

MOS MODELLING AT LOW CURRENT DENSITY

by H.J. Oguey and S. Cserveny
Centre Electronique Horloger S.A., Neuchâtel, Switzerland

SUMMARY

The low current static characteristics of MOSFETs are based on both drift and diffusion carrier transport. The long channel approximation of Pao and Sah gives an accurate implicit formulation of the current in an ideal situation. Modelling of this ideal case by an explicit system of equations is discussed and extended to real situations by a review of the most important effects, including mobility variation, non uniform doping, weak inversion, medium and short channels.

The adequacy and limits of this model are compared with other approaches in particular with two-dimensional simulations, and with experiments.

LIST OF SYMBOLS

a_h	parameter in f_h
a_w	parameter in f_w
b_h	parameter in f_h
b_w	parameter in f_w
B	effective gain defined by (64)
B_0	effective gain at low field defined by (65)
B_2	normalized body factor defined by (16)
C_D	depletion layer capacitance per unit area
C_{ox}	oxide capacitance per unit area
C_{ss}	surface state capacitance per unit area
C_w	parameter in f_w
d_{ox}	oxide thickness
D_I	implanted dose
D_n	electron diffusion coefficient
E	electric field
E_C	critical field at which carriers reach saturation velocity
f_h	strong inversion function defined by (46 and 47)
f_w	weak inversion function defined by (29 and 31)
F	charge sharing factor
F_1	normalized total charge density defined by (9)
F_2	auxiliary function $F_2 = F_1 \exp(-u_F/2)$
F_3	normalized mobile charge defined by (14)
I	channel current
I_{sat}	saturation current
I_T	normalization factor for currents defined by (15)
K	body factor defined by (23)
K_w	weak inversion parameter (see Appendix I)
l_{dep}	length of the depletion region between channel and drain
L	layout channel length
L_0	effective channel length before saturation
L_B	extrinsic Debye length for background doping before ion implantation
L_D	intrinsic Debye length

m	subthreshold parameter defined by (70)
n	subthreshold parameter defined by (71)
n	electron concentration
n_i	intrinsic carrier density
N_B	bulk doping concentration
T	absolute temperature
u	normalized potentials or voltages defined by (2)
u_C	normalized control voltage defined by (26)
u_D	normalized drain voltage
u_{Dsat}	normalized drain saturation voltage
u_F	normalized equilibrium Fermi potential defined by (3)
u'_G	normalized gate voltage defined by (19)
u_h	parameter in f_h
u_m	normalized surface potential at threshold
$u_{\Delta D}$	normalized surface potential at the drain
$u_{\Delta S}$	normalized surface potential at the source
u_S	normalized source to substrate voltage
u_{surf}	normalized surface potential
U_{TH}	thermodynamic potential kT/q
V	voltages
V_C	control voltage
V_D	drain to substrate voltage
V_D^*	expression corresponding to V_D limited at V_{Dsat} (see p. 21)
V_{Dsat}	drain to substrate saturation voltage
V_{FB}	flat-band voltage
V_G	gate to substrate voltage
V_S	source to substrate voltage
V_T	threshold voltage
V_{T0}	threshold voltage for $V_S = 0$
W	layout channel width
W_o	effective channel width
x	distance into substrate from Si-SiO ₂ interface
x_C	distance from the surface of the centroid of the depleted portion of the implanted dose
x_i	inversion layer depth
y	distance along the channel from the source
y	normalized current component

γ	function defined by (27)
ϵ_{ox}	permittivity of SiO_2
ϵ_s	permittivity of Si
η	parameter used to fit the transition formula (56)
Θ	parameter for mobility degradation due to the transversal field
λ_1	coefficient for l_{dep} adjustment used in (62)
λ_2	coefficient for mobile charge effect adjustment in l_{dep} used in(62)
μ	effective channel carrier mobility
μ_0	low field channel carrier mobility
ξ	normalized difference between electron and hole quasi-Fermi potentials
ρ	charge density
ϕ	potentials
ϕ_F	bulk Fermi potential

1. Introduction

Progress in the field of Very Large Scale Integration rests heavily on computer simulation. As MOSFETs play a central role in VLSI, a realistic simulation calls for adequate models. The broad field of VLSI can be divided into three categories, where quite different modeling philosophies are required:

1. Device optimization concentrates on a detailed analysis of device properties, especially of small devices. This is the domain of two- and three- dimensional simulations. Here, the long time necessary to obtain a single simulation is of secondary importance.
2. Digital system simulation, on the other hand, has to consider a limited number of situations (on-off), requires a limited accuracy, but needs a very simple analytical formulation of device properties in order to make possible the simulation of a complex system.
3. Analog circuit simulation lies in-between. It needs an accurate model of the MOSFET, valid in a wide range of situations (with respect to dimensions and currents), but still a simple analytical formulation of active devices and parasitic effects, compatible with computer simulation of complex analog circuits.

Consequently, there is no such thing as an universal MOSFET model, but more of a hierarchy of models, each presenting the proper trade-off between complexity on one side, accuracy and range of validity on the other side.

Today, most CAD programs in the categories 2 and 3 still include MOSFET models of limited validity. Weak inversion effects are poorly formulated, sometimes even ignored.

On the other hand, the operation of many circuits is critically dependent upon low level currents, and their optimization should rest on a valid formulation. A few examples of such circuits are easy to find: dynamic memories, low power digital and analog circuits for watches, pocket instruments, implanted medical electronics, switched capacitor filters and A/D converters for telecommunications, etc...

This paper concentrates on the low-current description of MOSFETs and on extensions of known models with a view to obtain an explicit and accurate formulation of the dc characteristics, valid in a wide range of voltages and currents. It starts with the long channel integral expression of Pao and Sah. Approximate explicit relations are proposed, valid in weak inversion, in strong inversion and, by an interpolation formula, also in the transition region; they are adjusted to fit with numerical solutions. The next step is a discussion of the most important deviations from the ideal situation, and of extension of the equations. The complete model finally is compared with two-dimensional simulations and with measurements.

2. The long channel model

Conduction in MOSFETs for gate voltages above threshold is controlled mainly by drift in a very narrow region called the channel. Under this channel area, we find a depleted region of the substrate which has to be taken into account in order to describe correctly the so called bulk effect.

For gate voltages slightly below threshold, the semiconductor is in weak inversion. Carrier transport takes place mainly by diffusion and is responsible for the small current observed under this condition.

Pao and Sah [66 Pa] presented the first analysis combining both drift and diffusion currents. Despite the fact that this paper has been widely commented in textbooks [69 Sz, 78 Mu, 81 Br], we will base our discussion mainly on the original paper.

2.1 Hypotheses about the physical structure

A classical idealized MOSFET structure is assumed (Fig. 1). In a p-type silicon substrate, two highly n-doped regions form the source and the drain. They are separated from the substrate (bulk) by abrupt junctions. The channel zone has a length L_0 and a width W_0 . Above it is an oxide of thickness d_{ox} and a gate electrode made of metal or polycrystalline silicon.

A point in the semiconductor is defined by its distance x with respect to the Si-SiO interface, and y with respect to source. The problem is two-dimensional, i.e. with no field component in the z direction.

The following assumptions are made:

1. In the channel region, the doping is uniform, in the order of 10^{21} to 10^{22} m^{-3} .
2. No generation or recombination effects are present.
3. The channel is long and wide ($\geq 20 \mu\text{m}$) with respect to d_{ox} ($\leq 0.1 \mu\text{m}$), allowing the "gradual channel assumption": the gradient of the electric field is assumed to be mainly in the x direction (perpendicular to the surface):

$$\left| \frac{\partial E_x}{\partial x} \right| \gg \left| \frac{\partial E_y}{\partial y} \right|$$

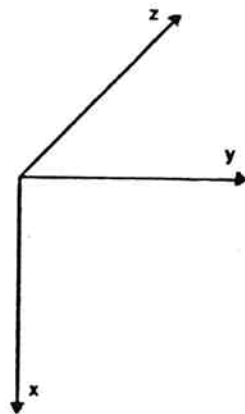
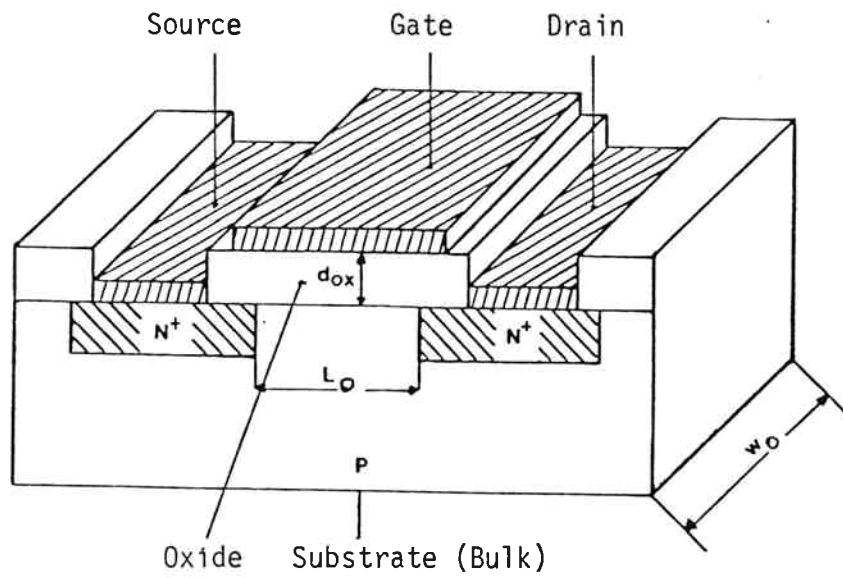


Fig. 1 Schematic cross-section of a MOS transistor.

4. Consequently, the flow of carriers is parallel to the surface, and the total current is constant along the channel, and is produced by a single type of carriers.
5. Mobility is a constant.
6. Absence of mobile charges in the oxide and of fast surface states. A fixed charge in the oxide or at the interface is accepted.
7. Thermodynamic quasi-equilibrium conditions (steady state currents and voltages, uniform temperature).

It is known that some of the assumptions are not accurate in real structures. Corrections will be discussed later.

2.2 Analysis of Pao and Sah

Some important properties of the current can be derived from the analysis of Pao and Sah, which will be discussed here in a slightly different way.

Let us define the source, drain and gate potentials V_S , V_D and V_G with respect to the substrate, and let us normalize all potentials with respect to the thermodynamic potential $U_{TH} = kT/q$:

$$u = \phi/U_{TH} \quad \text{or} \quad V/U_{TH} \quad (2)$$

In a n-channel transistor, the p-doped substrate (bulk) has a concentration

$$N_B = n_i \exp u_F \quad (3)$$

The carrier concentrations of the holes (p) and of the electrons (n) are related to the potential u and to the potential difference ξ of the quasi-Fermi levels $\xi = u_n - u_p$:

$$p = N_B \exp (-u) \quad (4)$$

$$n = N_B \exp (u - \xi - 2u_F) \quad (5)$$

Assuming the potential variation is mainly in the vertical direction x , the unidimensional Poisson equation has to be solved:

$$U_{TH} \frac{d^2 u}{dx^2} = - \frac{\rho}{\epsilon_s} = - \frac{q}{\epsilon_s} (p-n-N_B) \quad (6)$$

A first integration gives the normalized field

$$\frac{du}{dx} = - \frac{1}{L_D} F_1(u, \xi, u_F) \quad (7)$$

where $L_D = \sqrt{\frac{\epsilon_s U_{TH}}{2q n_i}}$ is the intrinsic Debye length, and (8)

$$F_1(u, \xi) = \frac{u}{|u|} \sqrt{e^{u-\xi-u_F} + e^{u_F-u} + (u-1)e^{u_F} - (u+e^{-\xi})e^{-u_F}} \quad (9)$$

is an auxiliary function (normalized total charge density).

The current density at any point x, y is given by

$$J(x, y) = -q D_n n \frac{d\xi}{dy} \quad (10)$$

The total channel current is obtained by integration

$$I = \frac{1}{L_0} \int_0^{L_0} D_n W_0 \frac{d\xi}{dy} \int_0^{x_i} q n(x, y) dx dy \quad (11)$$

with W_0 and L_0 the effective channel width and length.

The second integral represents the inversion charge density. Using (5) and (7), we can write

$$n dx = n \frac{dx}{du} du = \frac{n du}{\frac{du}{dx}} = - \frac{N_B}{L_D} \frac{\exp(u-\xi-2u_F)}{F_1(u, \xi)} du \quad (12)$$

the limits of integration are converted into the corresponding potentials u_F and u_{surf} , where u_{surf} is the surface potential.

The first integral can be similarly expressed in terms of the potential along the channel, with the boundary conditions u_S at the source and u_D at the drain. The current, normalized by a factor I_T , becomes

$$I = I_T \int_{u_S}^{u_D} F_3(u'_G, \xi) d\xi \quad (13)$$

with F_3 a function representing the normalized mobile charge:

$$F_3(u'_G, \xi) = \frac{1}{2} B_2 \int_{u_F}^{u_{surf}} \frac{\exp(u-2u_F-\xi)}{F_2(u, \xi)} du \quad (14)$$

$$I_T = \mu C_{ox} \frac{W_o}{L_o} U_{TH}^2 \quad (15)$$

$$B_2 = \frac{d_{ox}}{\epsilon_{ox}} \sqrt{\frac{2q N_B \epsilon_S}{U_{TH}}} \quad (16)$$

$$F_2 \approx \frac{u}{|u|} [u-1 + \exp(u-2u_F - \xi)]^{1/2} \quad (17)$$

The surface potential is given by the implicit relation relating it to the gate potential u'_G

$$u'_G = u_{surf} + B_2 F_2 (u_{surf}, \xi) \quad (18)$$

$$\text{where } u'_G = (V_G - V_{FB}) / U_{TH} \quad (19)$$

Finally, the general current-voltage relations are contained in eq. (13) with the boundary condition (18). The first definite integral (13) shows that the current is the difference between two functions $i(V_S)$ and $i(V_D)$. Each of these functions can be defined by choosing its asymptotic value equal to zero when the lower limit tends to a large value:

$$\int_{u_S}^{u_D} = \int_{u_S}^{\infty} - \int_{u_D}^{\infty}$$

We define a function $y(u'_G, \xi)$ with $\xi = u_S$ or u_D , in order to represent the two components of the normalized current:

$$y(u'_G, \xi) = \frac{I_x}{I_T} = \int_{\xi}^{\infty} F_3(u'_G, \xi) d\xi \quad (20)$$

Now the total current is obtained as a difference of two functions y :

$$I = I_T [y(u'_G, u_S) - y(u'_G, u_D)] \quad (21)$$

This property reminds the decomposition of the current in a forward and a reverse component in bipolar transistors.

An important special case is the grounded source saturation current, for which $u_S = 0$, $u_D = \infty$ and $I_{sat} = I_T y(u'_G, 0)$ (22)

This current is a function of u'_G only. Its knowledge is sufficient in many applications.

Numerical integration of (20) has been solved in order to have a basis of comparison for further simplifications. The set of normalized characteristics $y = f(u_G', \xi + 2u_F)$ is dependent upon a single parameter B_2 (eq. 16). This parameter is represented in fig. 2 as a function of d_{ox} and N . It is generally comprised between 3 and 7. Its value varies with temperature. It is related to the commonly defined bulk factor K :

$$K = B_2 \sqrt{U_{TH}} = \frac{d_{ox}}{\epsilon_{ox}} \sqrt{2 \epsilon_S q N_B} \quad (23)$$

Typical curves of $y = f(\xi)$ are plotted in fig. 3 to 5 on a semilog scale, with u_G' as a parameter, for $B_2 = 3, 5, 7$ and $u_F = 12$.

It appears clearly that, for $I_D < I_T$, these curves represent an exponential dependence of I_D with V_x , with a constant "slope". This is the weak inversion region. For a given value of B_2 , the characteristics can be obtained approximately by horizontal translation. For different values of B_2 , the curves are still of the same character in weak inversion, but are slightly translated in the vertical direction also, with minor changes in the transition between weak and strong inversion.

2.3 Explicit formulation of the Pao and Sah equations

Many authors tried to find explicit expressions of the Pao and Sah equations. Unfortunately, only approximations are possible. The problem can be decomposed into 4 phases:

- 1) Weak inversion approximation
- 2) Limit of the weak inversion region, corresponding to the threshold voltage
- 3) Strong inversion approximation
- 4) Analytical expression allowing a continuous interpolation.

2.3.1 Weak inversion approximation

The method used is a mere adaptation of Barron's [72 Ba].

Let us assume $\xi > 0$
 $u_F < u < 2u_F + \xi$

$$\text{Then } F_2 \cong \sqrt{u - 1} \quad (24)$$

The surface potential u_{surf} is obtained by solving the equation

$$u_G' = u_{surf} + B_2 \sqrt{u_{surf} - 1} \quad (25)$$

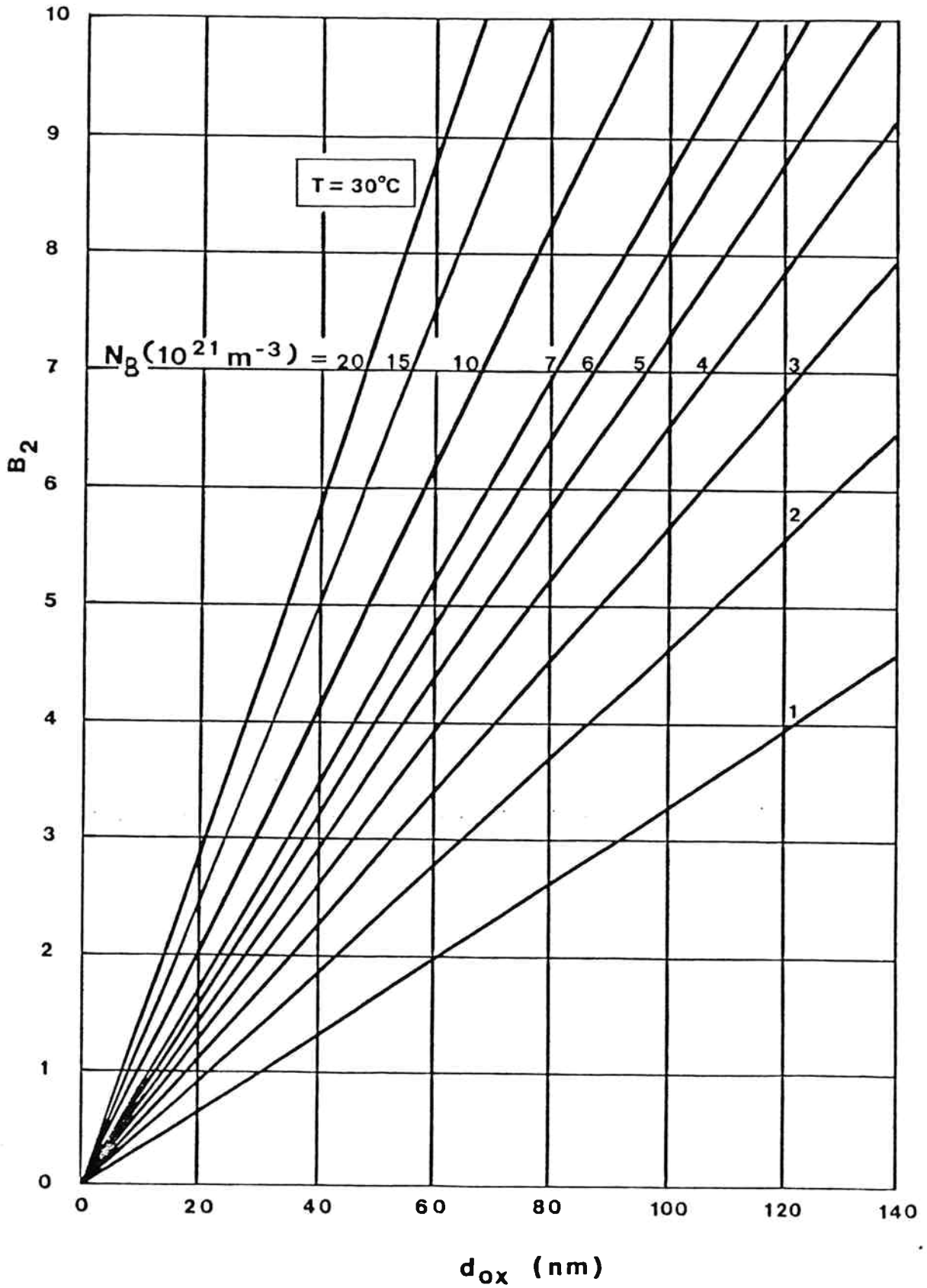


Fig. 2 Body factor $B_2 = K \sqrt{U_{TH}}$ as a function of oxide thickness d_{ox} for several substrate concentrations N_B .

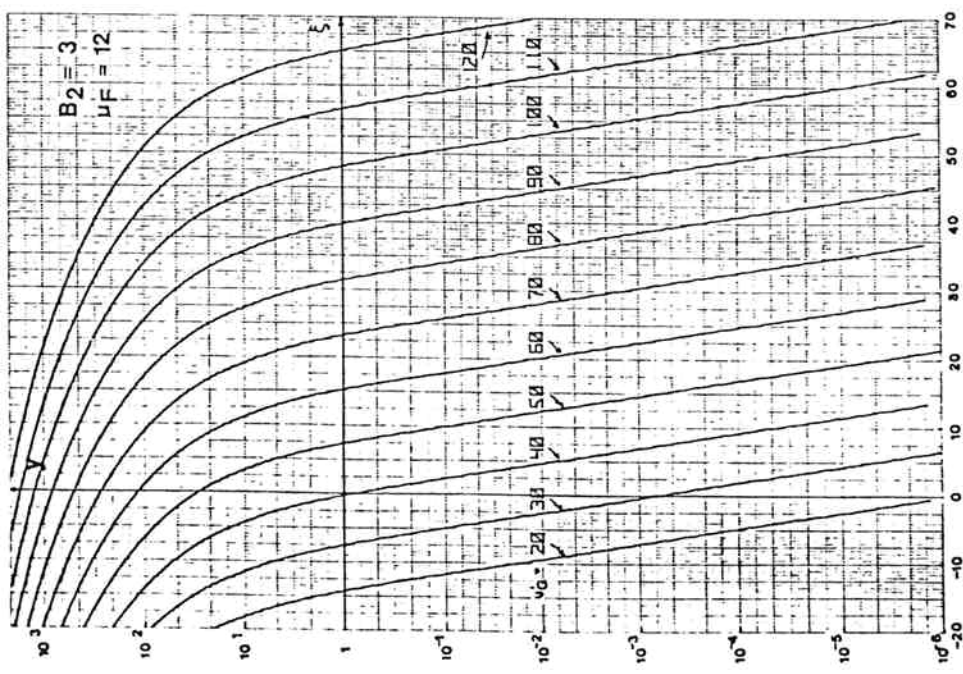
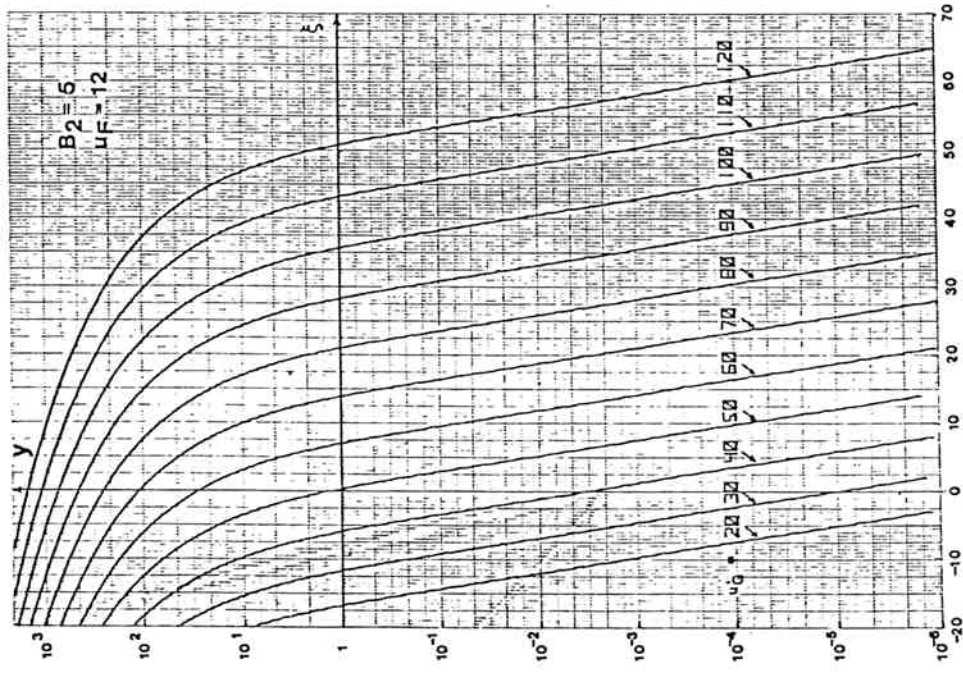
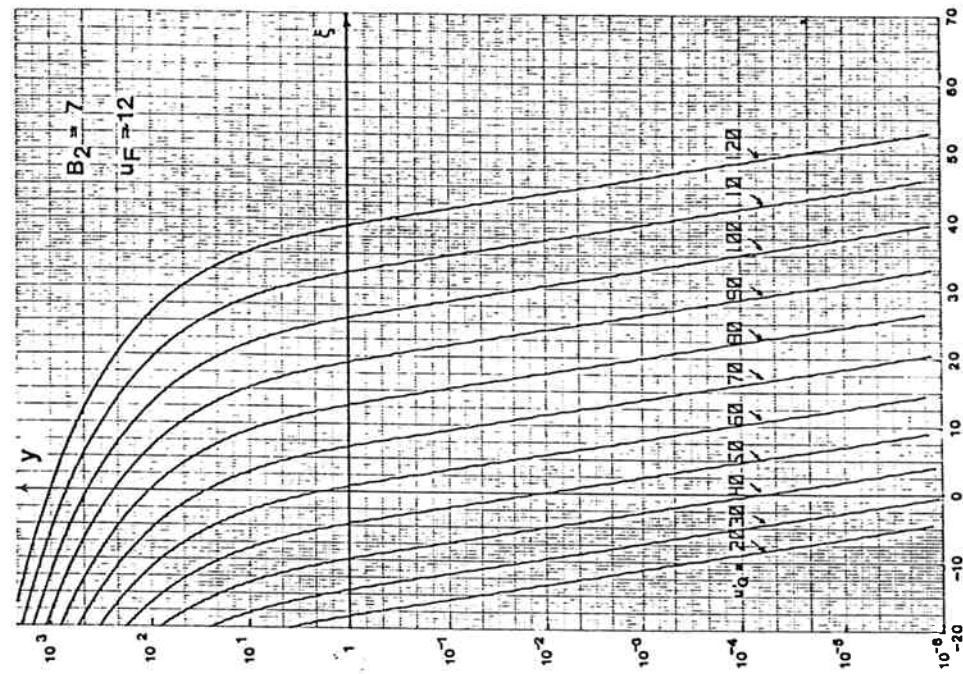


Fig. 3

Fig. 4

Fig. 5

Fig. 3-5. Normalized saturation current y as a function of the normalized channel potential ξ for 3 values of

$B_2 = K \sqrt{U_{TH}}$ and for $u_F = 12$. The same curves are valid for other values of u_F if they are shifted to maintain $2u_F + \xi$ constant.

It has been found useful to define a "control voltage" V_C or its equivalent normalized variable u_C . This voltage will be related to the surface potential u_{surf} in weak inversion but defined by the same mathematical function for the whole range of gate voltages:

$$u_C = u_{surf} \text{ weak inv.} - 2u_F$$

$$u_C = u_G' - 2u_F + \frac{B_2^2}{2} \left(1 - \sqrt{1 + \frac{4u_G' - 4}{B_2^2}} \right) \quad (26)$$

The derivative of this expression is also a meaningful quantity: it will be designed by γ :

$$\gamma = \frac{\partial u_C}{\partial u_G'} = 1 - \frac{1}{\sqrt{1 + \frac{4u_G' - 4}{B_2^2}}} \quad (27)$$

An important property of the surface potential in weak inversion is its independence from the carrier concentration, and from the potential ξ . Consequently, the inversion charge becomes

$$F_3 = \frac{1}{2} B_2 \exp(-2u_F - \xi) \int_{u_F}^{u_{surf}} \frac{\exp u \, du}{\sqrt{u - 1}}$$

Approximate integration is obtained after a series expansion of the denominator around $u = u_{surf}$.

The second integration in (20) is straightforward.

The current function becomes

$$y(u_C, \xi) = \frac{1}{2} B_2 \frac{\exp(u_C - \xi)}{\sqrt{u_C + 2u_F - 2}} \quad (28)$$

It is useful to introduce a "weak inversion function" f_w in accordance with the exponential character of y :

$$y = e^{2f_w} \quad (29)$$

$$\text{with } f_w = \frac{1}{2} \left[u_C - \xi - \frac{1}{2} \ln \frac{4(u_C + 2u_F - 2)}{B_2^2} \right] \quad (30)$$

In order to anticipate some discrepancy between the ideal structure assumed here and a real situation, we replace f_w by a general linear function of u_C and ξ :

$$f_w = a_w u_C - b_w \xi + c_w \quad (31)$$