Compact Modeling of Noise in the MOS Transistor

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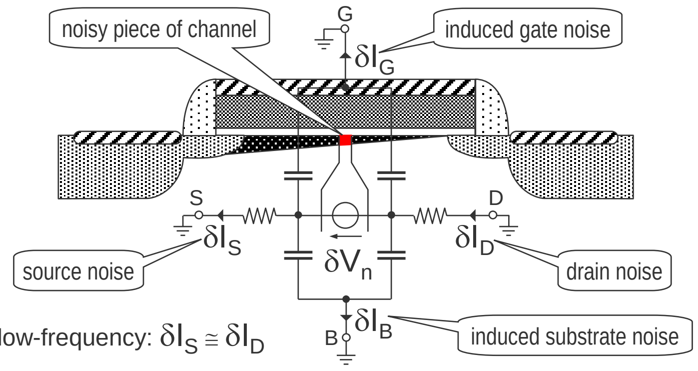
Outline

- Introduction (definitions)
- Effect of velocity saturation (VS)
- Effect of carrier heating (CH)
- Effect of mobility reduction due to vertical field (MRV)
- Effect of channel-length reduction (CLM)
- Conclusion

Introduction

- Noise sets the lower limit for signal amplification and detection, whereas upper limit is set by device non linearity
- Reduction of supply voltage in deep submicron CMOS technologies reduces upper limit and forces noise to become smaller at the cost of a higher power consumption
- Flicker noise largely dominates at low frequency (below the corner frequency), particularly for deep submicron CMOS
- Thermal noise dominates at HF and is hence important for RF IC design
- Thermal noise dominated by the intrinsic channel noise (~90%)
- It is therefore crucial to properly model thermal and flicker noise for analog and RF IC design

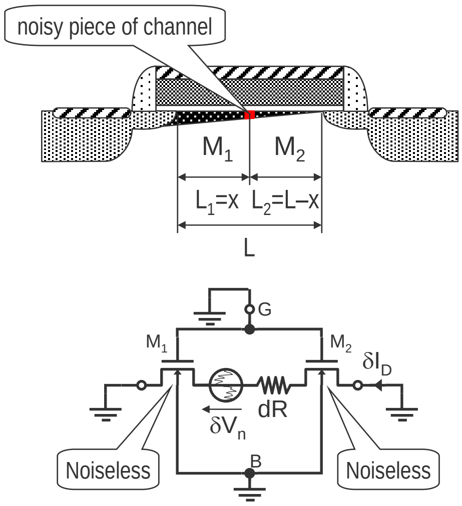
Channel Thermal Noise and Terminal Noise Currents



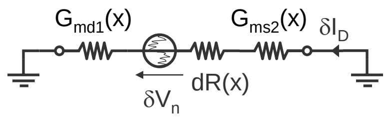
At low-frequency: $\delta I_S \cong \delta I_D$

- The thermal noise generated by voltage fluctuations in the channel appears at the drain, source, gate and bulk as terminal current fluctuations
- The channel voltage fluctuations are transferred to the drain and source through the (trans)conductances and to the gate and bulk by capacitive coupling

Two-Transistor Approach



Equivalent small-signal circuit



$$\delta I_D = G_{ch} \cdot \delta V_n$$

where G_{ch} is the channel conductance from point x

$$\frac{1}{G_{ch}} = \frac{1}{G_{md1}} + \frac{1}{G_{ms2}}$$

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Long-Channel Model

- PSD of drain current fluctuations δI_D due to δV_n

$$S_{\delta I_D} = G_{ch}^2 \cdot S_{\delta V_n} \quad \text{with} \quad S_{\delta V_n} = 4kT_C \cdot dR(x)$$

where T_C is the carrier temperature

- Long-channel → mobility μ_{eff} independent of lateral field E_x and no carrier heating (T_C=T_L) where T_L is the lattice temperature

$$dR = \frac{dx}{W \cdot (-Q_i) \cdot \mu_{eff}} \qquad G_{ch} = \frac{W}{L} \cdot (-Q_i) \cdot \mu_{eff} \qquad S_{\delta I_D} = 4kT_L \cdot \frac{W}{L^2} \cdot (-Q_i) \cdot \mu_{eff} \cdot dx$$

- Total noise PSD at the drain is then given by

$$S_{\Delta I_D} \equiv 4kT_L \cdot G_n \qquad G_n = \frac{W}{L^2} \cdot \mu_{eff} \cdot \int_0^L (-Q_i) \cdot dx = \frac{\mu_{eff}}{L^2} \cdot |Q_I|$$

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Short-Channel Model

- Mobility μ_{eff} depends on position and hence on the lateral field E_x
- Carrier get heated ($T_C \neq T_L$) and T_C depends also on position and hence on the lateral field E_x
- Has to be accounted for when evaluating dR and G_{ch}

$$dR = \frac{dx}{W \cdot (-Q_i) \cdot (\mu_{eff} + \mu'_{eff} \cdot E_x)} \qquad G_{ch} = \frac{W}{L} \cdot \frac{(-Q_i) \cdot \mu_{eff}}{1 + \frac{1}{L} \cdot \int_{V_S}^{V_D} \frac{\mu'_{eff} \cdot dV}{\mu_{eff} + \mu'_{eff} \cdot E_x}} \qquad \mu'_{eff} \equiv \frac{d\mu_{eff}}{dE_x}$$

- The PSD of the drain current fluctuations is now given by

$$G_n = \frac{W}{L^2} \cdot \frac{1}{\left(1 + \frac{1}{L} \cdot \int_{V_S}^{V_D} \frac{\mu'_{eff} \cdot dV}{E_x \cdot \mu'_{eff} + \mu_{eff}}\right)^2} \cdot \int_0^L \frac{\mu_{eff} \cdot (-Q_i)}{1 + \frac{E_x \cdot \mu'_{eff}}{\mu_{eff}}} \cdot \frac{T_C}{T_L} \cdot dx$$

Thermal Noise Parameter

- Several thermal noise excess factor can be defined
- The **thermal noise parameter** δ related to the drain is defined as

$$\delta \equiv \frac{G_n}{G_{dso}}$$

where G_{dso} is the **channel conductance** at V_{DS}=0

- Long-channel values δ = 1 for V_{DS} = 0 and δ_{sat} = 2/3 in saturation
- δ tells how much the thermal noise deviates from the value it takes when it operates like a resistor having a conductance G_{dso}
- δ compares noise at a given operating point to the noise at V_{DS}=0
- Mainly useful for **device modeling** but useless for circuit designers

Thermal Noise Excess Factor

- The thermal noise excess factor γ is defined as

$$\gamma \equiv \frac{G_n}{G_m}$$ where G_m is the gate transconductance

- γ shows how much noise is generated at the drain for a given G_m
- Contrary to δ, the noise conductance G_n and the transconductance G_m are evaluated at the same operating point
- γ is becoming large for small V_{DS}
- γ is related to δ by $\gamma \equiv \frac{G_n}{G_m} = \frac{G_n}{G_{dso}} \cdot \frac{G_{dso}}{G_m} = \delta \cdot \frac{n \cdot G_{dso}}{G_{ms} - G_{md}}$
- Where n is the slope factor with $n \cong 1.3 \ldots 1.6$
- In saturation $G_{md}=0$ and $G_{ms}=G_{dso}$ $\gamma_{sat} = n \cdot \delta_{sat} \cong 1.5 \cdot \frac{2}{3} = 1$

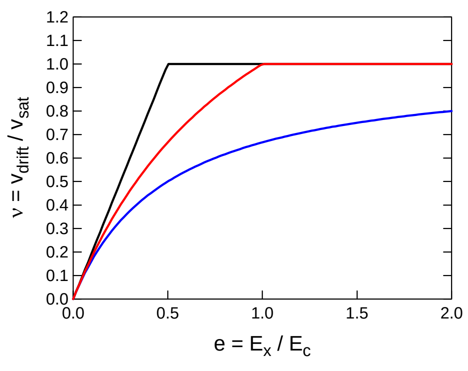
Values of δ Published in the Literature

- Very different values of δ for short channel devices have been published in the literature (old topic but still controversial today)
- Abidi (TED 86) presented measured values of δ as large as 7
- Scholten (IEDM 99) presented values of δ measured on several CMOS processes that are always smaller than 2
- Some authors pretend that velocity saturation is the only effect that should be accounted for and there is no need for carrier heating
- Chen (TED 02) attributes the increase of δ to the effect of channel length modulation
- Why bother?
  Noise factor strongly impacts noise performance of RF circuits (LNA)

Main Effects Affecting Channel Thermal Noise

- Better understand the different mechanisms affecting the MOST channel thermal noise
- The following effects will be considered:
  1. Velocity saturation (VS)
  2. Carrier heating (CH)
  3. Mobility reduction due to the vertical field (MRV)
  4. Channel length modulation (CLM)
- Evaluate the impact of each of these effects on δ and γ
- Each effect will be analyzed separately

VS and Velocity-Field Models



$$\nu \equiv \frac{v_{drift}}{v_{sat}} = \begin{cases} 2 \cdot e & for\ e < 0.5 \\ 1 & for\ e \geq 0.5 \end{cases}$$

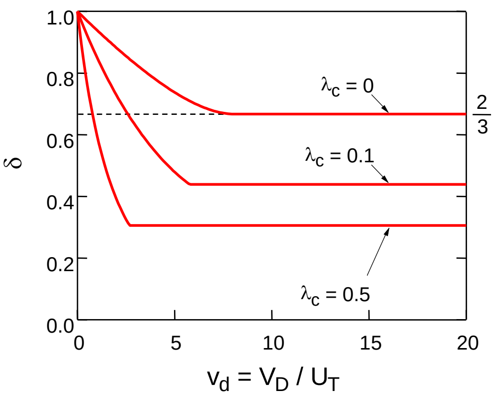
$$\nu \equiv \frac{v_{drift}}{v_{sat}} = \frac{2 \cdot e}{1 + 2 \cdot e}$$

$$\nu \equiv \frac{v_{drift}}{v_{sat}} = \begin{cases} \dfrac{2 \cdot e}{1 + e} & for\ e < 1 \\ 1 & for\ e \geq 1 \end{cases}$$

$$e \equiv \frac{E_x}{E_c} \qquad E_c \equiv \frac{2 \cdot v_{sat}}{\mu}$$

$$\lambda_c \equiv \frac{2 \cdot U_T}{E_c \cdot L} \cong 0.5\ for\ L = 0.1\mu m \qquad U_T \equiv \frac{k \cdot T}{q}$$

Effect of VS on δ



$$\delta \equiv \frac{G_n}{G_{dso}}$$

$$\lambda_c \equiv \frac{2 \cdot U_T}{E_c \cdot L}$$

- G_{dso} not affected by velocity because defined at $V_{DS}=0$
- δ becomes smaller than the long-channel value 2/3

1^{st}-Order Approximation of G_n

- From full expression

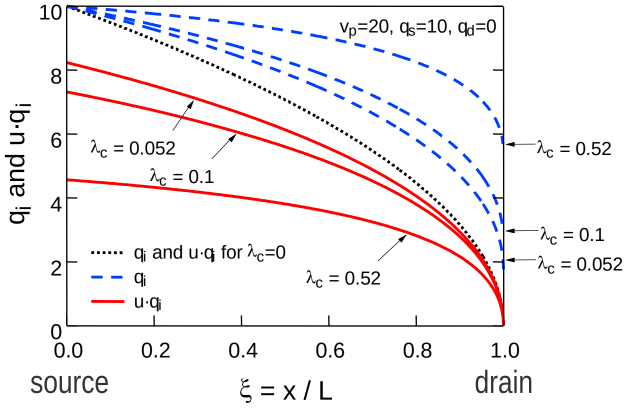
$$G_n = \frac{W}{L^2} \cdot \frac{1}{\left(1 + \frac{1}{L} \cdot \int_{V_S}^{V_D} \frac{\mu'_{eff} \cdot dV}{E_x \cdot \mu'_{eff} + \mu_{eff}}\right)^2} \cdot \int_0^L \frac{\mu_{eff} \cdot (-Q_i)}{1 + \frac{E_x \cdot \mu'_{eff}}{\mu_{eff}}} \cdot \frac{T_C}{T_L} \cdot dx$$

- Neglecting carrier heating ($T_C=T_L$) and the μ'_{eff} term, leads to

$$G_n \approx \frac{W}{L^2} \cdot \int_0^L \mu_{eff}(x) \cdot (-Q_i(x)) \cdot dx$$

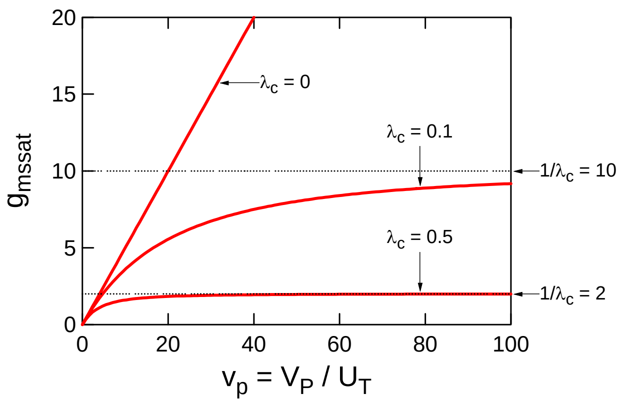
- Thermal noise conductance is the integral of the product of the mobility and the inversion charge along the channel

Effect of VS on Charge and Mobility Profiles



- Inversion charge increases due to velocity saturation
- The $\mu_{eff} \cdot Q_i$ product and hence G_n decreases due to VS

Effect of VS on Transconductances

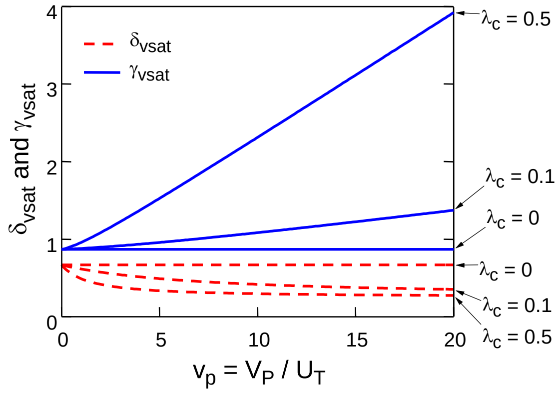


V_P pinch-off voltage

$$V_P \cong \frac{V_G - V_{T0}}{n}$$

- Transconductance is limited by velocity saturation
- This will affect the γ noise excess factor

Effect of VS on δ and γ (in saturation)



$$\gamma \equiv \frac{G_n}{G_m}$$

- Because of G_m degradation due to velocity saturation, the γ noise excess increases to values much larger than the long-channel value

Carrier Heating (CH)

- Effect of velocity saturation is closely linked to carrier heating
- Carrier temperature T_C corresponding to mobility model given by

$$\frac{T_C}{T_L} = \left(1 + \frac{E_x}{E_c}\right)^2 = (1+e)^2 \qquad \text{where} \qquad e \equiv \frac{E_x}{E_c}$$

- Introducing temperature dependance into the noise conductance, G_n can then be split into

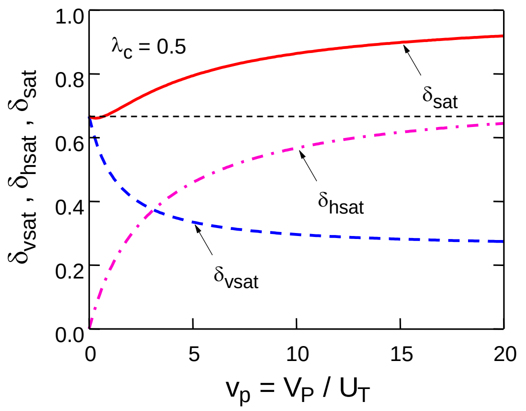
$$G_n = G_{nv} + G_{nh}$$

where G_{nv} accounts for VS only and G_{nh} for additional CH

- Same splitting can be applied to the δ and γ noise factors

$$\delta = \delta_v + \delta_h \qquad \gamma = \gamma_v + \gamma_h$$

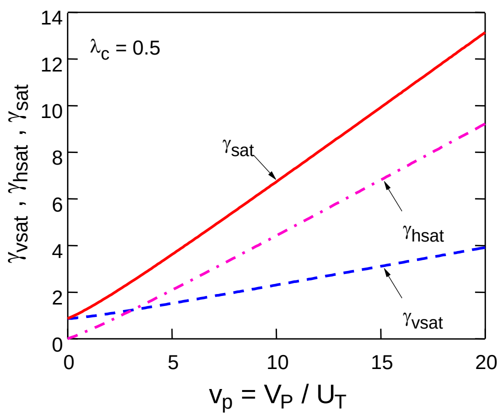
Combined Effects of VS and CH on δ (saturation)



$$\delta_{sat} = \delta_{vsat} + \delta_{hsat}$$

- Carrier heating increases δ_{sat} which gets back to a value slightly larger than the long-channel value 2/3

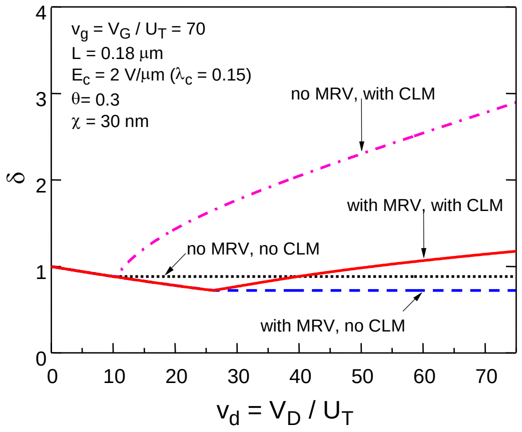
Combined Effects of VS and CH on γ (saturation)



$$\gamma_{sat} = \gamma_{vsat} + \gamma_{hsat}$$

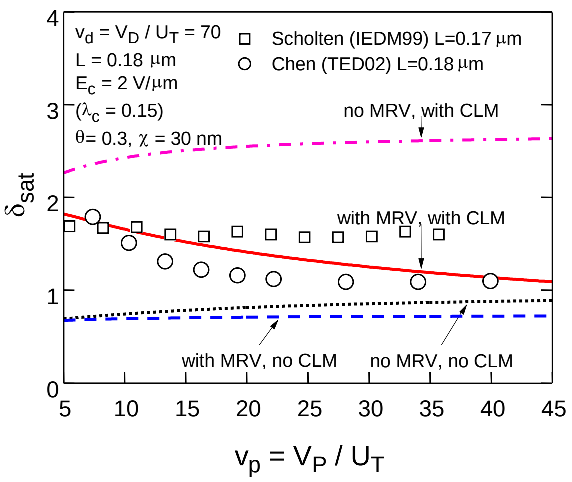
- Carrier heating even further increases γ_{sat} to values much larger than the long-channel value (n·2/3 ≈ 1)

MRV and CLM Effects on δ

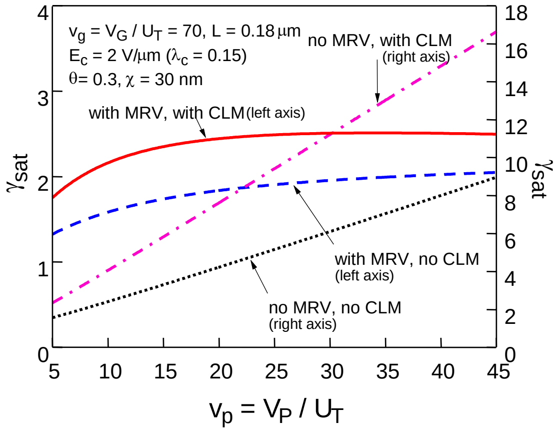


- MRV tends to decrease δ, whereas CLM increases δ, resulting in a value close to that obtained with VS and CH only

MRV and CLM Effects on δ (in saturation)



MRV and CLM Effects on γ (in saturation)



- CLM tends to increase γ further, whereas MRV reduces it back to values close to about 2

Conclusion

- VS, CH, MRV and CLM effects have been accounted for when evaluating the channel thermal noise and the noise factors
- Two different noise factors δ and γ to be considered that are differently affected by short channel effects
- δ is usefull for device modeling but useless for circuit design, whereas γ is more dedicated to circuit design
- VS and CH have opposite effects on δ but lead to an increase of γ
- MRV and CH have compensating effects on both δ and γ
- Accounting for all 4 effects leads to values of δ (~ 2) close to those measured by Scholten and Chen on several CMOS processes