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MODELLING AND CHARACTERIZATION OF NON-UNIFORM SUBSTRATE DOPING

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Abstract This article presents a new model for the effect of the transverse non-uniform substrate doping on the threshold voltage of MOS transistors. The new model is validated using 2D device simulations and measurements of a CMOS low-voltage process. A simple associated characterization method is also presented. The parameters related to the non-uniform doping are extracted from the pinch-off vs gate voltage characteristic, measured at constant current from a device biased in moderate inversion. © 1997 Elsevier Science Ltd

1. INTRODUCTION

Present-day CMOS processes use ion implantation to adjust the threshold voltages and to avoid the punchthrough effect. The substrate doping in the transistor can therefore no longer be considered as uniform, and accurate modelling of this effect is required for precise analog circuit simulation. The channel implant of *n*-channel transistors can be well approximated by a Gaussian distribution, however taking such a profile into account would lead to a complicated expression of the threshold voltage[1,2].

A compact MOST model for circuit simulation generally requires the drain current equations and their derivatives to be continuous with respect to variations of any terminal voltage over all regions of operation. The model should be computationally efficient, as well as accurate, which generally leads to a trade-off between the accuracy and speed of simulation. This remains true for an adequate modelling of the non-uniform substrate doping effect for circuit simulation, which requires the real doping profile to be suitably approximated, keeping a strong relation with the physical effects. Various approaches have been suggested to model the threshold voltage of enhancement type devices with a single equation[3–6]. Not all of these models are appropriate for all technologies, because of their semi-empirical or fully empirical nature.

Section 2 presents the physical definitions related to the non-uniform substrate doping, for both MOST models referred either to the source (SPICE-like models) or to the substrate (EKV MOST model). Existing models for the non-uniform substrate doping of particular interest are discussed and a new model[7] is proposed, and compared with 2D device simulations and measurements of a 2 μm CMOS low-voltage process in Section 3. Section 4 describes a simple method for the extraction of the parameters related to the non-uniform substrate doping, based on the measured pinch-off vs gate voltage characteristic.

2. MODELLING THE NON-UNIFORM SUBSTRATE DOPING

2.1. *General relationships*

The non-uniform doping mainly affects the relation between the depletion charge (per unit area) Q'_B and the channel voltage V_{ch} in strong inversion. This expression is given by;

$$-\frac{Q'_B}{C'_{ox}} = \gamma\sqrt{\Psi_s} \approx \gamma\sqrt{\Psi_0 + V_{ch}} \qquad (1)$$

where C'_{ox} is the gate oxide capacitance per unit area and Ψ_0 is the approximation of the surface potential Ψ_s in strong inversion and at equilibrium (i.e. for $V_{ch} = 0$), which is approximately given by;

$$\Psi_0 \cong 2\Phi_F + \text{several } U_T = 2U_T\ln(N_s/n_i) + 4U_T \qquad (2)$$

where $U_T \equiv kT/q$ is the thermodynamic voltage, Φ_F is the Fermi potential and N_s is the doping at the surface. The substrate factor γ is a function of the doping N_{sub} and is given by;

$$\gamma \equiv \gamma(N_{sub}) = \frac{\sqrt{2q\epsilon_0\epsilon_{si}N_{sub}}}{C'_{ox}} \qquad (3)$$

where N_{sub} is a function of the distance from the surface into the substrate. The effect of non-uniform substrate doping is best illustrated by considering the threshold voltage variation ΔV_{TB} defined as[7,8];

$$\Delta V_{TB} \equiv V_{TB}(V_{ch}) - V_{ch} - V_{T0}$$
$$= \left|\frac{Q'_B}{C'_{ox}}\right|\Bigg\|_{V_{ch}>0} - \left|\frac{Q'_B}{C'_{ox}}\right|\Bigg\|_{V_{ch}=0} \tag{4}$$
$$= \gamma\sqrt{\Psi_0 + V_{ch}} - \gamma_0\sqrt{\Psi_0},$$

where V_{TB} is the threshold voltage referred to the substrate, as are all voltages in the EKV MOST model[8]. V_{T0} is the threshold voltage corresponding to the gate voltage V_G for which the inversion charge Q'_{inv} forming the channel is zero at equilibrium (i.e. for $V_{ch} = 0$), and γ_0 is the substrate factor defined at equilibrium (i.e. for $(V_{ch} = 0)$). ΔV_{TB} can also be expressed as a variation of the more conventional threshold voltage V_{TH} referred to the source;

$$\Delta V_{TB} = V_{TH}(V_{ch}) - V_{T0}. \tag{5}$$

The EKV MOST model uses the so-called pinch-off voltage V_P to take into account the substrate effect[8]. It is a function of the gate voltage V_G and of the substrate factor γ[8]. It is defined as the particular value of the channel voltage V_{ch} for which the inversion charge Q'_{inv} becomes zero at a given gate voltage V_G larger than V_{T0}. This pinch-off voltage V_P can be seen as the voltage that has to be applied to the channel in order to cancel the effect caused by the gate voltage. It can therefore be interpreted as the effect of the gate voltage referred to the channel and the difference $V_P - V_{ch}$ is a measure of the inversion state of the channel. The slope of the V_G vs V_P characteristic corresponds to the slope factor $n \equiv \mathrm{d}V_G/\mathrm{d}V_P$, which is related to the inverse slope S ($S = 2.3nU_T$ V/decade) of the I_D vs V_G characteristic in weak inversion (in a log-lin scale).

The non-uniform doping will affect the relation existing between the pinch-off voltage and the gate voltage, mainly through the substrate factor γ. Consequently, the relation existing between the slope factor n and the gate voltage will also be affected. This effect can be modelled by changing the V_P vs V_G relation, which, in turn, can be measured using the extraction technique presented in Section 4. The effect of the non-uniform substrate doping on this relation is easiest to be observed by considering the variation of $V_G - V_{T0} - V_P$, which is nothing else than the ΔV_{TB} vs V_{ch} function given by (4), where V_{ch} is set to V_P[8]. It is also interesting to evaluate the effect of non-uniform doping on the slope factor n as a function of the pinch-off voltage.

2.2. *The step model and the Arora doping transformation model*

The real doping can be replaced by a step profile (doping N_s at the surface, and N_b in the bulk)[9] which leads to a simple expression of the threshold voltage, described by two parts, depending if the depletion depth, related to V_{ch}, is smaller or larger than the implant depth W_i. In VLSI devices, deep channel implants are needed such that the resulting implant depth is comparable to the depletion region depth in the back bias range of interest. Consequently the step model becomes inaccurate when the channel voltage is such that the depletion depth is around the implant depth. In addition, the discontinuity resulting from this profile is not adequate for circuit simulation because it may give rise to convergence problems.

A reformulation of the step profile was used in[10], where the depletion charge is calculated for both cases $V_{ch} \leqslant V_i$ and $V_{ch} > V_i$, and where the body effect factor is expressed as in eqn (3), with the N_{sub} term replaced by an effective doping N_{arora}, function of V_{ch}. The voltage V_i is defined as the particular value of V_{ch} for which the depletion depth is equal to the implant depth W_i,

$$V_i \equiv \frac{qN_sW_i^2}{2\epsilon_s} - \Psi_0. \tag{6}$$

The variation of the threshold voltage ΔV_{TB} is then given by;

$$\Delta V_{TB} = \gamma(N_{arora})\sqrt{\Psi_0 + V_{ch}} - \gamma_s\sqrt{\Psi_0}, \tag{7}$$

where γ_s is the substrate factor obtained by replacing N_{sub} in eqn (3) by the surface doping N_s. Note that γ_s in eqn (3) is constant, whereas $\gamma(N_{arora})$ depends on the channel voltage. This doping transformation model has very interesting features, but still requires two distinct sections. The resulting doping concentration is continuous, however its first derivative has a discontinuity. It can be shown that this solution is equivalent to the step profile model.

2.3. *The new model*

An approximation of the equivalent doping given by the Arora model was achieved by a simple but precise formulation using a single continuous function[7]. The inverse of the equivalent doping of the normalized Arora model N_s/N_{arora} vs the normalized voltage $x \equiv V_{ch}/V_i$ is nearly a linear function of x for $V_{ch} > V_i$ (or $x > 1$). An interpolation function[11] is then used to approximate the constant doping for the case $x < 1$ (Fig. 1). Finally, the

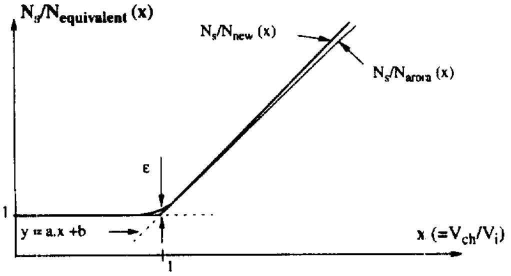

Fig. 1. Modelling of the inverse (normalized) non-uniform doping: The new model (N_{new}) compared with Arora's model (N_{arora}).

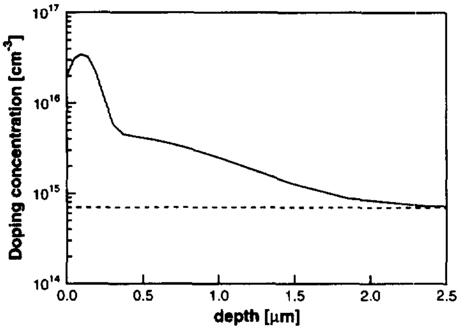

Fig. 2. Substrate doping profile with a double implant used for the 2D simulation.

non-uniform doping model, called N_{new}, can be expressed as;

$$N_{\mathrm{new}}(V_{\mathrm{ch}}) = \frac{N_{\mathrm{s}}}{1 + \frac{1}{2}(G + \sqrt{G^2 + 4\epsilon^2})} \tag{8}$$

with;

$$G = \alpha\left(\frac{V_{\mathrm{ch}}}{V_{\mathrm{i}}} - 1\right). \tag{9}$$

The parameter α is proportional to $\sqrt{N_{\mathrm{b}}/N_{\mathrm{s}}}$ and to the gate oxide thickness and can be adjusted by local optimization; ϵ is a fixed constant (typically equal to 1%) such that a smooth transition is obtained between the asymptotes around $x = 1$[11]. The variation of the threshold voltage ΔV_{TB} is expressed as in eqn (7), with N_{arora} replaced by N_{new}. Note that this new expression of the substrate factor $\gamma(N_{\mathrm{new}})$ can be used directly in any MOST model where the voltages are referred to the source (SPICE-like models).

3. RESULTS

3.1. Comparison of the new solution with a 2D simulation

The 2D simulator Medici was used to simulate a "measured" V_{P} vs V_{G} characteristic, using the doping profile shown in Fig. 2. The V_{P} vs V_{G} characteristic shown in Fig. 3 was computed using eqn (10) with the new modified model (see Appendix);

$$V_{\mathrm{P}}(V_{\mathrm{G}}) = V_{\mathrm{G}} - V_{\mathrm{T0}} + \gamma_{\mathrm{s}}\sqrt{\Psi_0} + \frac{\gamma(N^*_{\mathrm{new}})^2}{2} \tag{10}$$

$$-\gamma(N^*_{\mathrm{new}})\left[\sqrt{V_{\mathrm{G}} - V_{\mathrm{T0}} + \Psi_0 + \gamma_{\mathrm{s}}\sqrt{\Psi_0} + \left(\frac{\gamma(N^*_{\mathrm{new}})}{2}\right)^2}\right]$$

where $\gamma(N^*_{\mathrm{new}})$ is now depending on V_{G}, and where V_{i} (originally given by eqn (6)) is replaced by V_{i}^* (A.5), now referred to V_{G} (see Appendix). The ΔV_{TB} vs V_{P} simulated and "measured" characteristics and the slope factor n vs V_{P} are presented in Fig. 4 and Fig. 5 respectively. The extracted parameters V_{T0}, γ_{s}, Ψ_0, V_{i}^* and α, are given in Table 1.

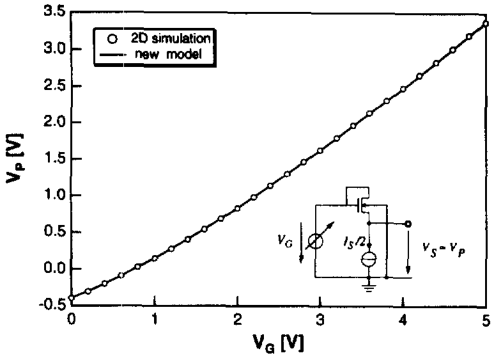

Fig. 3. V_{P} vs V_{G} characteristic (for an n-channel device)[15]. The inset shows the measurement setup used.

3.2. Validation

The measured and simulated characteristics of ΔV_{TB} vs V_{P} for an n-channel transistor of a 2 μm CMOS low-voltage process presenting a strongly non-uniform doping profile, are shown in Fig. 6. For this real case, the new solution gives an excellent agreement between simulation and measured characteristics. The extracted parameters are found in Table 2.

These results confirm the accuracy and validity of this model which is continuous and requires only two additional parameters, with respect to a formulation for uniform substrate doping.

4. THE PINCH-OFF VOLTAGE EXTRACTION METHOD

Conventional methods to extract the parameters related to the non-uniform doping are usually long and require several measurement steps[12]. A simple method to extract these parameters on the single V_{P} vs V_{G} characteristic is presented here. This measurement method is particularly well suited to the EKV MOST model[8,13,14], but can be adapted to other models.

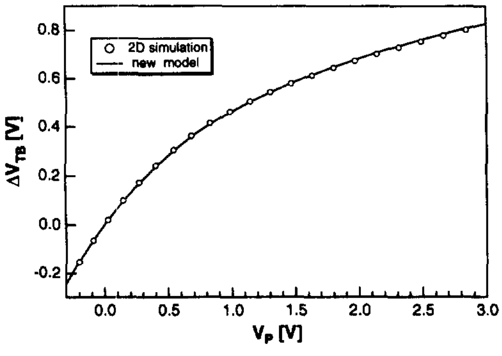

Fig. 4. ΔV_{TB} vs V_{P} characteristic derived from the V_{P} vs V_{G} characteristic.

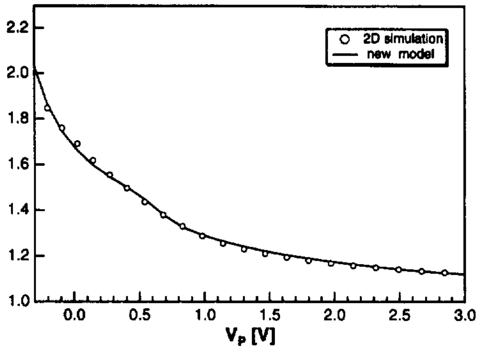

Fig. 5. Slope factor n vs V_P, derived from the V_P vs V_G characteristic.

Table 1. Parameter values for the new model, for the simulated double implant profile

V_{T0} (V)	γ_s ($V^{1/2}$)	Ψ_0 (V)	α (−)	V_t^* (V)
0.75	1.14	0.64	0.32	1.64

Table 2. Parameter values for the new model extracted for an n-channel transistor of a 2 μm CMOS low-voltage process

V_{T0} (V)	γ_s ($V^{1/2}$)	Ψ_0 (V)	α (−)	V_t^* (V)
0.77	0.75	0.46	0.089	1.56

4.1. *Pinch-off voltage measurement principle*

The pinch-off voltage can be measured at the source end of the device in saturation, for a particular value of the drain current approximately equal to half the specific current I_S defined as $I_S \equiv 2n\beta_{\text{eff}}U_T^2$ where $\beta_{\text{eff}} = \mu C'_{\text{ox}}(W_{\text{eff}}/L_{\text{eff}})$[15]. Note that with I_D equal to $I_S/2$, the transistor is biased in the middle of the moderate inversion region. The V_P vs V_G characteristic is simply obtained by sweeping the gate voltage and measuring the source voltage $V_S \approx V_P$.

4.2. *Parameter extraction*

The extraction of all parameters related to the new non-uniform doping model can be performed on the V_P vs V_G characteristic. V_{T0} is determined unambiguously with an unique value, without any need for extrapolation, as the particular value of V_G corresponding to the $V_P = 0$ cross point. γ_s and Ψ_0 are extracted by fitting eqn (10) to the measured V_P vs V_G characteristic in the region close to $V_P = 0$ and slightly above. V_t^* and α are determined in the upper region of the characteristic, maintaining the values of V_{T0}, γ_s and Ψ_0 previously obtained. An increased precision for the extraction on the doping parameters will be obtained by using the ΔV_{TB} vs V_P characteristic derived from the V_P vs V_G characteristic. The pinch-off voltage extraction method is comparable to other constant current methods[16] for the obtention of the threshold voltage except that the bias current is given a precise meaning here. The advantages of this method are its simplicity, efficiency and speed.

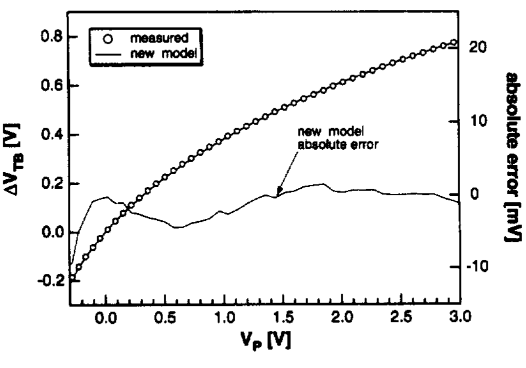

Fig. 6. ΔV_{TB} vs V_P characteristic, for an n-channel device of a CMOS low-voltage process.

5. CONCLUSION

A new approach for substrate non-uniform doping modelling has been discussed. The proposed solution is continuous, accurate, computationally efficient, and can be used for MOST models where voltages are referred either to the substrate (EKV MOST model) or to the source (SPICE-like models). Validation on the ΔV_{TB} vs V_P and the slope factor n vs V_P characteristics shows excellent agreement. A parameter extraction technique based on the measurement of the pinch-off voltage vs gate voltage characteristic in moderate inversion serves to extract the parameters related to the non-uniform doping in a straightforward way.

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APPENDIX

V_p vs V_G definitions

In the EKV MOST model, the pinch-off voltage V_P represents the voltage that should be applied to the channel (source and drain) to cancel the effect of the gate voltage. It can be interpreted as the effect of the gate voltage referred to the channel and is thus directly related to the gate voltage. In the case of a uniformly doped substrate, it is given by;

$$V_P = V_G - V_{T0} - \gamma\left[\sqrt{V_G - V_{T0} + \left(\sqrt{\Psi_0} + \frac{\gamma}{2}\right)^2} - \left(\sqrt{\Psi_0} + \frac{\gamma}{2}\right)\right] \quad (A.1)$$

or inversely;

$$V_G = V_{T0} + V_P + \gamma[\sqrt{\Psi_0 + V_P} - \sqrt{\Psi_0}] \quad (A.2)$$

where γ is constant and given by eqn (3). In the case of non-uniform substrate doping, equation (A.2) can be rewritten as:

$$V_G = V_{T0} + V_P + \gamma(N_{new})\sqrt{\Psi_0 + V_P} - \gamma_s\sqrt{\Psi_0} \quad (A.3)$$

Here, the substrate factor $\gamma(N_{new})$ is a function of the equivalent doping N_{new} depending on V_{ch} and given by eqn (8).

To invert (A.3) and obtain the new expression of the V_P vs V_G relation, modified by the non-uniform substrate doping effect, an approximation has to be made. The variation of $\gamma(N_{new})$ vs V_P with respect to γ_s is estimated to be small enough to consider it as constant during the inversion. Thus, in the inverted formulation (10), $\gamma(N_{new})$ is replaced by $\gamma(N^*_{new})$, where N^*_{new} is the same function as eqn (8) but with the term eqn (9) replaced by;

$$G = \alpha(V_G/V_i^* - 1) \quad (A.4)$$

with V_i replaced by V_i^*, now referred to V_G. V_i^* is defined as;

$$V_i^* = V_{T0} + V_i + \gamma_s(\sqrt{\Psi_0 + V_i} - \sqrt{\Psi_0}) \quad (A.5)$$

A numerical verification has shown that this approximation does not introduce any significant error compared to results obtained with the exact formulation.