EKV Workshop, June 30 2008

The EKV Charge-based Noise Model

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Outline

1. Introduction
2. The long-channel charge-based noise model
3. The extended charge-based noise model

Introduction

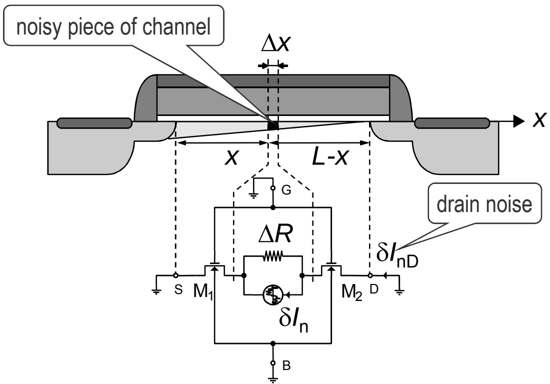
- Noise sets the lower limit for signal amplification and detection, whereas upper limit is set by device non linearity
- Reduction of supply voltage in deep submicron CMOS technologies reduces upper limit and forces noise to become smaller at the cost of a higher power consumption
- Flicker noise largely dominates at low frequency (below the corner frequency), particularly for deep submicron CMOS
- Thermal noise dominates at HF and is hence important for RF IC design
- It is dominated by the intrinsic channel thermal noise (~80-90%)
- It is therefore crucial to properly model thermal noise for RF IC design (for example LNA design)

The Long-channel Charge-based Noise Model

1. General noise calculation in MOSFETs
2. Channel thermal noise model
3. Flicker noise

The Long-channel Charge-based Noise Model

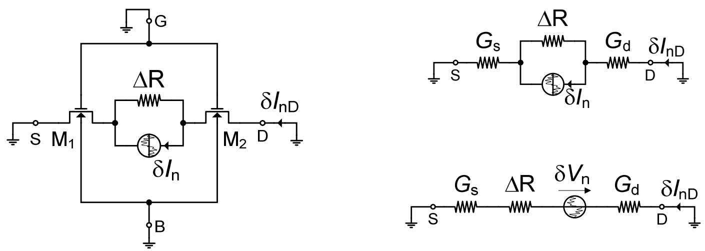
General MOST Noise Calculation



- Noiseless channel except for a slice of channel comprised between x and x+Δx and having a resistance ΔR
- Local noise (including both thermal and flicker noise) modeled by current source δI_n which induces a fluctuation of the drain current δI_{nD} through the (trans)conductance

The Long-channel Charge-based Noise Model

Two-Transistors Approach



- Drain current fluctuation due to local noise source

$$\delta I_{nD} = G_{ch} \cdot \Delta R \cdot \delta I_n = G_{ch} \cdot \delta V_n$$

where G_{ch} is the channel conductance seen from point x

$$\frac{1}{G_{ch}} = \frac{1}{G_s} + \frac{1}{G_d} \quad \text{with :} \quad G_s = G_{md1} \quad \text{and} \quad G_d = G_{ms2}$$

Total Noise Drain Current PSD

- PSD of drain current due to local fluctuation

$$S_{\delta I_{nD}^2}(\omega,x) = G_{ch}^2(x)\cdot \Delta R^2(x)\cdot S_{\delta I_n^2}(\omega,x) = G_{ch}^2(x)\cdot S_{\delta V_n^2}(\omega,x)$$

$$\text{with: } \quad S_{\delta V_n^2}(\omega,x) = \Delta R^2(x)\cdot S_{\delta I_n^2}(\omega,x)$$

- Total PSD of drain current assuming local contributions are uncorrelated along the channel

$$S_{\Delta I_D^2}(\omega) = \int_0^L G_{ch}^2(x)\cdot \Delta R^2(x)\cdot \frac{S_{\delta I_n^2}(\omega,x)}{\Delta x}\cdot dx$$

- Can also use local noise voltage source instead of current source

$$S_{\delta V_n^2} = \Delta R^2 \cdot S_{\delta I_n^2}$$

$$S_{\delta I_{nD}^2} = G_{ch}^2 \cdot S_{\delta V_n^2}$$

$$S_{\Delta I_D^2}(\omega) = \int_0^L G_{ch}^2(x)\cdot \frac{S_{\delta V_n^2}(\omega,x)}{\Delta x}\cdot dx$$

Long-Channel Approximation

- If mobility assumed constant, the conductance G_{ch} is given by

$$G_{ch}(x) = \frac{dI_D}{dV} = \mu\cdot(-Q_i(x))\cdot\frac{W}{L} = G_{spec}\cdot q_i(x) \qquad G_{spec} = \frac{I_{spec}}{U_T} = 2n\beta U_T$$

whereas the noise resistance is given by

$$\Delta R(x) = \frac{\Delta V}{I_D} = \frac{\Delta x}{W\cdot\mu\cdot(-Q_i(x))}$$

hence

$$G_{ch}\cdot\Delta R = \cancel{\mu}\cdot(-\cancel{Q_i}(x))\cdot\frac{\cancel{W}}{L}\cdot\frac{\Delta x}{\cancel{W}\cdot\cancel{\mu}\cdot(-\cancel{Q_i}(x))} = \frac{\Delta x}{L}$$

$$S_{\delta I_{nD}^2}(\omega,x) = \left(\frac{\Delta x}{L}\right)^2\cdot S_{\delta I_n^2}(\omega,x)$$

$$S_{\Delta I_{nD}^2}(\omega) = \int_0^L\left(\frac{\Delta x}{L}\right)^2\cdot\frac{S_{\delta I_n^2}(\omega,x)}{\Delta x}\cdot dx = \frac{1}{L^2}\cdot\int_0^L \Delta x\cdot S_{\delta I_n^2}(\omega,x)\cdot dx$$

Long-Channel Thermal Noise

- The PSD of the local noise source is given by

$$S_{\delta I_n^2}(x) = \frac{4kT}{\Delta R(x)} = 4kT \cdot \frac{W \cdot \mu \cdot (-Q_i(x))}{\Delta x}$$

- The PSD of the total noise at the drain is then given by

$$S_{\Delta I_{nD}^2} = 4kT \cdot G_{nD}$$

where G_{nD} is the thermal noise conductance given by

$$G_{nD} = \frac{1}{L^2} \cdot \int_0^L W \cdot \mu \cdot (-Q_i(x)) \cdot dx = \mu \cdot \frac{W}{L^2} \cdot \int_0^L -Q_i(x) \cdot dx = \frac{\mu}{L^2} \cdot |Q_I|$$

- The thermal noise PSD and conductance at the drain is proportional to the total charge stored in the channel

$$|Q_I| = W \cdot \int_0^L -Q_i(x) \cdot dx \qquad \text{(long-channel)}$$

Thermal Noise Conductance Calculation

- Normalized thermal noise conductance defined by

$$g_{nD} = \frac{G_{nD}}{G_{spec}} = \int_0^1 q_i(\xi) \cdot d\xi = q_I \quad \text{with} \quad G_{spec} \triangleq \frac{I_{spec}}{U_T} = 2n\beta U_T$$

- Using the charge-based drain current expression

$$i_d = -(2q_i + 1) \cdot \frac{dq_i}{d\xi} \quad \Rightarrow \quad d\xi = -\frac{2q_i + 1}{i_d} \cdot dq_i$$

- Allows to perform the integration in the charge domain

$$g_{nD} = q_I = -\frac{1}{i_d} \cdot \int_{q_s}^{q_d} q_i \cdot (2q_i + 1) \cdot dq_i = \frac{1}{6} \cdot \frac{4q_s^2 + 3q_s + 4q_s q_d + 3q_d + 4q_d^2}{q_s + q_d + 1}$$

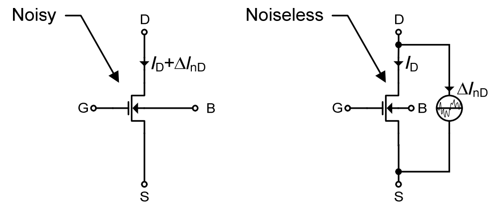
where the following expression is used for the drain current

$$i_d = \left(q_s^2 + q_s\right) - \left(q_d^2 + q_d\right)$$

The Long-channel Charge-based Noise Model

Long-Channel Thermal Noise

- The thermal noise at low-frequency can be modeled as a current source between source and drain having a PSD given by



$$S_{\Delta I_{nD}^2} = 4kT \cdot G_{nD}$$

$$G_{nD} = \frac{\mu}{L^2} \cdot |Q_I| = G_{spec} \cdot q_I$$

where q_I is the total normalized inversion charge given by

$$q_I = \frac{1}{6} \cdot \frac{4q_s^2 + 3q_s + 4q_s q_d + 3q_d + 4q_d^2}{q_s + q_d + 1} = \begin{cases} q_s & \text{linear region with } q_s = q_d, i_f = i_r \\ q_s \cdot \dfrac{\frac{2}{3} \cdot q_s + \frac{1}{2}}{q_s + 1} & \text{saturation } (q_s >> q_d, i_f >> i_r) \end{cases}$$

The Long-channel Charge-based Noise Model

Channel Thermal Noise in Weak Inversion

- The total normalized inversion charge in weak inversion is given by

$$q_I \cong \frac{q_s + q_d}{2} = \frac{i_f + i_r}{2}$$

the noise PSD can be rewritten as

$$G_{nD} = G_{spec} \cdot q_I = \frac{I_{spec}}{U_T} \cdot \frac{i_f + i_r}{2} = \frac{I_F + I_R}{2U_T}$$

and therefore

$$S_{\Delta I_D^2} = 4kT \cdot G_{nD} = 4kT \cdot \frac{I_F + I_R}{2U_T} = 2q \cdot (I_F + I_R)$$

which corresponds to full shot noise of both forward and reverse components

- This result is consistent with the fact that the current in weak inversion is dominated by the diffusion current

The Long-channel Charge-based Noise Model

Thermal Noise Parameter

- The thermal noise parameter δ_{nD} related to the drain is defined as

$$\delta_{nD} \equiv \frac{G_{nD}}{G_{dso}} \qquad G_{dso} = G_{ms} = G_{spec} \cdot q_s$$

  where G_{dso} is the channel conductance at V_{DS}=0 which is equal to the source transconductance G_{ms}
- δ_{nD} tells how much the thermal noise deviates from the value it takes when it operates like a resistor having a conductance G_{dso}
- δ_{nD} compares noise at a given operating point to the noise at V_{DS}=0
- Mainly useful for device modeling but useless for circuit designers
- The noise conductance can then be expressed as a function of the source transconductance as

$$G_{nD} = \delta_{nD} \cdot G_{ms}$$

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Thermal Noise Parameter

- For long-channel devices, we have

$$\delta_{nD} = \frac{G_{spec} \cdot q_I}{G_{spec} \cdot q_s} = \frac{q_I}{q_s} = 1 \quad \text{linear region with } q_s = q_d, i_f = i_r$$

$$\delta_{nD} = \frac{G_{spec} \cdot q_I}{G_{spec} \cdot q_s} = \frac{q_I}{q_s} = \frac{2}{3} \cdot \frac{q_s + 3/4}{q_s + 1} = \begin{cases} \frac{1}{2} & \text{WI and sat.} \quad (1 >> q_s >> q_d, 1 >> i_f >> i_r) \\ \frac{2}{3} & \text{SI and sat.} \quad (q_s >> 1 >> q_d, i_f >> 1 >> i_r) \end{cases}$$

- The thermal noise conductance is then given by

$$G_{nD} = \begin{cases} G_{ms} & \text{linear region with } V_{DS} = 0 \\ \frac{1}{2} \cdot G_{ms} & \text{WI and sat.} \\ \frac{2}{3} \cdot G_{ms} & \text{SI and sat.} \end{cases}$$

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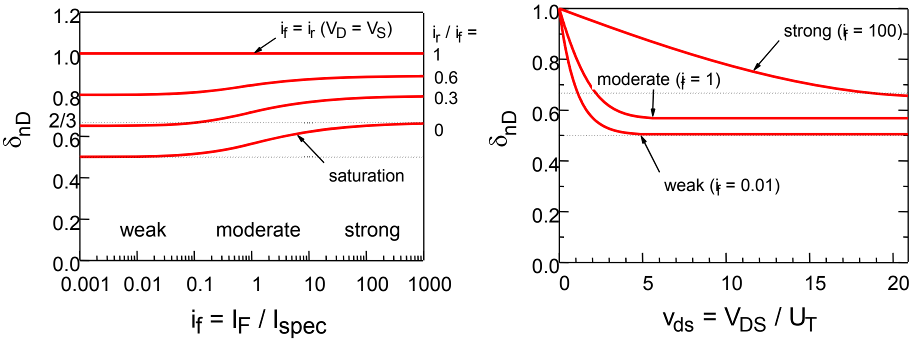
The Long-channel Charge-based Noise Model

Thermal Noise Parameter



The Long-channel Charge-based Noise Model

Thermal Noise Excess Factor

- The thermal noise excess factor γ_{nD} is defined as

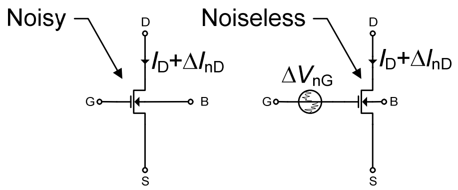
$$\gamma_{nD} \equiv \frac{G_{nD}}{G_m}$$ where G_m is the gate transconductance

- γ_{nD} shows how much noise is generated at the drain for a given G_m
- Contrary to δ_{nD}, the noise conductance G_n and the transconductance G_m are evaluated at the same operating point
- Since $G_m \rightarrow 0$ for $V_{DS} \rightarrow 0$, γ_{nD} is becoming large for small V_{DS}
- γ_{nD} is related to δ_{nD} by $\gamma_{nD} \equiv \frac{G_{nD}}{G_m} = \frac{G_{nD}}{G_{ms}} \cdot \frac{G_{ms}}{G_m} = \delta_{nD} \cdot \frac{n \cdot G_{ms}}{G_{ms} - G_{md}} = \delta_{nD} \cdot \frac{n \cdot q_s}{q_s - q_d}$
- For long-channel devices in saturation G_{md}=0 and hence $\gamma_{nD} = n \cdot \delta_{nD}$
- The thermal noise conductance (in saturation) is the given by

$$G_{nD} = \gamma_{nD} \cdot G_m = \begin{cases} \frac{n}{2} \cdot G_m & \text{WI and sat.} \\ \frac{2n}{3} \cdot G_m \cong G_m & \text{SI and sat.} \end{cases}$$

The Long-channel Charge-based Noise Model

Input Gate Referred Thermal Noise



- For $G_m \neq 0$ and in particular in saturation, the thermal noise can also be referred to the gate as a voltage source having a PSD given by

$$S_{\Delta V_{nG}^2} = \frac{S_{\Delta I_{nD}^2}}{G_m^2} = 4kT \cdot R_{nG}$$

where the input (or gate) referred thermal noise resistance R_{nG} is given by

$$R_{nG} = \frac{G_{nD}}{G_m^2} = \frac{\gamma_{nD}}{G_m} = \begin{cases} \frac{1}{2} \cdot \frac{n}{G_m} & \text{WI and sat.} \\ \frac{2}{3} \cdot \frac{n}{G_m} \cong \frac{1}{G_m} & \text{SI and sat.} \end{cases}$$

The Long-channel Charge-based Noise Model

Flicker Noise

- Basically two main causes to this 1/f noise:
  - Carrier number fluctuation ΔN (Mc Worther model): trapping of mobile charge in traps located in the oxide close to the Si-SiO2 interface resulting in fluctuations of the inversion charge
  - Carrier mobility fluctuation $\Delta\mu$ (Hooge model)
- The PSD of the input referred gate voltage fluctuations is given by

$$S_{\Delta V_{nG}^2}(f) = S_{\Delta V_{nG}^2}(f)\Big|_{\Delta N} + S_{\Delta V_{nG}^2}(f)\Big|_{\Delta\mu}$$

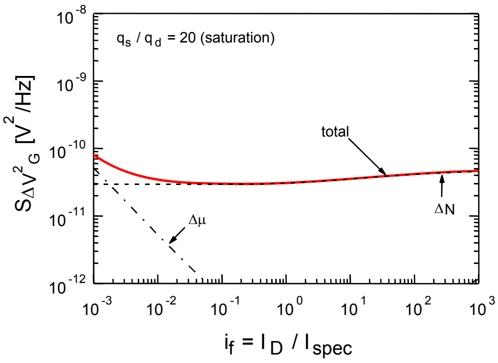
where $S_{\Delta V_{nG}^2}(f)\Big|_{\Delta N} \cong \frac{K_{\Delta N}}{W \cdot L \cdot C_{ox}^2 \cdot f}$ and $S_{V_{nG}^2}\Big|_{\Delta\mu} \cong \frac{K_{\Delta\mu}}{W \cdot L \cdot C_{ox} \cdot f}$

- Inversely proportional to frequency and to gate area
- Note that $K_{\Delta N}$ and $K_{\Delta\mu}$ are slightly bias dependent

The Long-channel Charge-based Noise Model

Flicker Noise – Total Flicker Noise

$$S_{\Delta V_{nG}^2} = S_{\Delta V_{nG}^2}\Big|_{\Delta N} + S_{\Delta V_{nG}^2}\Big|_{\Delta\mu}$$

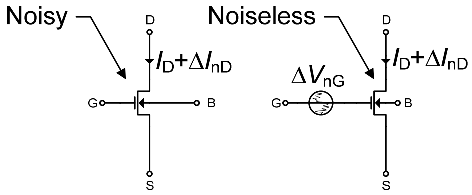


- Usually number fluctuation dominates over mobility fluctuation
- For design purpose, the gate referred noise PSD can be considered at first order as bias independent

The Long-channel Charge-based Noise Model

Flicker Noise – Total Flicker Noise

- Simple model for design

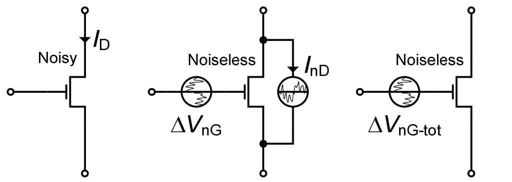


- If number fluctuation dominates mobility fluctuation

$$S_{\Delta V_{nG}^2} \cong \frac{KF}{C_{ox}^2 \cdot W \cdot L \cdot f} = 4kT \cdot R_{nG}(f)$$

with $R_{nG}(f) = \dfrac{\rho}{W \cdot L \cdot f}$ and $\rho = \dfrac{KF}{4kT \cdot C_{ox}^2}$

Total MOST Noise (in saturation)



Thermal noise: $S_{\Delta I_D^2} = 4kT \cdot G_{nD}$ $\quad G_{nD} = \delta_{nD} \cdot G_{ms} = \gamma_{nD} \cdot G_m$ $\quad \delta_{nD} = \begin{cases} \frac{1}{2} & \text{WI} \\ \frac{2}{3} & \text{SI} \end{cases}$

$$\gamma_{nD} = n \cdot \delta_{nD}$$

Flicker noise: $S_{\Delta V_{nG}^2} \cong \dfrac{KF}{C_{ox}^2 \cdot W \cdot L \cdot f} = 4kT \cdot R_{nG}(f)$ $\quad R_{nG}(f) = \dfrac{\rho}{W \cdot L \cdot f}$

$$\rho = \frac{KF}{4kT \cdot C_{ox}^2}$$

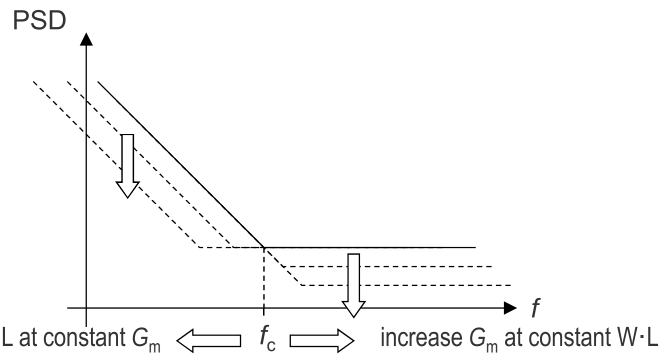
Total input referred noise: $S_{\Delta V_{nG-tot}^2} = 4kT \cdot R_{nG-tot}$ $\quad R_{nG-tot} = \dfrac{\rho}{W \cdot L \cdot f} + \dfrac{\gamma_{nD}}{G_m}$

Corner Frequency

- The corner frequency is defined as the frequency for which the 1/f noise PSD is equal to the thermal noise PSD

$$\frac{\rho}{W \cdot L \cdot f_c} = \frac{\gamma_{nD}}{G_m} \quad \Rightarrow \quad f_c = \frac{G_m \cdot \rho}{\gamma_{nD} \cdot W \cdot L} = \frac{G_m \cdot KF}{4kT \cdot C_{ox}^2 \cdot \gamma_{nD} \cdot W \cdot L} = \frac{\mu_{eff} \cdot KF}{4kT \cdot C_{ox} \cdot \gamma_{nD}} \cdot \frac{V_P - V_S}{L^2}$$

- If n=1, the corner frequency scales approximately as $1/(C_{ox} L^2)$



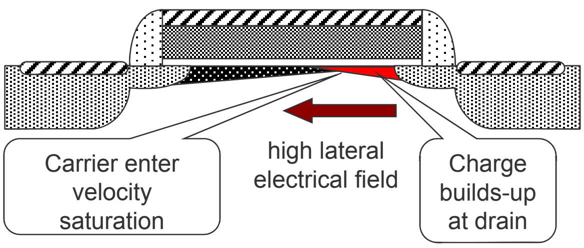
The Extended Charge-based Model

Short-channel Effects on Thermal Noise

- Thermal noise is affected by following effects:
  1. Velocity saturation (VS)
  2. Carrier heating (CH)
  3. Mobility reduction due to the vertical field (MRV)
  4. Channel length modulation (CLM)
- Evaluate the impact of each of these effects on δ_{nD} and γ_{nD}
- Each effect will be analyzed separately

The Extended Charge-based Model

Effect of Velocity Saturation

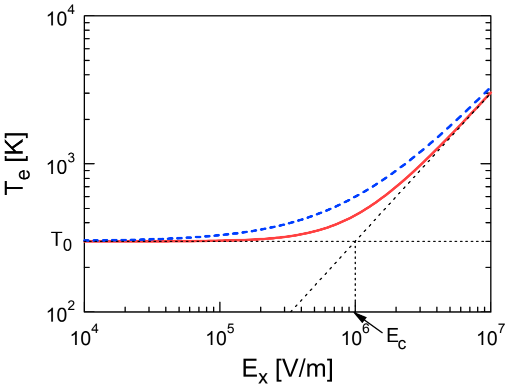


- For short-channel devices in SI and saturation → lateral electrical field larger than critical field → carrier velocity saturation
- Carrier velocity limited → additional charge builds up close to the drain → additional thermal noise without increase of G_m → increase of δ_{nDsat} compared to the long-channel value 2/3

The Extended Charge-based Model

Hot Carriers and Effective Temperature

- High lateral electric field → carrier not in thermal equilibrium with lattice → higher carrier temperature → higher thermal noise



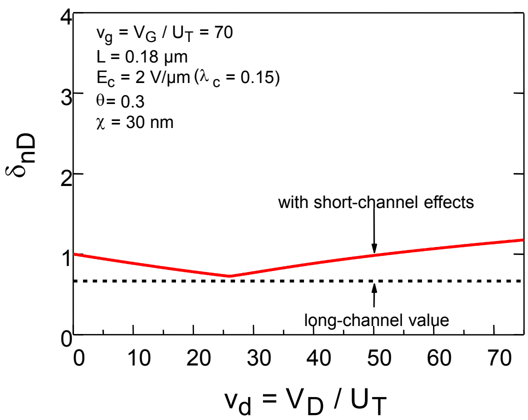
$$T_e = T_0 \cdot \left(1 + \frac{E_x}{E_c}\right)^m$$

$$m = 1 \ldots 3$$

P. Klein, EDL, Aug. 1999.

The Extended Charge-based Model

Short-channel Effects on δ_{nD}



A. S. Roy and C. C. Enz, TED, April 2005.

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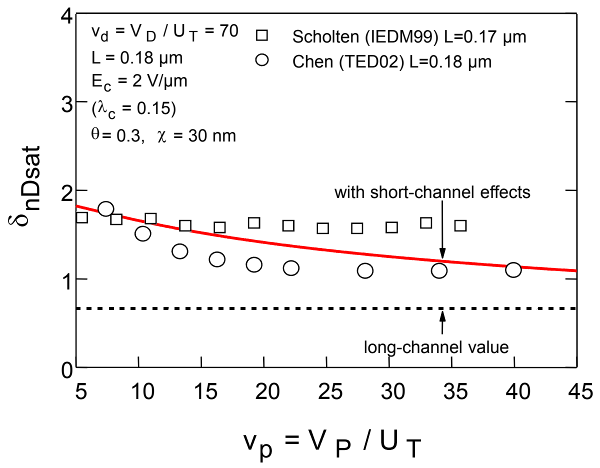
Short-channel Effects on δ_{nDsat} (in saturation)



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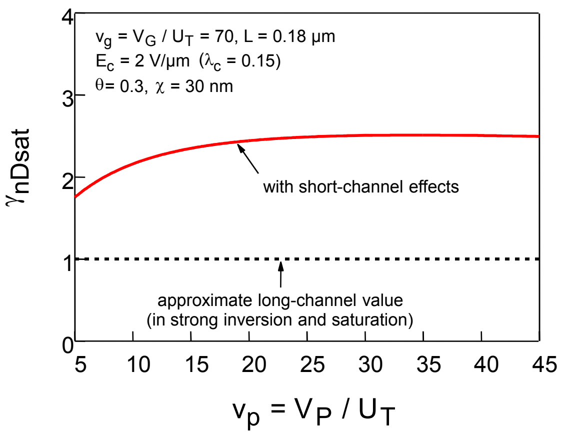
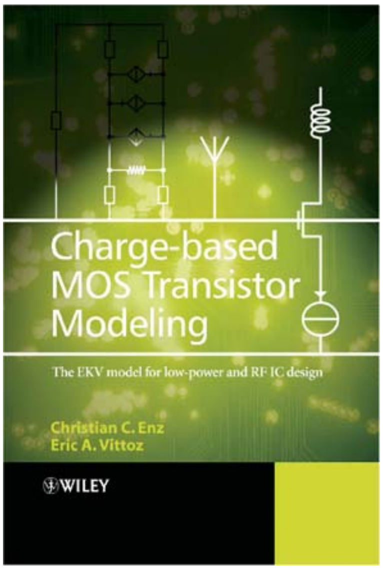
Short-channel Effects on γ_{nD} (in saturation)



A. S. Roy and C. C. Enz, TED, April 2005.

Conclusions



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The EKV model for low-power and RF IC design

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