

Physics-based model for partially and fully ballistic nanoscale MOSFETs

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Motivation

Traditional models lack the description of the **transition from drift-diffusion to ballistic transport regime**. Indeed, the far from equilibrium transport is commonly described by the introduction of electron heating, but this approach is not valid for totally ballistic MOSFETs, where the carriers fly through the channel without scattering at all .

We expect that ballistic or partially ballistic transport regime will be important in deca-nanometers devices and it is already important for high mobility FETs.

Another aspect that we will consider is **degeneracy**. It is remarkable that the **Fermi-Dirac statistics** is not considered in any MOSFET model as well, but it can be important for very thin devices and low temperatures.

A remark

EKV-like models (linearized charge models and opposite current fluxes)

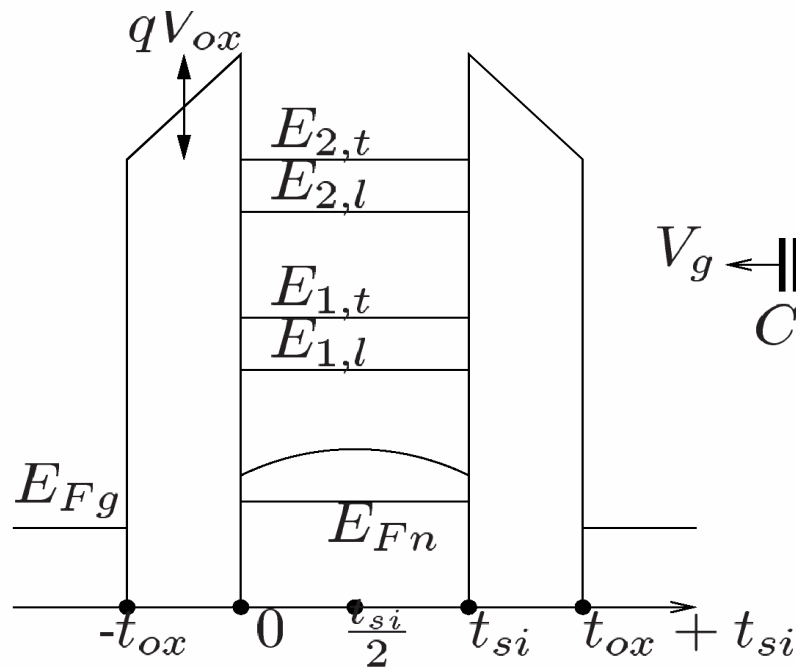
- **EKV**
- **ACM**
- **USIM**
- **UCCM**
- **Maher-Mead model**
- **and so on!...**

Ballistic DGMOSFET in a closed form

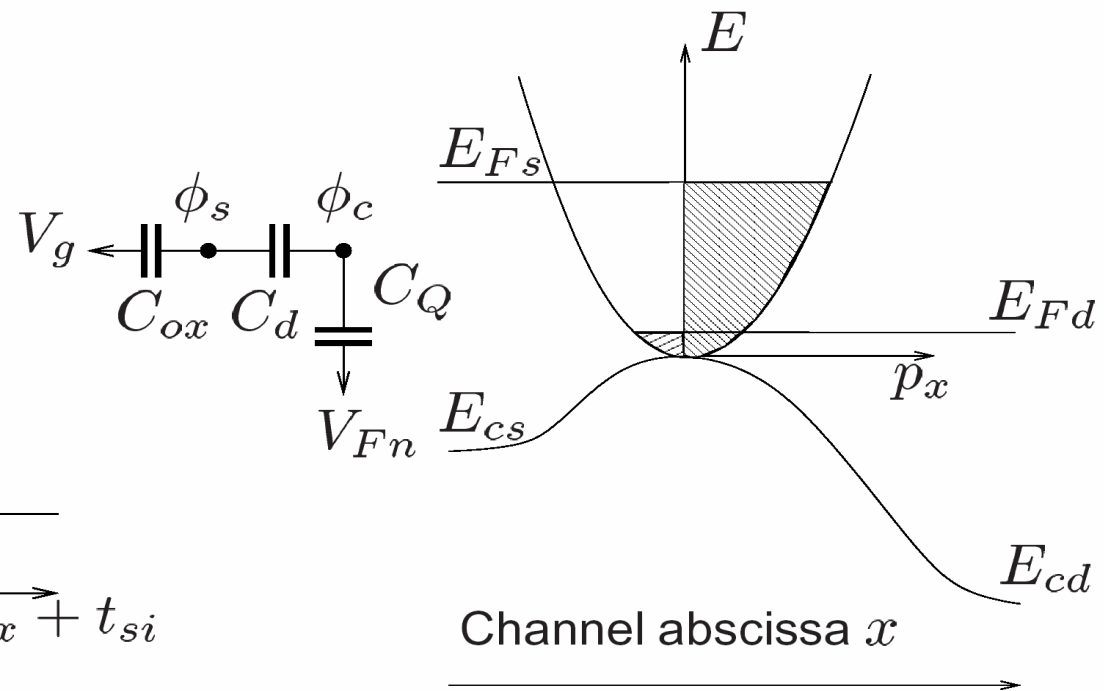
Starting from the Natori theory [Natori, 1994] of ballistic FETs, two hemi-maxwellian carrier populations are present on the peak of the barrier:

$$2C_g(V_g - \phi_m + \chi - \phi_c) - Q_b = \frac{qN_c}{2} \left[e^{\frac{\phi_c - V_s}{\phi_t}} + e^{\frac{\phi_c - V_d}{\phi_t}} \right]$$

$$I_{ds} = I_f - I_r = \frac{qN_c}{2} v_{th} \left[e^{\frac{\phi_c - V_s}{\phi_t}} - e^{\frac{\phi_c - V_d}{\phi_t}} \right]$$



Rectangular quantum well!



The total density of electron states in all conduction sub-bands:

$$N_c \equiv \sum_{n=1} \sum_{k=t,l} N_{n,k} \exp \left(-\frac{\epsilon_{n,k}}{\phi_t} \right)$$

Interestingly the vertical electrostatics can be written in a form that is similar to the EKV-like electrostatics:

$$V_g - V_T - V_m = \frac{Q_m}{2C_g} + \phi_t \log \frac{Q_m}{qN_c}$$

Where:

$$V_m \equiv -\phi_t \log \left(\frac{\exp \left(-\frac{V_s}{\phi_t} \right) + \exp \left(-\frac{V_d}{\phi_t} \right)}{2} \right)$$

Analytic solution through the Lambert-function [Corless,1993]:

$$\phi_c = V_g - V_0 - \phi_t W \left[\frac{qN_c}{Q_n} \exp \left(\frac{V_g - V_{Fn} - V_0}{\phi_t} \right) \right]$$

$$V_T \equiv V_0 + \frac{k_B T}{q} \log \left(\frac{Q_n}{qN_c} \right), V_0 = \phi_m - \chi + \frac{Q_b}{2C_g}$$

that gives:

Q_m on the peak:

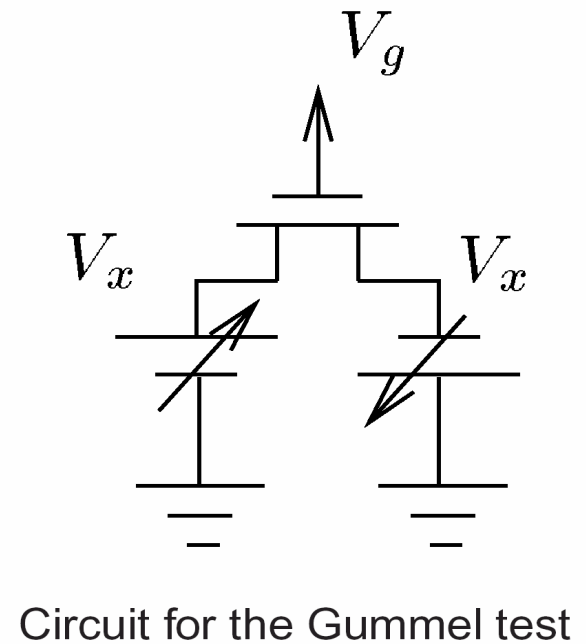
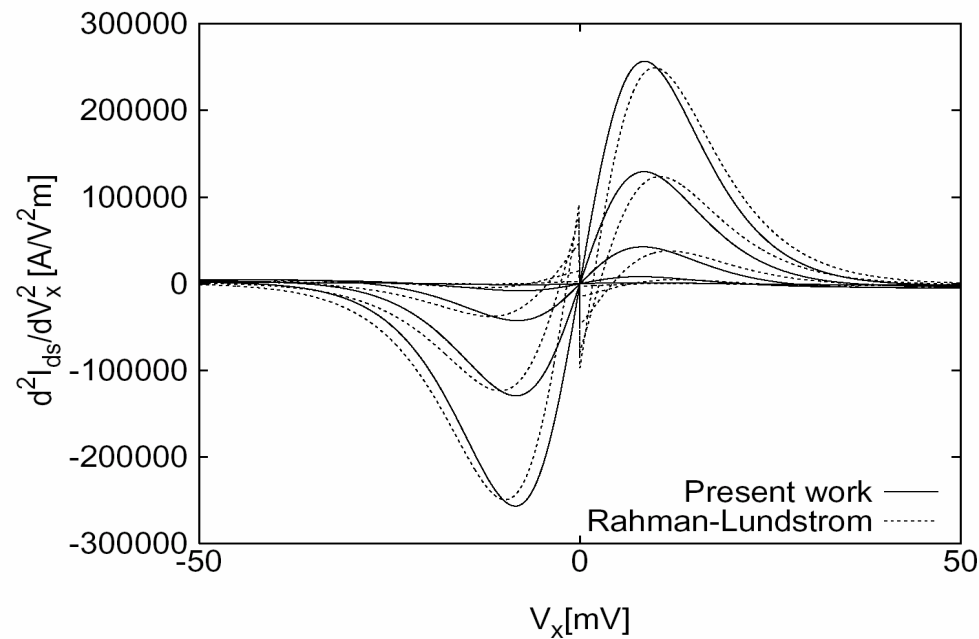
$$Q_m = Q_n W \left(\frac{e^{\frac{V_g - V_s - V_T}{\phi_t}} + e^{\frac{V_g - V_d - V_T}{\phi_t}}}{2} \right)$$

current I_{ds} :

$$I_{ds} = Q_n W \left(\frac{e^{\frac{V_g - V_s - V_T}{\phi_t}} + e^{\frac{V_g - V_d - V_T}{\phi_t}}}{2} \right) v_{th} \tanh \left(\frac{V_{ds}}{2\phi_t} \right)$$

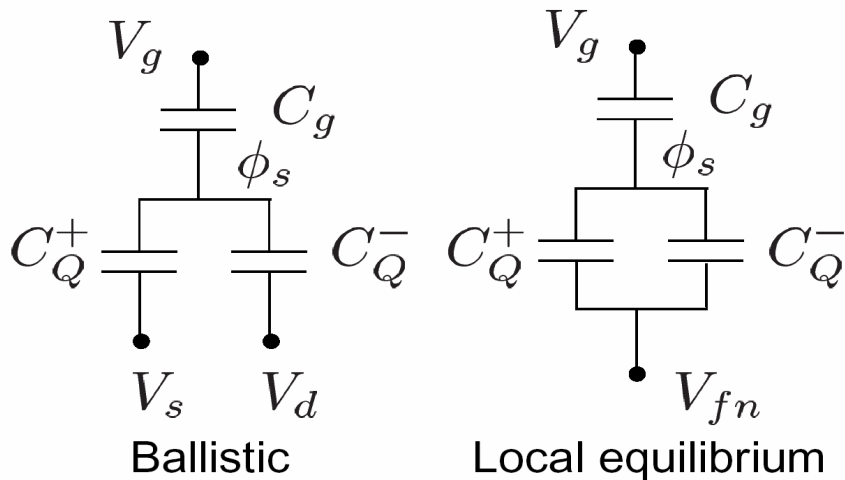
New ballistic MOSFET model is similar to Natori-Lundstrom model. Some benefits:

- Explicit equations
- fully symmetrical form. The Natori-Lundstrom model does not pass the Gummel symmetry test:



Vertical electrostatics and local equilibrium

The vertical ballistic electrostatics is fully consistent with local equilibrium transport (i.e. drift-diffusion transport with uniform mobility).



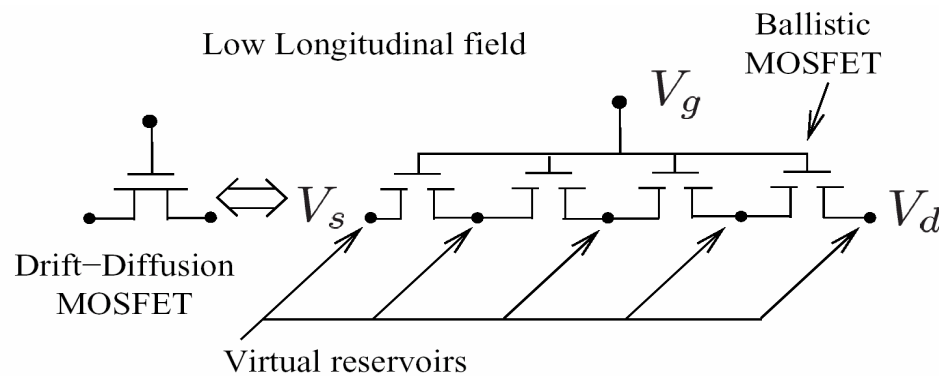
If $V_s = V_d = V_{Fn}$ (V_{Fn} the local quasi Fermi potential):

$$V_g - V_T - V_{Fn} = \frac{Q_m}{2C_g} + \phi_t \log \frac{Q_m}{qN_c}$$

same electrostatics used in the EKV model and similar models!

Ballistic Segmentation of a drift-diffusion channel

Consistently with the approach of Buttiker probes [Buttiker,1986], a drift-diffusion MOSFET can be interpreted as a long enough chain of ballistic transistors. Fermi potentials are defined *only* at the k -th contact.



If N is the number of MOSFETs in the chain, current continuity imposes N equations for the N unknowns:

$$N - 1 \text{ internal Fermi levels } V_k (k = 1..N - 1) \\ + \\ 1 \text{ the unknown current } I_{ds}$$

For the k -th ballistic MOSFET:

$$I_{ds} = v_{th} Q_n W \left[\frac{e^{\frac{V_g - V_k - V_T}{\phi_t}} + e^{\frac{V_g - V_{k-1} - V_T}{\phi_t}}}{2} \right] \tanh \left(\frac{\Delta V_k}{2\phi_t} \right) \text{ with } k = 1..N$$

If N is large enough and $\Delta V_k = V_k - V_{k-1}$ is small enough:

- (discrete) $V_k \longrightarrow$ (continuous) V_{Fn}
- (nonlinear) $\tanh(x) \longrightarrow$ (linear) x
- (discrete) $Q_{m,k} v_{th} \tanh \left(\frac{\Delta V_k}{2\phi_t} \right) \longrightarrow$ (continuous and linearized!) $\mu_n Q_m \nabla V_{Fn}$

$$\mu_{no} \equiv \frac{\lambda v_{th}}{2\phi_t}, \text{ where } \lambda \text{ is the mean free path.}$$

For a long channel drift-diffusion MOSFET:

$$V_g - V_T - V_s = \frac{Q_{ms}}{2C_g} + \phi_t \log \frac{Q_{ms}}{qN_c}$$

$$V_g - V_T - V_d = \frac{Q_{md}}{2C_g} + \phi_t \log \frac{Q_{md}}{qN_c}$$

$$I_d = \frac{\mu_n Q_n \phi_t}{L} \left(\frac{Q_{ms}^2 - Q_{md}^2}{2Q_n^2} + \frac{Q_{ms} - Q_{md}}{Q_n} \right)$$

***EKV-like model for non-degenerate long DGMOSFETs
subject to rectangular quantum confinement!***

Effects of non-linear ballistic transport on the mobility

If $\Delta V_k \gg \phi_t$ somewhere in the chain



the nonlinear $\tanh(x)$ can not be linearized! (ballistic limit)



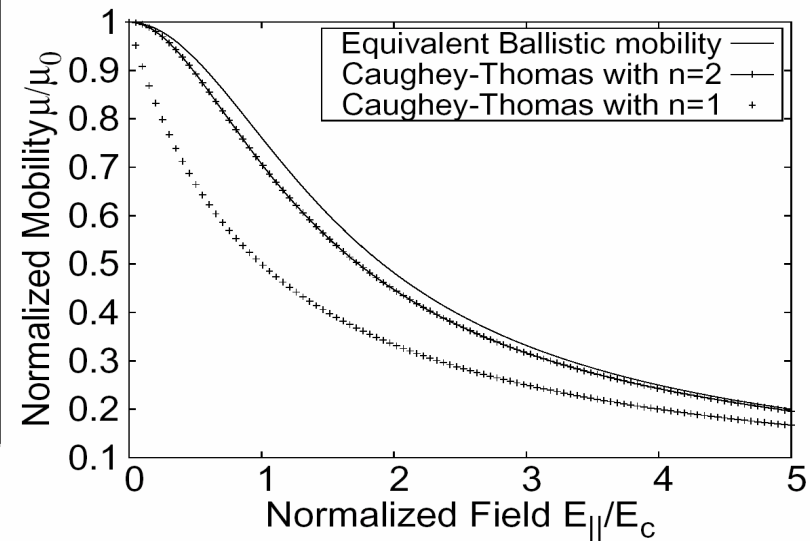
Limitation of the maximum current



Reduction of the effective mobility μ_n

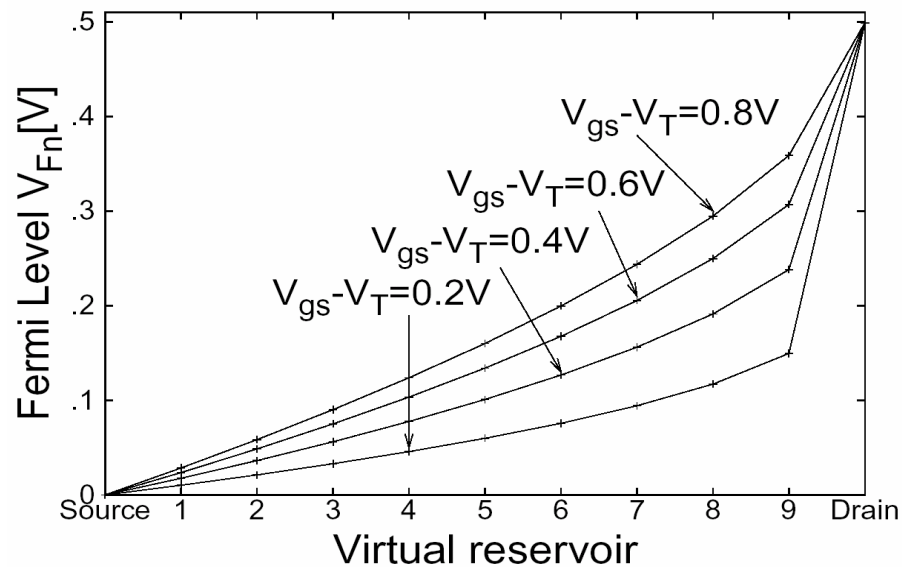
$$\mu_n = v_{th} \frac{\tanh\left(\frac{\lambda}{2\phi_t} \frac{dV_{Fn}}{dx}\right)}{\frac{dV_{Fn}}{dx}}$$

$\frac{2\phi_t}{\lambda}$ can be interpreted as a “critical field”, analogous to the critical field in the saturation velocity effect.



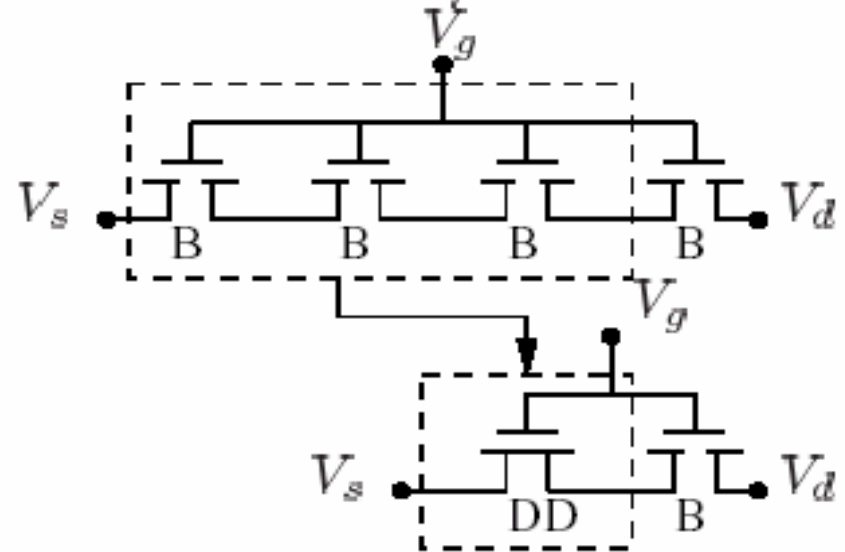
Compact macromodel for the ballistic chain

$$N = L/\lambda = 10, V_{ds} = 0.5V$$

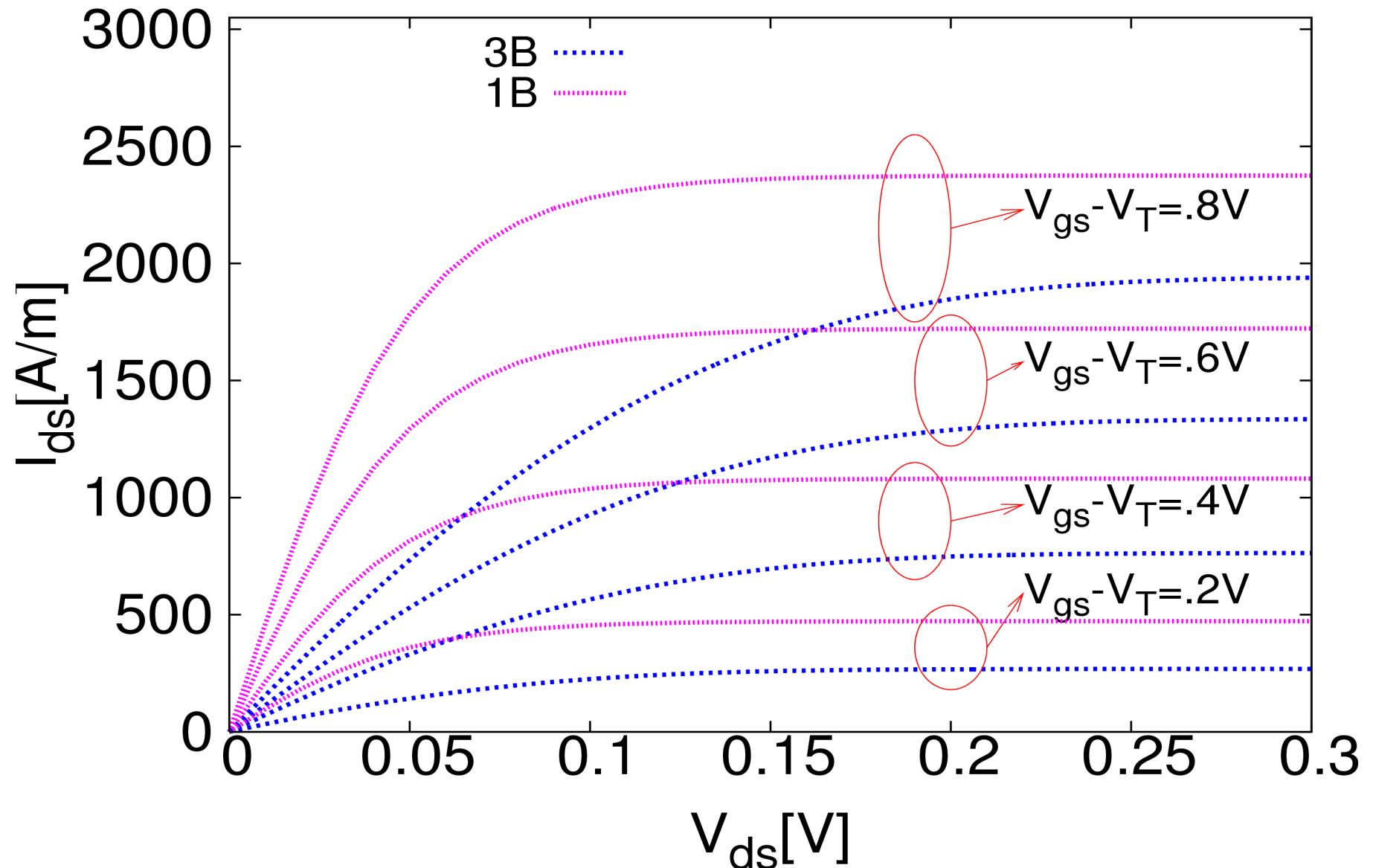


When non-linear transport emerges, it manifests its effects mainly in the last ballistic transistor of the chain.

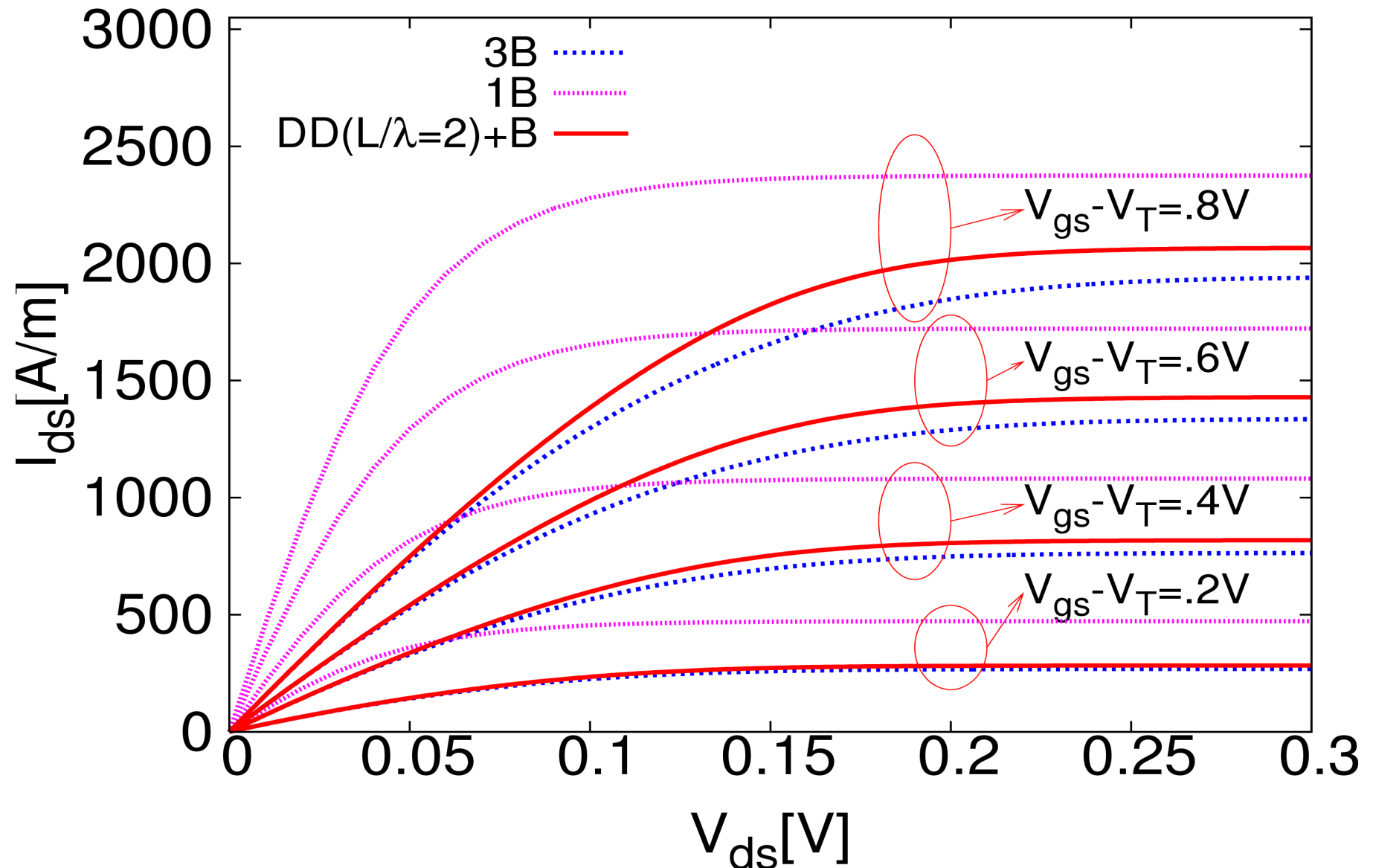
The first $N - 1$ ballistic transistors can be aggregated in an equivalent drift-diffusion transistor with ratio $L/\lambda = N - 1$.



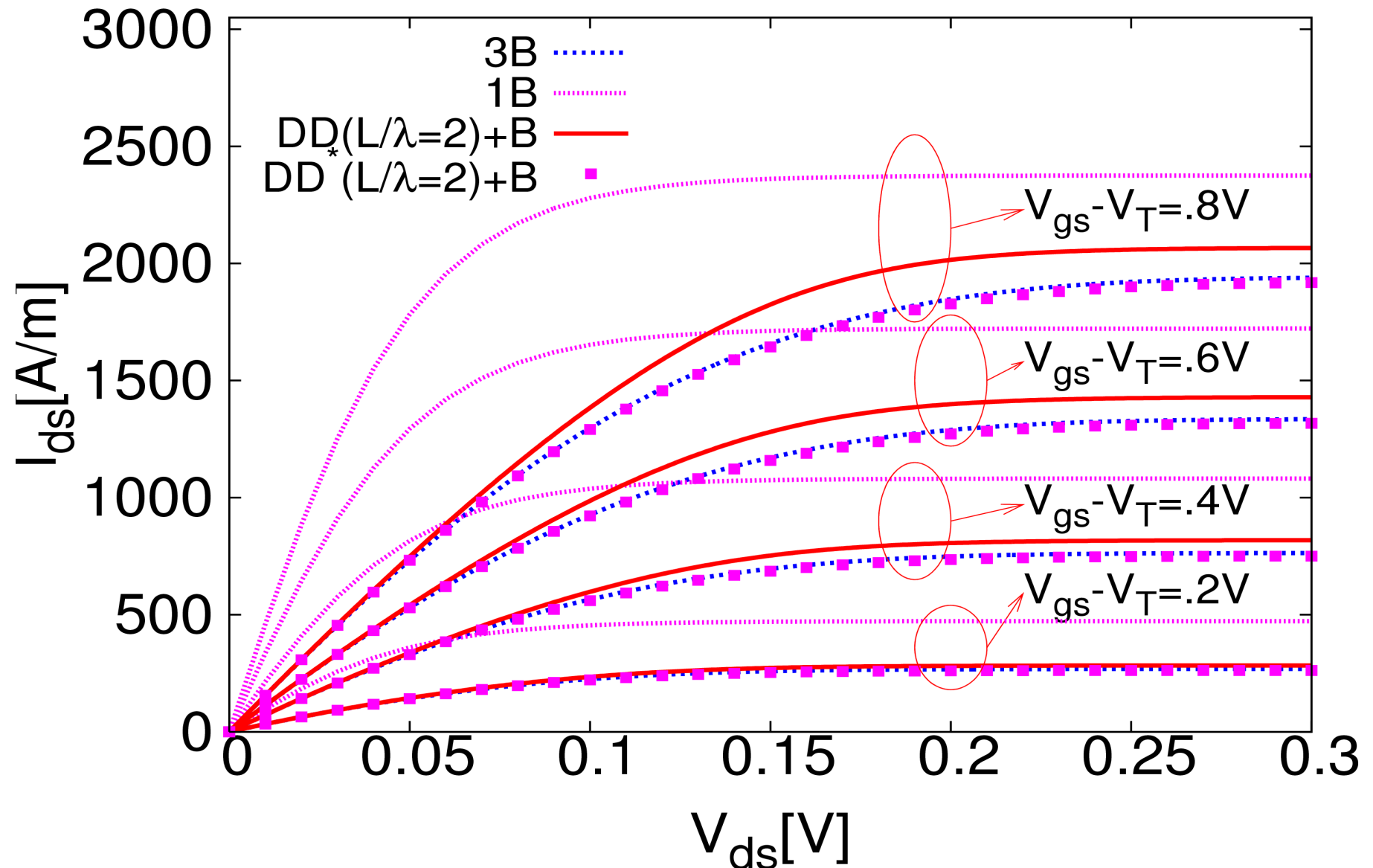
Comparison of the macromodel with the ballistic chain



Comparison of the macromodel with the ballistic chain



Comparison of the macromodel with the ballistic chain



The model can be improved if the weakly non-linear transport in the drift-diffusion section is considered.

- The mobility

$$\mu_n = v_{th} \frac{\tanh\left(\frac{\lambda}{2\phi_t} \frac{dV_{Fn}}{dx}\right)}{\frac{dV_{Fn}}{dx}}$$

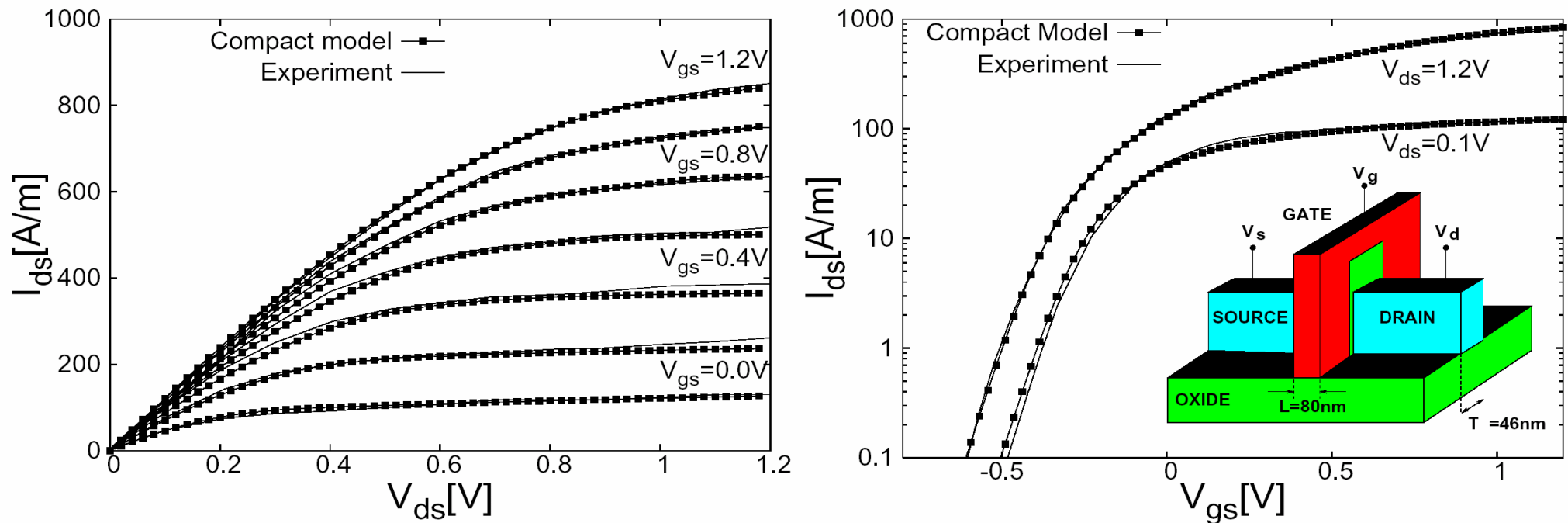
is introduced in the DD section *without any clamping function!*

- The ballistic section acts as the smoothed clamping in traditional models.

Other 2-nd order effects considered:

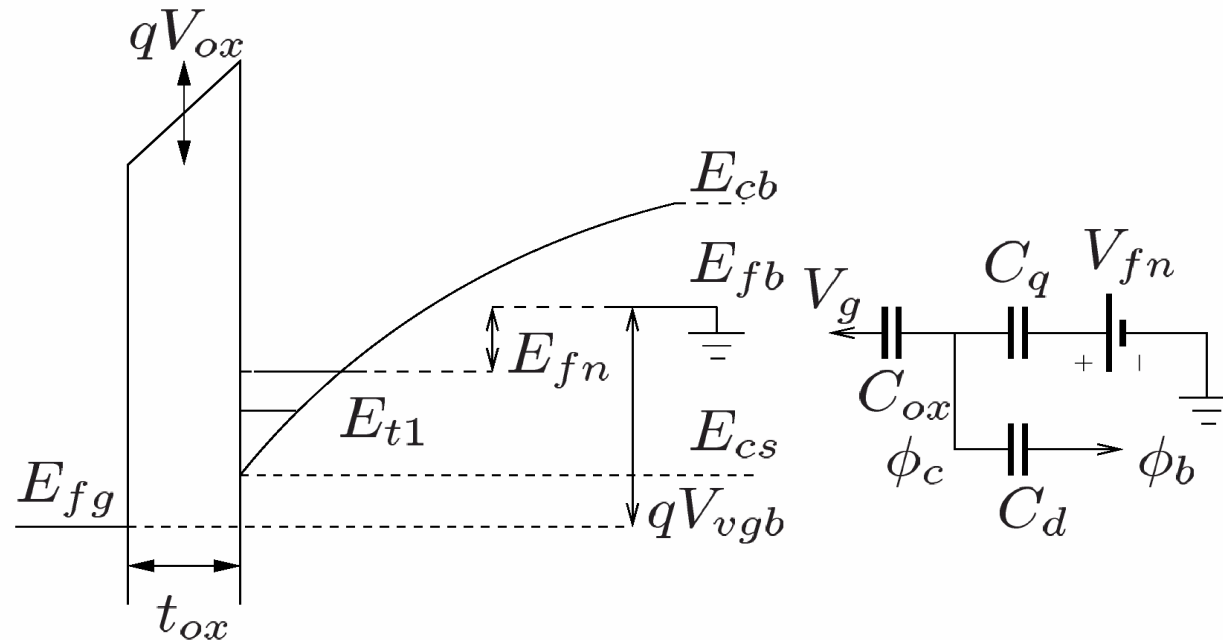
- 2D geometrical capacitances for DIBL and short channel effects (2 or 4 parameters);
- degradation of mobility in the DD section, caused by high vertical fields (1 parameter)[Iniguez et al.,1996];
- polysilicon depletion effect (1 parameter)[Lallement et al.,2000]

Fitting of experimental curves of a FinFET



Experimental curves of a FinFET [Yu et al.,2002] with 80 nm

Bulk MOSFET under the effects of Fermi-Dirac statistics



FD statistics degrades vertical electrostatics \implies quantum capacitance, [Luryi, 1988]:

$$V_g - \phi_c - (\phi_m - \chi) = \frac{qN_{1,l}}{C_{ox}} \left[\frac{\mathfrak{F}_0(\eta_{Fs}) + \mathfrak{F}_0(\eta_{Fd})}{2} \right] + \gamma \sqrt{\phi_c - \phi_b} \quad \text{B section}$$

$$V_g - \phi_c - (\phi_m - \chi) = \frac{qN_{1,l}}{C_{ox}} \mathfrak{F}_0(\eta_{Fn}) + \gamma \sqrt{\phi_c - \phi_b} \quad \text{DD section}$$

It is verified that the DD+B decomposition still works

Ballistic section

$$I_{ds} = \frac{qN_{1l}}{2} v_{th} \left[\mathfrak{S}_{\frac{1}{2}}(\eta_{Fs}) - \mathfrak{S}_{\frac{1}{2}}(\eta_{Fd}) \right].$$



Büttiker Probes Approach



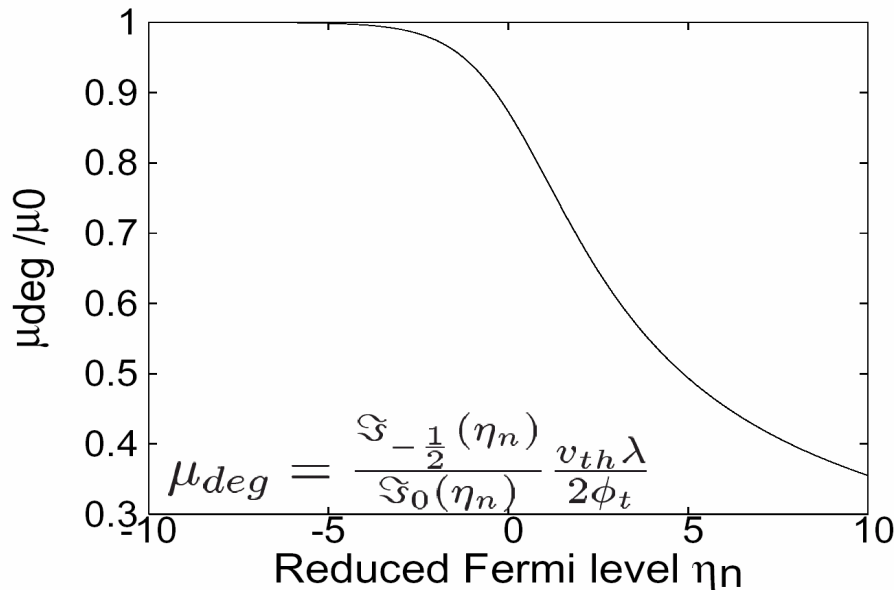
Drift-Diffusion section

$$I_{ds}^{(DD)} = \frac{v_{th}}{2\phi_t} \left(\frac{\lambda}{L} \right) \int_{V_s}^{V_d} \frac{\mathfrak{S}_{-\frac{1}{2}}(\eta_F)}{\mathfrak{S}_0(\eta_F)} Q_m(V_{Fn}) dV_{Fn}$$

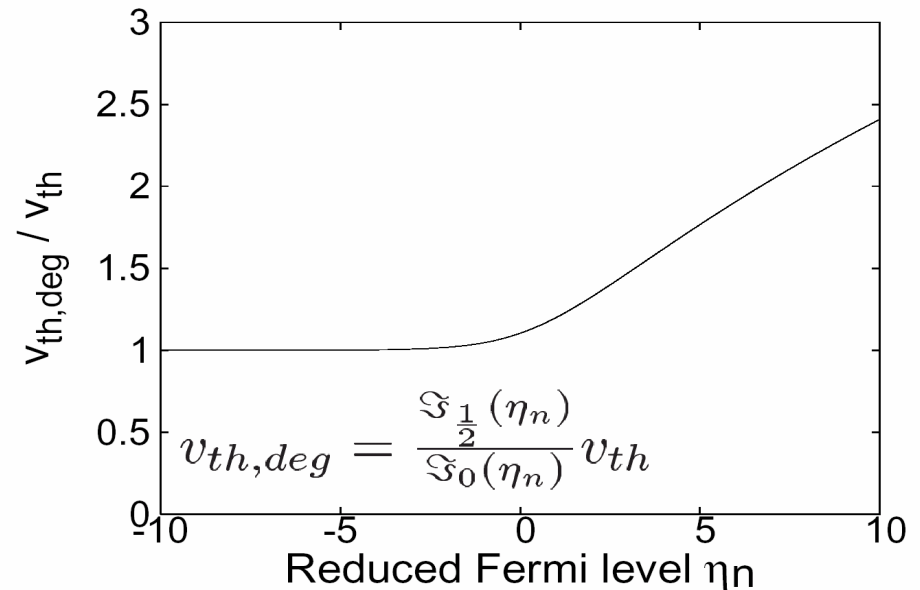
The transcendental integrals for the DD section can be simplified with a linearization near a suitable point: $\phi_c^* = \frac{\phi_{c,s} + \phi_{c,d}}{2} \iff$ Symmetrical linearization *a la* Goldenblat et al.

$$\mu_{deg} = \frac{\mathfrak{S}_{-\frac{1}{2}}(\eta_n)}{\mathfrak{S}_0(\eta_n)} \frac{v_{th}\lambda}{2\phi_t} = \frac{\mathfrak{S}_{-\frac{1}{2}}(\eta_n)}{\mathfrak{S}_0(\eta_n)} \mu_{clas}$$

If an uniform λ along the channel is assumed:



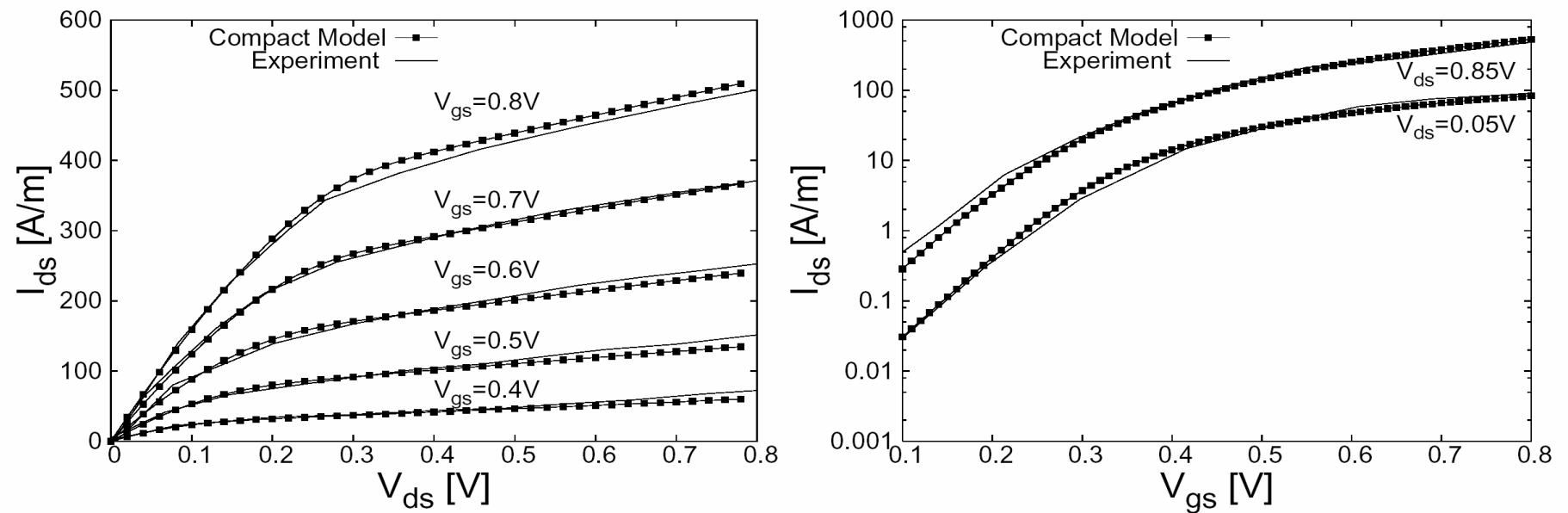
FD statistics *degrades* the mobility of DD section!



FD statistics *improves* the thermal velocity of B section!

$$\text{Reduced Fermi level } \eta_n = \frac{E_{Fn} - E_c - \Delta E_{1,l}}{\phi_t}$$

Comparison between compact model and experimental output characteristics from a Bulk MOSFET with $L=30\text{nm}$ [Doyle et al.,2002]



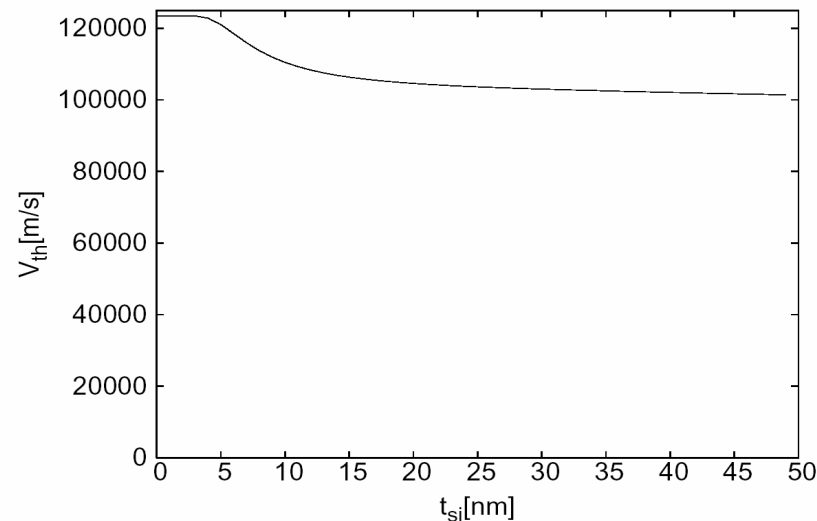
Conclusion

- **We have proposed an analytical model with the following features:**
 - **Description of transport from ballistic to drift-diffusion**
 - **Inclusion of Fermi-Dirac statistics**
 - **Different device architectures**
 - **Alternative interpretation and implementation of the saturation velocity effect.**
- **Work in progress and perspectives**
 - **Noise models in nanoMOSFETs in the case of partially ballistic transport**
 - **Analytical model of transport in nanowire transistors**
 - **Quantum confinement in not ultra thin body MOSFETs**

Effect of the anisotropy of the effective mass tensor

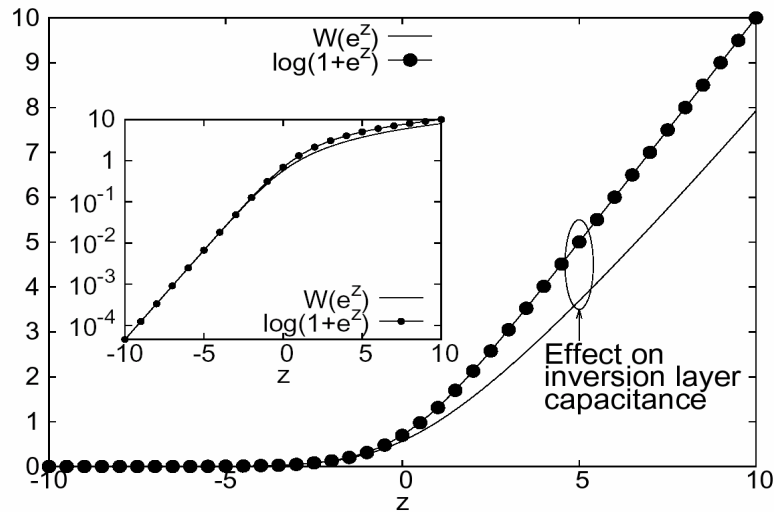
If the anisotropy of the effective mass tensor is considered, the unidirectional thermal velocity is found to vary slowly with the silicon thickness:

$$v_{th}^* \equiv \frac{N_{1,l} \sqrt{\frac{2kT}{\pi m_t}} \sum_n e^{-\frac{\varepsilon_{n,l}}{\phi_t}} + N_{1,t} \left(\sqrt{\frac{2kT}{\pi m_t}} + \sqrt{\frac{2kT}{\pi m_l}} \right) \sum_n e^{-\frac{\varepsilon_{n,t}}{\phi_t}}}{N_{1,l} \sum_n e^{-\frac{\varepsilon_{n,l}}{\phi_t}} + 2N_{1,t} \sum_n e^{-\frac{\varepsilon_{n,t}}{\phi_t}}}$$



Remarkably, v_{th} does not depend on bias!

On Lambert W-function



Behavior of function $W(\exp(z))$ compared with $\log(1 + \exp(z))$ used in the Baccarani-Reggiani compact model [Baccarani, 1999]

The Lambert $W(x)$ function satisfies:

$$W e^W = x$$

and then

$$W(e^z) + \log W(e^z) = z$$

An useful property:

$$\int W(e^z) dz = \frac{W^2(e^z)}{2} + (e^z)$$