The EKV MOSFET Model v3.0

Polysilicon Depletion & Quantum Effects

Polysilicon Gate Depletion:

- Potential & Charge Balance in the MOSFET Including Poly Depletion
- Pinch-Off Surface Potential & Inversion Charge
- Linearisation of the Inversion Charge vs. Surface Potential
- Poly Depletion in Accumulation (MOSCAP's)

Quantum Mechanical Effects:

- Total Charge in the Semiconductor Including Quantum Effects, "Bandgap Widening"
- Extension of the General Charge-Voltage Expression
- Evaluation of the Quantum Correction
- The "Electric" Oxide Capacitance
- Extension of the EKV Current-Voltage Expression

Summary

Charge Balance in the MOS Structure

$$Q_g = -(Q_{ox} + Q_i + Q_b)$$

- for an NMOS in inversion (N-type polysilicon gate), $Q_g > 0$ is entirely produced by the fixed charges in the depleted layer at the lower face of the gate

- potential drop $\Delta\Psi_g$ across the depleted gate layer can be evaluated using the depletion approximation:

$$Q_g = C_{ox} \cdot \Gamma_g \cdot \sqrt{\Delta\Psi_g}$$

$$\Gamma_g = \frac{\sqrt{2 \cdot q \cdot \varepsilon_{si} \cdot N_g}}{C_{ox}}$$: gate modulation factor

that is:

$$\Delta\Psi_g = \left(\frac{Q_g}{\Gamma_g \cdot C_{ox}}\right)^2$$

Potential Balance in the MOS Structure

$$V_G = \Phi_{ms} + \Psi_s + \Delta\Psi_{ox} + \Delta\Psi_g$$

(voltages referred to bulk)

- replacing :

$$V_G = \Phi_{ms} + \Psi_s + \frac{Q_g}{C_{ox}} + \left(\frac{Q_g}{\Gamma_g \cdot C_{ox}}\right)^2 = \Phi_{ms} + \Psi_s - \frac{Q_{ox} + Q_i + Q_b}{C_{ox}} + \left(\frac{Q_{ox} + Q_i + Q_b}{\Gamma_g \cdot C_{ox}}\right)^2$$

- rearranging :

$$V_G - \underbrace{\left[\Phi_{ms} - \frac{Q_{ox}}{C_{ox}} + \left(\frac{Q_{ox}}{\Gamma_g \cdot C_{ox}}\right)^2\right]}_{V_{FB'}} = \Psi_s - \frac{Q_i + Q_b}{C_{ox}} \cdot \left(1 - \frac{2 \cdot Q_{ox}}{\Gamma_g^{\,2} \cdot C_{ox}}\right) + \left(\frac{Q_i + Q_b}{\Gamma_g \cdot C_{ox}}\right)^2$$

Pinch-Off Surface Potential

- $Q_i(\Psi_P) = 0$ by definition of Ψ_P. Then :

$$V_G - V_{FB'} = \Psi_P - \frac{Q_b(\Psi_P)}{C_{ox}} \cdot \left(1 - \frac{2 \cdot Q_{ox}}{\Gamma_g^{\,2} \cdot C_{ox}}\right) + \left(\frac{Q_b(\Psi_P)}{\Gamma_g \cdot C_{ox}}\right)^2$$

- uniformly doped substrate $\Rightarrow Q_b(\Psi_P) = -\Gamma_b \cdot C_{ox} \cdot \sqrt{\Psi_P}$:

$$V_G - V_{FB'} = \Psi_P \cdot \left[1 + \left(\frac{\Gamma_b}{\Gamma_g}\right)^2\right] + \Gamma_b \cdot \sqrt{\Psi_P} \cdot \left(1 - \frac{2 \cdot Q_{ox}}{\Gamma_g^{\,2} \cdot C_{ox}}\right) + \left(\frac{Q_b(\Psi_P)}{\Gamma_g \cdot C_{ox}}\right)^2$$

then :

$$\Psi_P = \left[\sqrt{\frac{V_G - V_{FB'}}{1 + \left(\frac{\Gamma_b}{\Gamma_g}\right)^2} + \left(\frac{\Gamma_{eq}}{2}\right)^2} - \frac{\Gamma_{eq}}{2}\right]^2$$

with $\Gamma_{eq} = \Gamma_b \cdot \left(\dfrac{\Gamma_g^{\,2} - 2 \cdot \dfrac{Q_{ox}}{C_{ox}}}{\Gamma_g^{\,2} + \Gamma_b^{\,2}}\right)$

Inversion Charge vs. Surface Potential

- Once Ψ_P is known $Q_i(\Psi_s, \Psi_P)$ is :

$$-\frac{Q_i}{C_{ox}} = \frac{(1+\delta_{pd})\cdot(\Psi_P-\Psi_s)+\Gamma_b\cdot\left(\sqrt{\Psi_P}-\sqrt{\Psi_s}\right)}{\left(\frac{1}{2}-\delta_{ox}+\delta_{pd}\cdot\frac{\sqrt{\Psi_s}}{\Gamma_b}\right)+\sqrt{\left(\frac{1}{2}-\delta_{ox}+\delta_{pd}\cdot\frac{\sqrt{\Psi_s}}{\Gamma_b}\right)^2+\delta_{pd}\cdot\left[(1+\delta_{pd})\cdot\frac{\Psi_P-\Psi_s}{\Gamma_b{}^2}+\frac{\sqrt{\Psi_P}-\sqrt{\Psi_s}}{\Gamma_b}\right]}}$$

with $\delta_{ox} = \frac{Q_{ox}}{\Gamma_g{}^2\cdot C_{ox}}$ and $\delta_{pd} = \left(\frac{\Gamma_b}{\Gamma_g}\right)^2$

- Note that without PD (Γ_g >> 1), one simply has :

$$-\frac{Q_i}{C_{ox}} = (\Psi_P-\Psi_s)+\Gamma_b\cdot\left(\sqrt{\Psi_P}-\sqrt{\Psi_s}\right)$$

Charge Slope Factor n_q

- Rewriting $-\dfrac{Q_i}{C_{ox}}$ as :

$$-\frac{Q_i}{C_{ox}} = n_q \cdot (\Psi_P - \Psi_s)$$

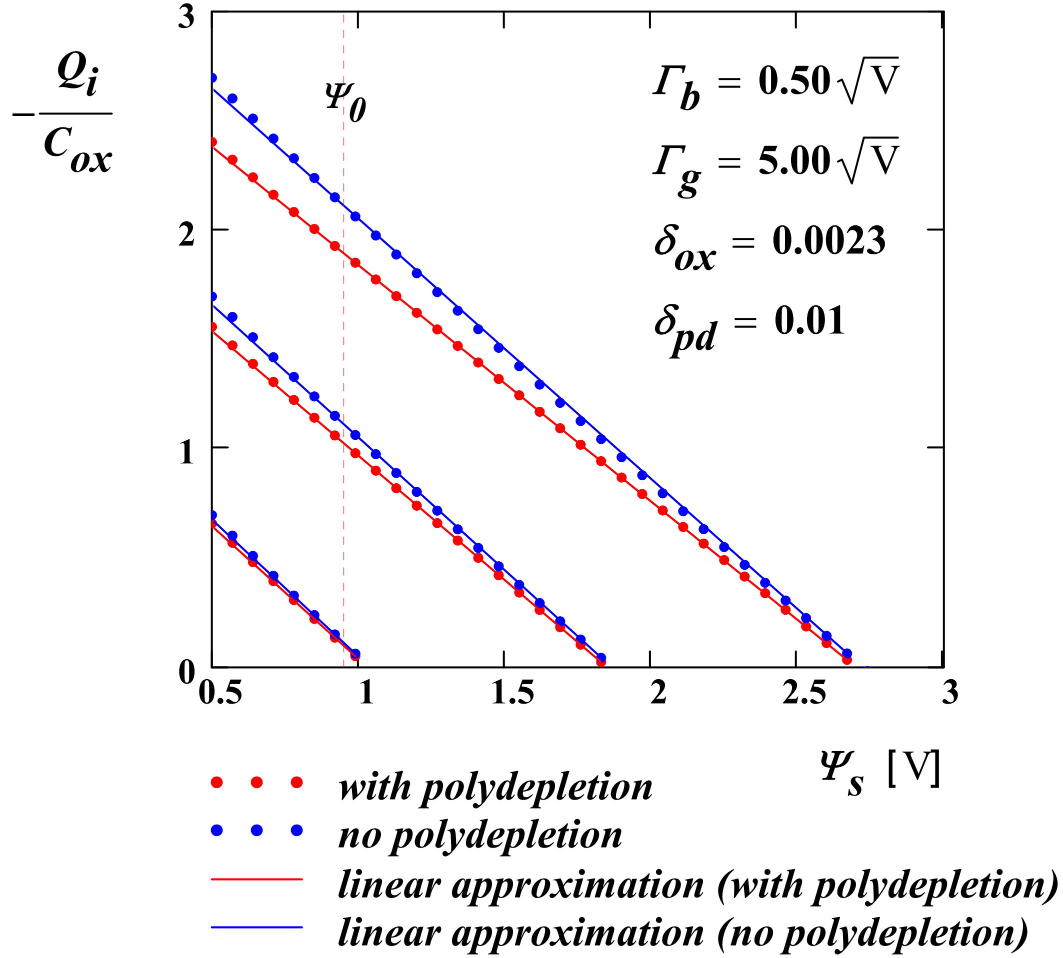
yields, by identification :

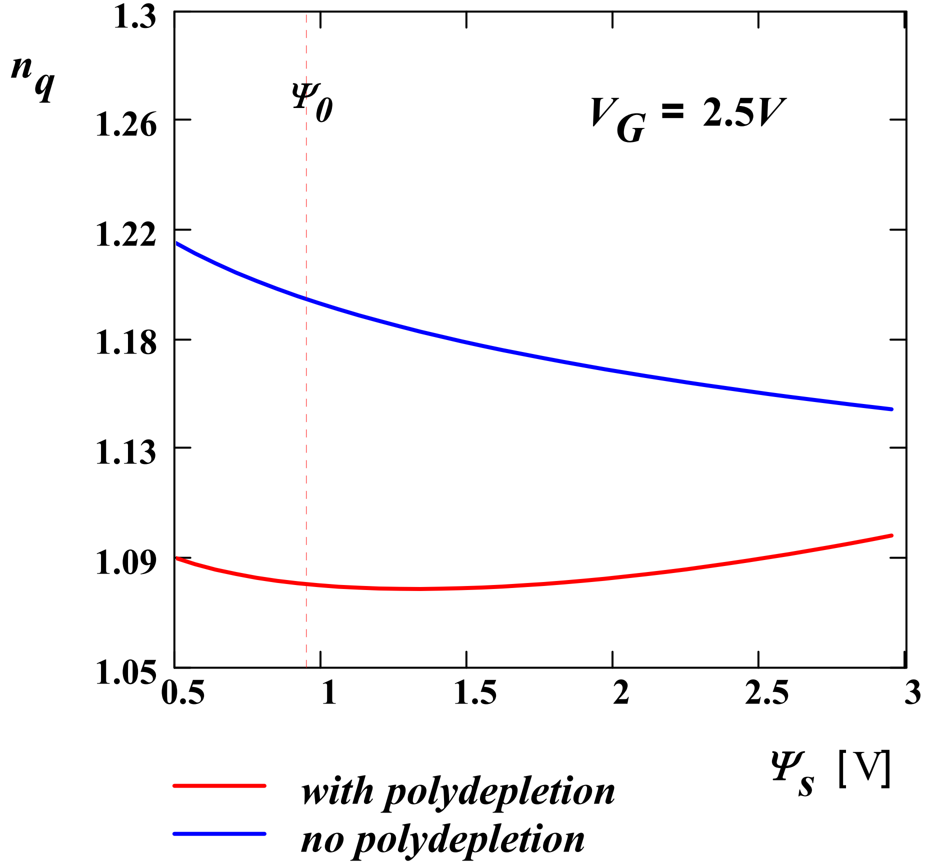
$$n_q(\Psi_P, \Psi_s) = \frac{1 + \delta_{pd} + \dfrac{\Gamma_b}{\sqrt{\Psi_P} + \sqrt{\Psi_s}}}{\left(\dfrac{1}{2} - \delta_{ox} + \delta_{pd} \cdot \dfrac{\sqrt{\Psi_s}}{\Gamma_b}\right) + \sqrt{\left(\dfrac{1}{2} - \delta_{ox} + \delta_{pd} \cdot \dfrac{\sqrt{\Psi_s}}{\Gamma_b}\right)^2 + \left(1 + \delta_{pd} + \dfrac{\Gamma_b}{\sqrt{\Psi_P} + \sqrt{\Psi_s}}\right) \cdot \dfrac{\Psi_P - \Psi_s}{\Gamma_g^{\ 2}}}}$$

Linearisation of the Inversion Charge vs. Surface Potential 2/2

$$V_G \equiv 0.5\text{V}, 1.5\text{V} .. 2.5\text{V}$$

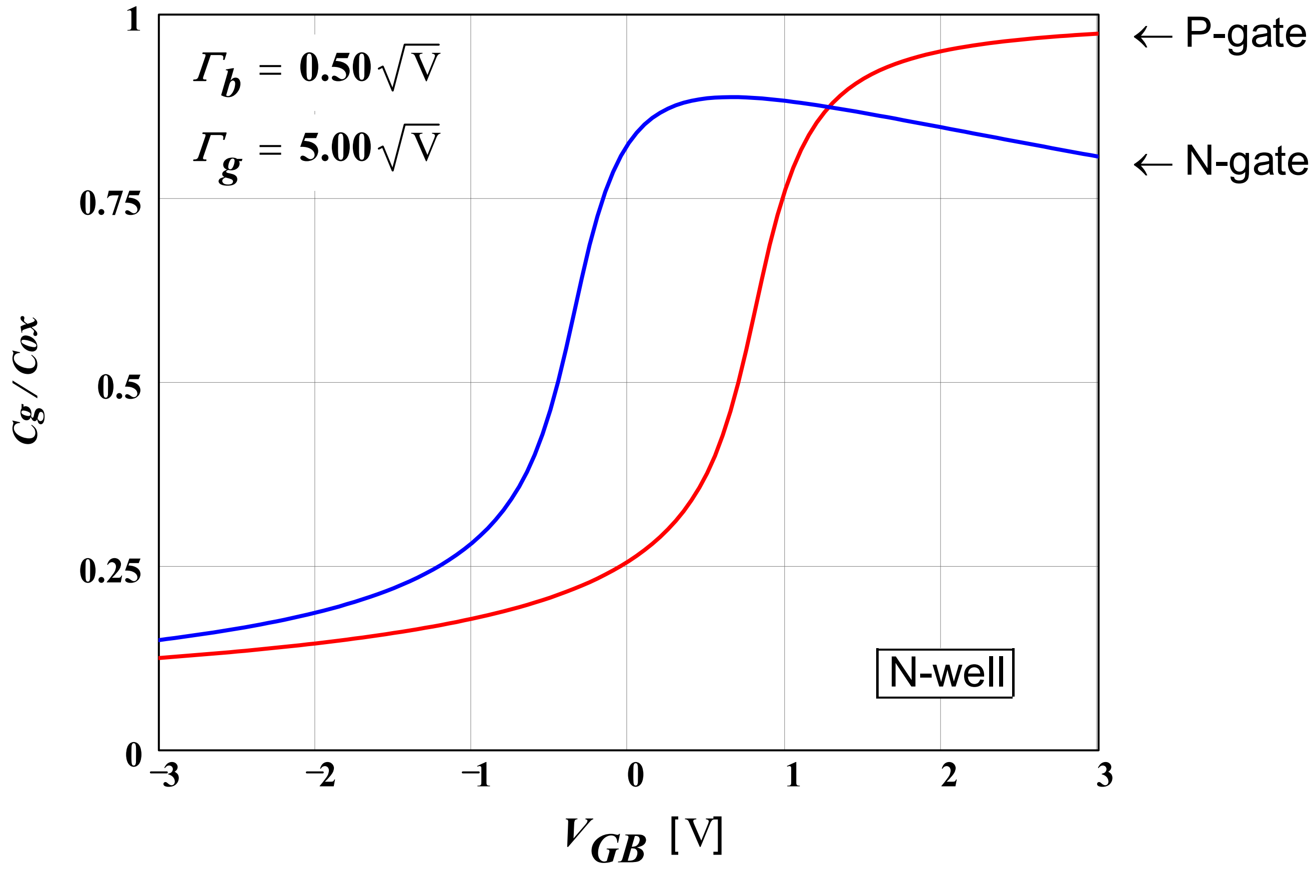
$$n_q = -\frac{Q_i}{C_{ox} \cdot (\Psi_P - \Psi_s)}$$





MOSCAP's & Poly Depletion in Accumulation 1/2

Case study: N-well MOSCAP, N-gate vs. P-gate



Total Charge in the Semiconductor Including Quantum Effects

- quantum confinement due to sharp band bending near the oxide (high vertical field, high bulk doping) :

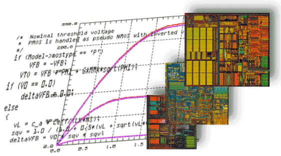
$$Q_{Si} = -sign(\Psi_s)\cdot\Gamma_b\cdot C_{ox}\cdot\sqrt{U_T\cdot\left(e^{-\frac{\Psi_s+\Delta\Psi_v}{U_T}}-1\right)+U_T\cdot e^{-\frac{2\cdot\Phi_F+V}{U_T}}\cdot\left(e^{\frac{\Psi_s-\Delta\Psi_c}{U_T}}-1\right)+\Psi_s}$$

|-------- holes --------| |------------ electrons ------------| |--- fixed charges

where $\Delta\Psi_c$ and $\Delta\Psi_v$ correspond to the difference between the first allowed energy level and the bottom of the conduction band (depletion & inversion), and top of the valence band (accumulation), respectively

- $\Delta\Psi_c$ and $\Delta\Psi_v$ are both positive and can be related to the electric field (i.e. the charges) in the silicon :

$$\Delta\Psi_c = \begin{cases}\Delta\Psi_c(-Q_b,-Q_i) > 0 & if\ \ \Psi_s > 0\\ 0 & if\ \ \Psi_s \le 0\end{cases} \qquad \Delta\Psi_v = \begin{cases}0 & if\ \ \Psi_s \ge 0\\ \Delta\Psi_v(Q_b) > 0 & if\ \ \Psi_s < 0\end{cases}$$



Extension of the General Charge-Voltage Relationship 1/2

Analysis in Depletion & Inversion

- charge sheet approximation
- linearisation of the inversion charge wrt. surface potential :

$$-Q_i = n_q \cdot C_{ox} \cdot (\Psi_P - \Psi_s) \qquad \Rightarrow \qquad \Psi_s = \Psi_P - \frac{-Q_i}{n_q \cdot C_{ox}}$$

- quantum correction linearised in terms of $-Q_i$ around $Q_i = 0$:

$$\Delta\Psi_c(-Q_b, -Q_i) = \Delta\Psi_c(\Psi_P, Q_i = 0) + \frac{d\Delta\Psi_c(\Psi_P, -Q_i)}{d(-Q_i)} \cdot (-Q_i) = \Delta\Psi_{c.p} + \delta_{qm} \cdot \frac{-Q_i}{n_q \cdot C_{ox}}$$

- from $Q_{Si} = Q_b + Q_i = F(V_G, \Psi_s, V)$, the charge-voltage relationship is derived:

$$2 \cdot \frac{-Q_i \cdot (1 + \delta_{qm})}{2 \cdot n_q \cdot C_{ox} \cdot U_T} + ln\left[\frac{-Q_i \cdot (1 + \delta_{qm})}{2 \cdot n_q \cdot C_{ox} \cdot U_T}\right] = \frac{\Psi_P - 2 \cdot \Phi_F - V}{U_T} - \frac{\Delta\Psi_{c.p}}{U_T} - ln\left(\frac{4 \cdot n_q}{1 + \delta_{qm}} \cdot \frac{\sqrt{\Psi_P}}{\Gamma_b}\right)$$

Extending the EKV Formalism to Include QM

- extension of the pinch-off voltage definition :

$$V'_P = \Psi_P - \left(\Psi_0 + \Delta\Psi_0\right)$$

that is, due to quantum effects, Ψ_0 is increased by:

$$\Delta\Psi_0 = \Delta\Psi_{c.p} - U_T \cdot ln\left(1 + \delta_{qm}\right) \approx \Delta\Psi_{c.p} - \delta_{qm} \cdot U_T$$

- the "electric" oxide capacitance :

$$C'_{ox} = C'_{ox}\left(\Psi_P\right) = \frac{C_{ox}}{1 + \delta_{qm}\left(\Psi_P\right)} < C_{ox}$$

- normalisation:

$$2 \cdot \underbrace{\frac{-Q_i}{2 \cdot n_q \cdot C'_{ox} \cdot U_T}}_{q'} + ln\left(\underbrace{\frac{-Q_i}{2 \cdot n_q \cdot C'_{ox} \cdot U_T}}_{q'}\right) = \frac{V'_P - V}{U_T}$$

or

$$2 \cdot q' + ln(q') = v'_p - v$$

Bandgap Widening

- $\Delta\Psi_c$ is primarily a function of the effective vertical electric field :

$$\Delta\Psi_c = A_{qm}\cdot\left(\left|Q_b + \eta\cdot Q_i\right|\right)^{\frac{2}{3}} = A_{qm}\cdot\left[-Q_b\left(\Psi_P - \frac{-Q_i}{n_q\cdot C_{ox}}\right) + \eta\cdot(-Q_i)\right]^{\frac{2}{3}} = \Delta\Psi_c(\Psi_P, -Q_i)$$

- simplified linear approximation around $Q_i = 0$:

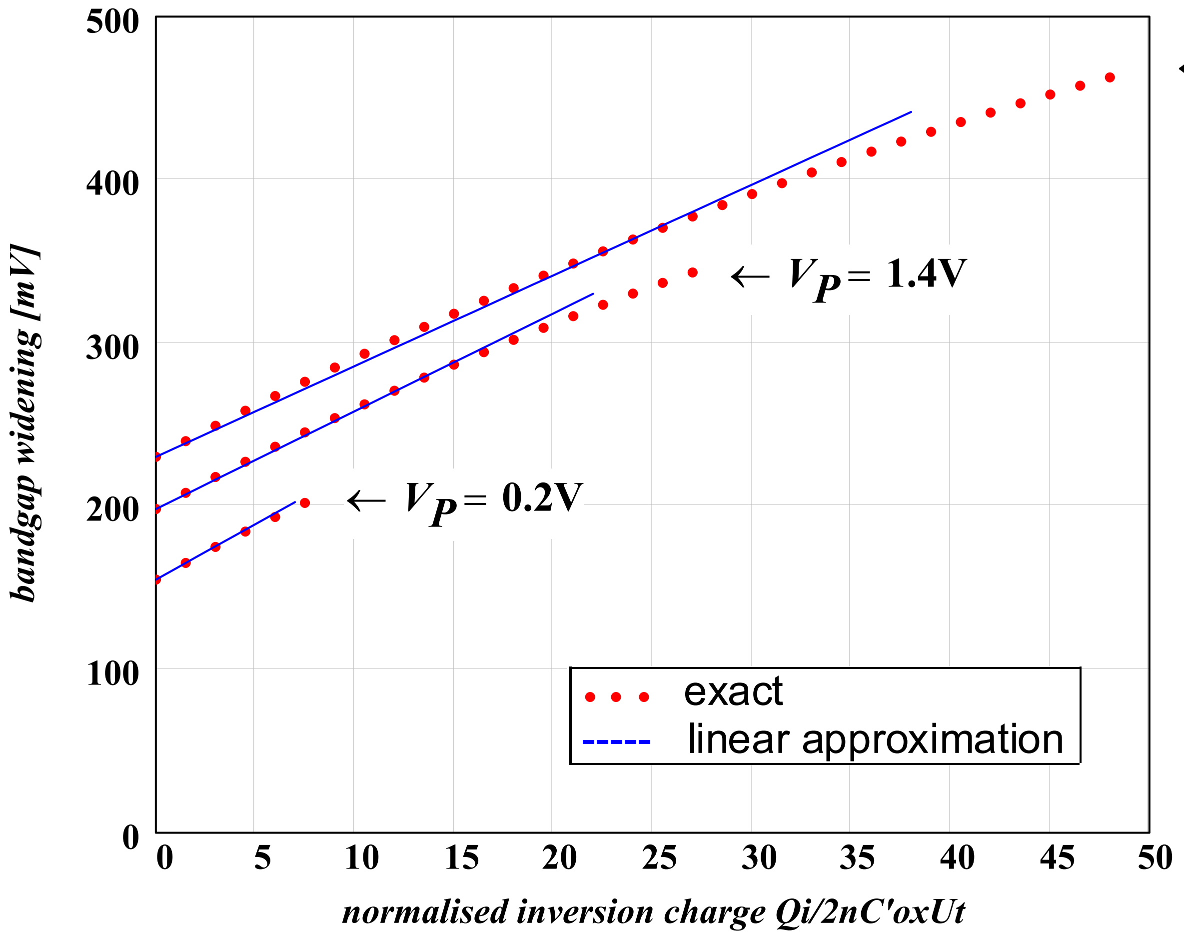
$$\Delta\Psi_c = A_{qm}\cdot\left(\Gamma_b\cdot C_{ox}\cdot\sqrt{\Psi_P}\right)^{\frac{2}{3}} + \delta_{qm}\cdot\frac{-Q_i}{n_q\cdot C_{ox}}$$

with

$$\delta_{qm} = \frac{2}{3}\cdot A_{qm}\cdot C_{ox}\cdot\left(\eta\cdot n_{q0} - \frac{\Gamma_b}{2\sqrt{2\cdot\Phi_F}}\right)\cdot\left(\Gamma_b\cdot C_{ox}\cdot\sqrt{\Psi_P}\right)^{-\frac{1}{3}}$$

Evaluation of the Quantum Correction

Bandgap Widening (cont'd)



$2 \cdot \Phi_F = 0.90\,V$

$t_{ox} = 2\,nm$

$\Gamma_b = 0.5\sqrt{V}$

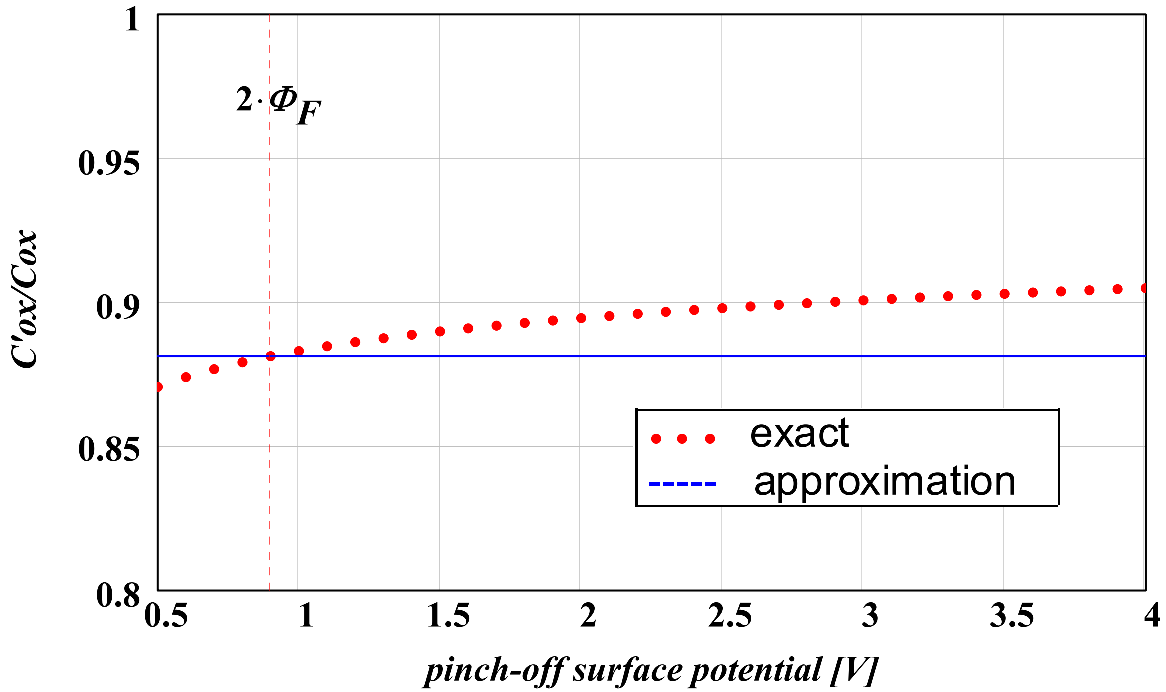
$\delta_{pd} = 0.0$

$\eta = 0.75$

Evaluation of the Quantum Correction

The Apparent Oxide Capacitance

$$C'_{ox} = \frac{C_{ox}}{1+\delta_{qm}} = \frac{C_{ox}}{1+\frac{2}{3}\cdot A_{qm}\cdot C_{ox}\cdot\left(\eta\cdot n_{q0}-\frac{\Gamma_b}{2\sqrt{2\cdot\Phi_F}}\right)\cdot\left(\Gamma_b\cdot C_{ox}\cdot\sqrt{\Psi_P}\right)^{-\frac{1}{3}}}$$



Extension of the EKV Current-Voltage Expression

- Drift-diffusion transport equation :

$$I_D = I_{drift} + I_{diff} = \mu \cdot W \cdot \left(Q_i \cdot E(x) + U_T \cdot \frac{dQ_i}{dx} \right)$$

- Electric field experienced by the mobile carriers :

$$E(x) = -\frac{d}{dx}\left(\Psi_s - \Delta\Psi_c\right) = -\frac{d\Psi_s}{dx} - \left(\frac{\delta_{qm}}{n_q \cdot C_{ox}}\right) \cdot \frac{dQ_i}{dx}$$

- Linearised relation between the inversion charge and the surface potential:

$$-Q_i = n_q \cdot C_{ox} \cdot \left(\Psi_P - \Psi_s\right) \qquad \text{i.e.} \qquad dQ_i = n_q \cdot C_{ox} \cdot d\Psi_s$$

then :

$$I_D = \mu \cdot W \cdot \left[\frac{-Q_i}{n_q \cdot C_{ox}} \cdot \left(1 + \delta_{qm}\right) + U_T\right] \cdot \frac{dQ_i}{dx} = \mu \cdot W \cdot \left(\frac{-Q_i}{n_q \cdot C'_{ox}} + U_T\right) \cdot \frac{dQ_i}{dx}$$

Polysilicon Depletion

- occurs in depletion and inversion (normal MOSFET: type of gate opposite to that of bulk) or in accumulation (MOSCAP's: gate same type as bulk)

- pinch-off voltage is reduced, equivalent to :

$$V_G - V_{FB} \Rightarrow \frac{V_G - V_{FB} - \left(\dfrac{Q_{ox}}{\Gamma_g \cdot C_{ox}}\right)^2}{1 + \left(\dfrac{\Gamma_b}{\Gamma_g}\right)^2} \qquad \Gamma_b \Rightarrow \Gamma_b \cdot \left(\frac{\Gamma_g^{\ 2} - 2 \cdot \dfrac{Q_{ox}}{C_{ox}}}{\Gamma_g^{\ 2} + \Gamma_b^{\ 2}}\right)$$

- slope factor n_q can be strongly modified, but the charge-voltage linearisation scheme remains valid

- PD reduces the MOS capacitances due to an increased apparent dielectric thickness

- Adds one model parameter: `Ngate` or equivalently `GammaG`

Quantum Effects

- bandgap widening is accounted for with $\Delta\Psi_0$ (reduced V_P for a given Ψ_p) and "electric" capacitance $C'_{ox} < C_{ox}$

- the EKV charge-voltage & current-voltage relations remain valid with the following normalisation :

$$q = \frac{-Q_i}{2 \cdot n_q \cdot C'_{ox} \cdot U_T} \quad \text{and} \quad i = \frac{I_d}{2 \cdot n_q \cdot \frac{W}{L} \cdot \mu_0 \cdot C'_{ox} \cdot U_T^2}$$

- QM impacts both accumulation and inversion

- Adds no model parameter, but a flag to turn off QM for convenience

Summary

$$t_{ox} = 2\,nm \qquad \Gamma_b = 0.37\sqrt{V} \qquad \Gamma_g = 5.0\sqrt{V}$$