanical models and the pushover obtained by non-linear static analyses.
Displacement based vulnerability method (DBV)

Figure 4-7. Building n°1 - M6 class; a) 3Muri modeling; b) Comparison between the capacity curves (displacement-based method) and the pushover from 3Muri.

Figure 4-8. Building n°2 - M6 class; a) 3Muri modeling; b) Comparison between the capacity curves (displacement-based method) and the pushover from 3Muri.
Displacement based vulnerability method (DBV)

Figure 4-9. Building n°3 - M6 class; a) 3Muri modeling; b) Comparison between the capacity curves (displacement-based method) and the pushover from 3Muri.

Figure 4-10. Building n°4 - M6 class; a) 3Muri modeling; b) Comparison between the capacity curves (displacement-based method) and the pushover from 3Muri.
Displacement based vulnerability method (DBV)

Figure 4-11. Building n°5 - M6 class; a) 3Muri modeling; b) Comparison between the capacity curves (displacement-based method) and the pushover from 3Muri.

Figure 4-12. Building n°6 - M6 class; a) 3Muri modeling; b) Comparison between the capacity curves (displacement-based method) and the pushover from 3Muri.
Displacement based vulnerability method (DBV)

Figure 4-13. Building n°7 - M6 class; a) 3Muri modeling; b) Comparison between the capacity curves (displacement-based method) and the pushover from 3Muri.

Figure 4-14. Building n°8 - M6 class; a) 3Muri modeling; b) Comparison between the capacity curves (displacement-based method) and the pushover from 3Muri.
Displacement based vulnerability method (DBV)

Figure 4-15. Building n°9 - M6 class; a) 3Muri modeling; b) Comparison between the capacity curves (displacement-based method) and the pushover from 3Muri.

Figure 4-16. Building n°10 - M6 class; a) 3Muri modeling; b) Comparison between the capacity curves (displacement-based method) and the pushover from 3Muri.
Displacement based vulnerability method (DBV)

Table 4-1. Corrective factors for the buildings analyzed when a Uniform collapse mode is activated.

<table>
<thead>
<tr>
<th>Building</th>
<th>Analysis direction</th>
<th>$\zeta_1$</th>
<th>$\zeta_2$</th>
<th>$\zeta_3$</th>
<th>$\zeta_4$</th>
<th>$\zeta_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Build. n°1</td>
<td>X</td>
<td>0.89</td>
<td>0.80</td>
<td>0.50</td>
<td>0.45</td>
<td>0.30</td>
</tr>
<tr>
<td>Build. n°2</td>
<td>X</td>
<td>0.88</td>
<td>0.80</td>
<td>0.50</td>
<td>0.61</td>
<td>0.30</td>
</tr>
<tr>
<td>Build. n°3</td>
<td>X</td>
<td>0.81</td>
<td>0.82</td>
<td>0.70</td>
<td>0.67</td>
<td>0.30</td>
</tr>
<tr>
<td>Build. n°4</td>
<td>X</td>
<td>0.91</td>
<td>0.89</td>
<td>0.50</td>
<td>0.68</td>
<td>0.30</td>
</tr>
<tr>
<td>Build. n°5</td>
<td>X</td>
<td>0.85</td>
<td>0.80</td>
<td>0.70</td>
<td>0.61</td>
<td>0.40</td>
</tr>
<tr>
<td>Build. n°6</td>
<td>X</td>
<td>0.87</td>
<td>0.81</td>
<td>0.60</td>
<td>0.71</td>
<td>0.40</td>
</tr>
<tr>
<td>Build. n°7</td>
<td>X</td>
<td>0.80</td>
<td>0.80</td>
<td>0.70</td>
<td>0.68</td>
<td>0.30</td>
</tr>
<tr>
<td>Build. n°8</td>
<td>X</td>
<td>0.88</td>
<td>0.80</td>
<td>0.50</td>
<td>0.65</td>
<td>0.30</td>
</tr>
<tr>
<td>Build. n°9</td>
<td>X</td>
<td>0.85</td>
<td>0.80</td>
<td>0.60</td>
<td>0.53</td>
<td>0.30</td>
</tr>
<tr>
<td>Build. n°10</td>
<td>X</td>
<td>0.89</td>
<td>0.89</td>
<td>0.50</td>
<td>0.51</td>
<td>0.30</td>
</tr>
<tr>
<td>Build. n°11</td>
<td>Y</td>
<td>0.89</td>
<td>0.99</td>
<td>0.60</td>
<td>0.71</td>
<td>0.40</td>
</tr>
<tr>
<td>Build. n°12</td>
<td>Y</td>
<td>0.87</td>
<td>0.96</td>
<td>0.50</td>
<td>0.64</td>
<td>0.30</td>
</tr>
</tbody>
</table>
Displacement based vulnerability method (DBV)

Table 4-2. Corrective factors for the buildings analyzed when a Soft Story collapse mode is activated.

<table>
<thead>
<tr>
<th>Building</th>
<th>Analysis direction</th>
<th>$\zeta_1$</th>
<th>$\zeta_2$</th>
<th>$\zeta_3$</th>
<th>$\zeta_4$</th>
<th>$\zeta_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Build. n°1</td>
<td>Y</td>
<td>0.91</td>
<td>0.80</td>
<td>0.90</td>
<td>0.67</td>
<td>0.80</td>
</tr>
<tr>
<td>Build. n°2</td>
<td>Y</td>
<td>0.87</td>
<td>1.00</td>
<td>1.00</td>
<td>0.72</td>
<td>0.80</td>
</tr>
<tr>
<td>Build. n°3</td>
<td>Y</td>
<td>0.85</td>
<td>0.95</td>
<td>0.80</td>
<td>0.44</td>
<td>0.80</td>
</tr>
<tr>
<td>Build. n°4</td>
<td>Y</td>
<td>0.88</td>
<td>0.75</td>
<td>0.80</td>
<td>0.72</td>
<td>0.80</td>
</tr>
<tr>
<td>Build. n°5</td>
<td>Y</td>
<td>0.87</td>
<td>0.95</td>
<td>0.80</td>
<td>0.63</td>
<td>0.80</td>
</tr>
<tr>
<td>Build. n°7</td>
<td>Y</td>
<td>0.85</td>
<td>1.00</td>
<td>0.80</td>
<td>0.61</td>
<td>0.80</td>
</tr>
<tr>
<td>Build. n°9</td>
<td>Y</td>
<td>0.85</td>
<td>0.75</td>
<td>0.80</td>
<td>0.61</td>
<td>0.80</td>
</tr>
<tr>
<td>Build. n°10</td>
<td>Y</td>
<td>0.85</td>
<td>0.95</td>
<td>0.80</td>
<td>0.60</td>
<td>0.80</td>
</tr>
</tbody>
</table>
Displacement based vulnerability method (DBV)

Table 4-3. Corrective factors for M6 typology.

<table>
<thead>
<tr>
<th>Corrective factors</th>
<th>Effect</th>
<th>Uniform</th>
<th>Soft Story</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\zeta_1$</td>
<td>Non homogeneous size of piers</td>
<td>0.87</td>
<td>0.86</td>
</tr>
<tr>
<td>$\zeta_2$</td>
<td>Plan irregularities</td>
<td>0.85</td>
<td>0.89</td>
</tr>
<tr>
<td>$\zeta_3$</td>
<td>Spandrels stiffness</td>
<td>0.58</td>
<td>0.84</td>
</tr>
<tr>
<td>$\zeta_4$</td>
<td>Flexural deformation</td>
<td>0.62</td>
<td>0.63</td>
</tr>
<tr>
<td>$\zeta_5$</td>
<td>Spandrels stiffness</td>
<td>0.33</td>
<td>0.80</td>
</tr>
</tbody>
</table>
Content

• Capacity curve

• **Seismic damage**
  
  • Acceleration-Displacement Response Spectrum (ADRS)
  
  • Performance Point
  
  • Large-scale vulnerability assessment
Seismic Damage
Seismic Damage

- **Damage-based design/assessment:**
  - Related to social and economical costs of damage/mitigating measures
## Classification of damage to masonry buildings

<table>
<thead>
<tr>
<th>Grade</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grade 1</td>
<td>Negligible to slight damage (no structural damage, slight non-structural damage)</td>
</tr>
<tr>
<td></td>
<td>Hair-line cracks in very few walls.</td>
</tr>
<tr>
<td></td>
<td>Fall of small pieces of plaster only.</td>
</tr>
<tr>
<td></td>
<td>Fall of loose stones from upper parts of buildings in very few cases.</td>
</tr>
<tr>
<td>Grade 2</td>
<td>Moderate damage (slight structural damage, moderate non-structural damage)</td>
</tr>
<tr>
<td></td>
<td>Cracks in many walls.</td>
</tr>
<tr>
<td></td>
<td>Fall of fairly large pieces of plaster.</td>
</tr>
<tr>
<td></td>
<td>Partial collapse of chimneys.</td>
</tr>
<tr>
<td>Grade 3</td>
<td>Substantial to heavy damage (moderate structural damage, heavy non-structural damage)</td>
</tr>
<tr>
<td></td>
<td>Large and extensive cracks in most walls.</td>
</tr>
<tr>
<td></td>
<td>Roof tiles detach.</td>
</tr>
<tr>
<td></td>
<td>Chimneys fracture at the roof line; failure of individual non-structural elements (partitions, gable walls).</td>
</tr>
<tr>
<td>Grade 4</td>
<td>Very heavy damage (heavy structural damage, very heavy non-structural damage)</td>
</tr>
<tr>
<td></td>
<td>Serious failure of walls; partial structural failure of roofs and floors.</td>
</tr>
<tr>
<td>Grade 5</td>
<td>Destruction (very heavy structural damage)</td>
</tr>
<tr>
<td></td>
<td>Total or near total collapse.</td>
</tr>
</tbody>
</table>

## Classification of damage to buildings of reinforced concrete

<table>
<thead>
<tr>
<th>Grade</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grade 1</td>
<td>Negligible to slight damage (no structural damage, slight non-structural damage)</td>
</tr>
<tr>
<td></td>
<td>Fine cracks in plaster over frame members or in walls at the base.</td>
</tr>
<tr>
<td></td>
<td>Fine cracks in partitions and infills.</td>
</tr>
<tr>
<td>Grade 2</td>
<td>Moderate damage (slight structural damage, moderate non-structural damage)</td>
</tr>
<tr>
<td></td>
<td>Cracks in columns and beams of frames and in structural walls.</td>
</tr>
<tr>
<td></td>
<td>Cracks in partition and infill walls; fall of brittle cladding and plaster.</td>
</tr>
<tr>
<td></td>
<td>Falling mortar from the joints of wall panels.</td>
</tr>
<tr>
<td>Grade 3</td>
<td>Substantial to heavy damage (moderate structural damage, heavy non-structural damage)</td>
</tr>
<tr>
<td></td>
<td>Cracks in columns and beam column joints of frames at the base and at joints of coupled walls.</td>
</tr>
<tr>
<td></td>
<td>Spalling of concrete cover, buckling of reinforced rods.</td>
</tr>
<tr>
<td></td>
<td>Large cracks in partition and infill walls; failure of individual infill panels.</td>
</tr>
<tr>
<td>Grade 4</td>
<td>Very heavy damage (heavy structural damage, very heavy non-structural damage)</td>
</tr>
<tr>
<td></td>
<td>Large cracks in structural elements with compression failure of concrete and fracture of rebars; bond failure of beam reinforced bars; tilting of columns.</td>
</tr>
<tr>
<td></td>
<td>Collapse of a few columns or of a single upper floor.</td>
</tr>
<tr>
<td>Grade 5</td>
<td>Destruction (very heavy structural damage)</td>
</tr>
<tr>
<td></td>
<td>Collapse of ground floor or parts (e.g. wings) of buildings.</td>
</tr>
</tbody>
</table>

---

**EMS98 damage grades**
Content

• Capacity curve
• Seismic damage
• **Acceleration-Displacement Response Spectrum (ADRS)**
  • Performance Point
  • Large-scale vulnerability assessment
Response Spectrum

ADSR spectrum

Sa

Sd
Response Spectrum

accelerogramme

\[ a_{\text{max}} = 3.66 \text{ m/s}^2 \]
\[ a_{\text{min}} = -2.62 \text{ m/s}^2 \]

... linear behavior ...

... damping ratio: \( \zeta \) ...

\[ S_a = 2.31 \text{ m/s}^2 \]
\[ S_a = 7.00 \text{ m/s}^2 \]
\[ S_a = 11.33 \text{ m/s}^2 \]

\( \zeta = 5\% \)

response spectrum
Response Spectrum

\[ S_d \approx \frac{S_a}{\omega_n^2} \]

\[ \omega_n \approx \sqrt{\frac{k}{m}} \]
Acceleration-Displacement Response Spectrum (ADRS)
Acceleration-Displacement Response Spectrum (ADRS)

**ADRS**
Acceleration Displacement Response Spectrum

### Accelerogram

- **Time (s):** 0 to 10
- **Acceleration (m/s^2):** -2.0 to 2.0

### Response Spectrum $S_a$

- **Frequency (f):** 0.1 to 100 Hz
- **Acceleration (m/s^2):** 0.0 to 5.0

- $S_a = S_a / \omega^2$

- **Damping Ratio ($\xi$):** 5%

### SDOF

- **Frequency ($f$) variable**
- **Damping Ratio ($\xi$):** 5% (choice)
SIA norm ADSR

Figure 5.6: Spectres de réponse de norme (EC8, SIA 261): spectre de réponse élastique de l’accélération (a) et spectre de réponse élastique du déplacement relatif (b).

- $T_B < T < T_C$: constant acceleration = $a_{gd} \times 3$ factor (2.5; $S$; $\eta$)
- $T < T_B$: increasing acceleration (tending for $T=0$ to $a_{gd} \times S$)
- $T < T_C$: decreasing acceleration
- $T < T_D$: constant displacement
### Classe de terrain de fondation

<table>
<thead>
<tr>
<th>Classe</th>
<th>Description du profil stratigraphique</th>
<th>( v_{s,30} ) [m/s]</th>
<th>( N_{cy,50} ) [nombre de coups ( /0,3 \mathrm{m} )]</th>
<th>( c_u ) [kN/m²]</th>
<th>( S )</th>
<th>( T_D ) [s]</th>
<th>( T_C ) [s]</th>
<th>( I_g ) [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Rocher ou formation géologique similaire avec une couverture de terrain meuble d’au plus 5 m d’épaisseur en surface</td>
<td>&gt; 800</td>
<td>—</td>
<td>1,00</td>
<td>0,15</td>
<td>0,4</td>
<td>2,0</td>
<td>600</td>
</tr>
<tr>
<td>B</td>
<td>Dépôts de sable et gravier très compact ou d’argile très ferme, d’une épaisseur d’au moins quelques dizaines de mètres, caractérisées par une augmentation progressive des propriétés mécaniques avec la profondeur</td>
<td>500…800</td>
<td>&gt; 50</td>
<td>&gt; 250</td>
<td>1,20</td>
<td>0,15</td>
<td>0,5</td>
<td>2,0</td>
</tr>
<tr>
<td>C</td>
<td>Dépôts de sable et gravier moyennement compact à compact ou d’argile ferme, d’une épaisseur de quelques dizaines à plusieurs centaines de mètres</td>
<td>300…500</td>
<td>15…50</td>
<td>70…250</td>
<td>1,15</td>
<td>0,20</td>
<td>0,6</td>
<td>2,0</td>
</tr>
<tr>
<td>D</td>
<td>Dépôts de terrain meuble non cohésif lâche à moyennement compact (avec ou sans couches cohérentes tendres) ou à prédominance de terrain meuble cohésif de consistance tendre à ferme</td>
<td>&lt; 300</td>
<td>&lt; 15</td>
<td>&lt; 70</td>
<td>1,35</td>
<td>0,20</td>
<td>0,8</td>
<td>2,0</td>
</tr>
<tr>
<td>E</td>
<td>Couche superficielle de terrain meuble avec des valeurs de ( v_s ) correspondant à la classe C ou D et une épaisseur comprise entre 5 m et 20 m, reposant sur un matériau plus ferme avec une valeur ( v_{s,30} &gt; 800 ) m/s</td>
<td>—</td>
<td>—</td>
<td>1,40</td>
<td>0,15</td>
<td>0,5</td>
<td>2,0</td>
<td>500</td>
</tr>
</tbody>
</table>

Figure 5.7: Définition des classes de sol de fondation (tableau 24 de la norme SIA 261 [5.5]).
SIA norm ADSR

Tableau 5.1: SIA 261, valeur de calcul de l’accélération du sol en fonction de la zone d’aléa sismique [5.5].

<table>
<thead>
<tr>
<th>Zone</th>
<th>$a_{gd}$ [m/s²]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z1</td>
<td>0.6</td>
</tr>
<tr>
<td>Z2</td>
<td>1.0</td>
</tr>
<tr>
<td>Z3a</td>
<td>1.3</td>
</tr>
<tr>
<td>Z3b</td>
<td>1.6</td>
</tr>
</tbody>
</table>

\[
S_a = a_{gd} \cdot S \cdot \left[ 1 + \frac{(2.5\eta - 1) \cdot T}{T_B} \right] \quad 0 \leq T \leq T_B
\]

\[
S_a = 2.5 \cdot a_{gd} \cdot S \cdot \eta \quad T_B \leq T \leq T_B
\]

\[
S_a = 2.5 \cdot a_{gd} \cdot S \cdot \eta \cdot \frac{T_C}{T} \quad T_C \leq T \leq T_D
\]

\[
S_a = 2.5 \cdot a_{gd} \cdot S \cdot \eta \cdot \frac{T_C \cdot T_D}{T^2} \quad T_D \leq T
\]
Content

• Pushover technique – capacity curve
• Seismic damage
• Acceleration-Displacement Response Spectrum (ADRS)
• **Performance Point**
• Large-scale vulnerability assessment
Determination of the performance point

Two of the main static non linear methods to determine the performance point (examples covered here)

- Equivalent linearization (FEMA 440)
  - Improved Spectrum Method of ATC40

- N2 method (equal displacement rule, EC8 approach)
Equivalent Linearization Method

A little bit of background...

- First introduced in 1970 in a pilot project as a rapid evaluation tool (Freeman et al. 1975)
- Basis of the simplified analysis methodology in ATC-40 (1996)
- Improved later in FEMA 440 document (2005)
Equivalent Linearization Method

Based on equivalent linearization. The displacement demand of a non-linear SDOF system is estimated from the displacement demand of a linear-elastic SDOF system. The elastic SDOF system, referred to as an equivalent system, has a period and a damping ratio larger than those of the initial non-linear system (ATC, 2005).
Equivalent Linearization

Basic equations...
Equivalent Linearization - Performance Point

\[ S_d = S_d(T_{eq}; \zeta_{eq}) = S_d(T_{eq}; \zeta=5\%) \cdot \eta = S_d(T_{eq}; \zeta=5\%) \cdot \frac{T_{eq}^2}{4\pi^2} \cdot \eta \]

Source: FEMA 440
Equivalent Linearization - Performance Point

\[ S_d = S_d(T_{eq}; \zeta_{eq}) = S_d(T_{eq}; \zeta=5\%) \cdot \eta = S_d(T_{eq}; \zeta=5\%) \cdot \frac{T_{eq}^2}{4\pi^2} \cdot \eta \]

\( S_d(T_{eq}; \zeta_{eq}) \)  \quad \text{Spectral displacement of the equivalent system}

\( T_{eq} \)  \quad \text{Equivalent period of vibration}

\( \zeta_{eq} \)  \quad \text{Equivalent viscous damping ratio}

\( S_d(T_{eq}; \zeta=5\%) \)  \quad \text{Displacement demand of the linear system with 5%-damping elastic ratio}

\( \eta \)  \quad \text{Reduction factor depending from the damping modification factor}

\[ \eta = \sqrt{\frac{1}{0.5+10\zeta_{eq}}} \]
Equivalent Linearization—Performance Point

\[ S_d = S_d(T_{eq}; \zeta_{eq}) = S_d(T_{eq}; \zeta=5\%) \cdot \eta = S_a(T_{eq}; \zeta=5\%) \cdot \frac{T_{eq}^2}{4\pi^2} \cdot \eta \]

The equivalent period and the equivalent damping ratio are functions of the strength reduction factor of the non-linear SDOF system and, respectively, of the initial period of vibration and of the damping ratio. The various equivalent linear methods differ from each other mainly for functions used to compute \( T_{eq} \) and \( \zeta_{eq} \).

In their work (2008), Lin & Miranda give the equivalent period and the equivalent damping ratio as follows:

\[ T_{eq} = \left[ 1 + \frac{m_1}{T^2} \cdot (R_\mu^{1.8} - 1) \right] \cdot T \]
\[ \zeta_{eq} = \zeta=5\% + \frac{n_1}{Tn_2} \cdot (R_\mu - 1) \]
Equivalent Linearization-Performance Point

\[ S_d = S_d(T_{eq}; \zeta_{eq}) = S_d(T_{eq}; \zeta=5\%) \cdot \eta = S_a(T_{eq}; \zeta=5\%) \cdot \frac{T_{eq}^2}{4\pi^2} \cdot \eta \]

\[ T_{eq} = \left[ 1 + \frac{m_1}{T^2} \cdot (R\mu^{1.8} - 1) \right] \cdot T \]

\[ \zeta_{eq} = \zeta=5\% + \frac{n_1}{Tn_2} \cdot (R\mu - 1) \]

<table>
<thead>
<tr>
<th>(\alpha)</th>
<th>(m_1)</th>
<th>(m_2)</th>
<th>(n_1)</th>
<th>(n_2)</th>
</tr>
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<tr>
<td>0%</td>
<td>0.026</td>
<td>0.87</td>
<td>0.016</td>
<td>0.84</td>
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<tr>
<td>5%</td>
<td>0.027</td>
<td>0.65</td>
<td>0.027</td>
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<tr>
<td>10%</td>
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<td>0.51</td>
<td>0.031</td>
<td>0.39</td>
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<tr>
<td>20%</td>
<td>0.024</td>
<td>0.36</td>
<td>0.030</td>
<td>0.24</td>
</tr>
</tbody>
</table>

*Table 1* Coefficients for calculating equivalent period and damping (Lin & Miranda, 2008)
Equivalent Linearization

• Advantages:
  – Linear computation
  – Use of pushover analysis

• Drawbacks:
  – Value of damping
  – Not always conservative
N2 Method

A little bit of background...

• Started in the mid 1980’s (Fajfar and Fischinger 1987, 1989)
• A variant of the Capacity Spectrum Method (ATC-40)
• Based on inelastic spectra rather than elastic spectra
N2 Method Procedure

I. DATA
a) Structure
b) Elastic acceleration spectrum $S_{ae}$

II. DEMAND SPECTRA IN AD FORMAT
a) Determine elastic spectrum in AD format

$$S_{de} = \frac{T^2}{4 \pi^2} S_{ae}$$

b) Determine inelastic spectra for constant ductilities

$$S_a = \frac{S_{ae}}{R_\mu}, \quad S_d = \frac{\mu}{R_\mu} S_{de}$$

$$R_\mu = (\mu - 1) \frac{T}{T_C} + 1 \quad T < T_C$$

$$R_\mu = \mu \quad T \geq T_C$$
N2 Method Procedure

\[ S_a = \frac{S^{ae}}{R_\mu}, \quad S_d = \frac{\mu}{R_\mu} S^{de} \]

**Ductility factor**

\[ R_\mu = (\mu - 1) \frac{T}{T_C} + 1 \quad T < T_C \]

\[ R_\mu = \mu \quad T \geq T_C \]

**Reduction factor**

“Vidic et al. 1992”

Inelastic response spectrum (ADRS)
III. PUSHOVER ANALYSIS
a) Assume displacement shape \( \{\Phi\} \)

b) Determine vertical distribution of lateral forces
\[
\{P\} = [M] \{\Phi\}, \quad P_i = m_i \Phi_i
\]
c) Determine base shear \( (V) \) – top displacement \( (D_t) \) relationship

IV. EQUIVALENT SDOF MODEL
a) Determine mass \( m^* \)
\[
m^* = \sum m_i \Phi_i
\]
Note: \( \Phi_n = 1.0 \), \( n \) denotes roof level

b) Transform MDOF quantities \( (Q) \) to SDOF quantities \( (Q^*) \)
\[
Q^* = \frac{Q}{\Gamma}, \quad \Gamma = \frac{m^*}{\sum m_i \Phi_i^2}
\]
c) Determine an approximate elasto-plastic force – displacement relationship
d) Determine strength \( F_y^* \), yield displacement \( D_y^* \), and period \( T^* \)
\[
T^* = 2\pi \sqrt{\frac{m^* D_y^*}{F_y^*}}
\]
e) Determine capacity diagram (acceleration versus displacement)
\[
S_a = \frac{F_y^*}{m^*}
\]
V. SEISMIC DEMAND FOR SDOF MODEL

a) Determine reduction factor $R_\mu$

$$R_\mu = \frac{S_{ae}}{S_{ay}}$$

b) Determine displacement demand $S_d = D^*$

$$S_d = \frac{S_{de}}{R_\mu} \left(1 + \left(R_\mu - 1\right) \frac{T_C}{T^*}\right) \quad T^* < T_C$$

$$S_d = S_{de} \quad T^* \geq T_C$$
N2 Method Procedure (3)- Performance Point

if \( T < T_c \):

Basic equations...
N2 Method Procedure (3)-
Performance Point

\[ S_d = S_{de} R_{\mu} \left[ 1 + \left( R_{\mu} - 1 \right) \frac{T_C}{T^*} \right] \]

**Diagram:**
- \( S_a \) and \( S_{ae} \) represent system responses.
- \( S_{ay} \) indicates a lower bound.
- \( S_{de} \) and \( S_d \) indicate different states.
- \( S_d < T < T_C \) for elastic behavior.
- \( \mu = 1 \) (elastic behavior).

**Equation:**
\[ S_d = \frac{S_{de}}{R_{\mu}} \left[ 1 + \left( R_{\mu} - 1 \right) \frac{T_C}{T^*} \right] \]
N2 Method Procedure (3)-Performance Point

if $T \geq T_c$ :

Basic equations...
N2 Method Procedure (3) - Performance Point

\[ S_d = S_{de} \]

\( S_a \)

\( S_{ae} \)

\( S_{ay} \)

\( S_{ad} \)

\( D_d^* \)

\( D_y^* \)

\( S_d = S_{de} \)

\( \mu = 1 \) (elastic)
N2 Method Procedure

VI. GLOBAL SEISMIC DEMAND FOR MDOF MODEL

a) Transform SDOF displacement demand to the top displacement of the MDOF model

\[ D_t = \Gamma S_d \]

VII. LOCAL SEISMIC DEMANDS

a) Perform pushover analysis of MDOF model up to the top displacement \( D_t \) (or to an amplified value of \( D_t \))

b) Determine local quantities (e.g. story drifts, rotations \( \Theta \)), corresponding to \( D_t \)

VIII. PERFORMANCE EVALUATION

a) Compare local and global seismic demands with the capacities for the relevant performance level
Content

- Capacity curve
- Seismic damage
- Acceleration-Displacement Response Spectrum (ADRS)
- Performance Point
- Large-scale vulnerability assessment