AVI as a mechanical tool for studying thin-shells based on Kirchhoff-Love constraints

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ABSTRACT

Thin-shell and rod theory using discrete mechanics applied to structures in civil engineering. The aim is to apply structure preserving algorithms to concrete problems in construction. The major objectives of this interdisciplinary work is the research and the development of a practical tool to study irregular surfaces.

NOTATION AND DEFINITIONS

In this paper we shall regard a body $B \subset \mathbb{R}^3$ as a smooth orientable Riemannian manifold endowed with a Riemannian metric $g$. The space $S \subset \mathbb{R}^3$ in which the body moves is also taken to be a smooth orientable Riemannian manifold $g$. A configuration of $\phi : B \to \mathbb{R}^3$ is, by definition, an orientation preserving diffeomorphism between $B$ and its embedded image $\phi(B) \subset S$. The configuration space is defined to be $\mathcal{C} = \{ \phi : B \to \mathbb{R}^3 : \phi \text{ a C}^\infty$ embedding $\}$. The deformed body $\phi(B)$ inherits the Riemannian structure of $S$. We shall call $\phi$ the reference configuration and $S$ the ambient space. Let $TS, T^*S$ be the tangent bundles of $S$ and $S$, respectively, and let $T^*B, T^*S$ be their cotangent bundles. Let $\xi^\alpha$ denote the Euclidean coordinates of a point $x \in S$ relative to the standard basis $\{ e_i \}$ of $\mathbb{R}^3$. Similarly, $\{ x^\alpha \}$ are the Euclidean coordinates of a point $x \in S$ relative to the standard basis $\{ e_i \}$ of $\mathbb{R}^3$.

Kirchhoff-Love assumptions for thin-shell

According to standard Kirchhoff-Love assumptions, we take the reference shell director $\mathbf{T}$ and the deformed shell director $\mathbf{t}$ equal to the third basis vector respectively

$E_t = E_\mathbf{t} = E_{\mathbf{t}} + X_{\mathbf{t}}$ and $0 \leq \theta_0 \leq 1$.

(5)

Denote by $\phi_{\text{K-L}}$ the standard inner product in $\mathbb{R}^3$ for vectors based at $x \in S = \mathbb{R}^3$ and by $\langle \cdot, \cdot \rangle_\mathcal{C}$ the standard inner product in $\mathcal{C}$ for vectors based at $x \in S$. The components $g_{\alpha\beta}$ of the metric tensor on $\phi(B)$ (obtained by pulling back by the inclusion map) the inner product $\langle \cdot, \cdot \rangle_\mathcal{C}$ on $\mathcal{C}$ are defined by $g_{\alpha\beta}(x) = \langle \xi^\alpha(x), \xi^\beta(x) \rangle_\mathcal{C}$. Similarly, we define the components $g_{\alpha\beta}$ of the metric on $S$ by $g_{\alpha\beta}(x) = \langle \xi^\alpha(x), \xi^\beta(x) \rangle_S$ and $\mathcal{C} = [g_{\alpha\beta}]$. The stress measure relative to the dual spatial basis $\mathcal{S}$ is given by $\mathcal{S}(x) = 1/2 (\mathcal{T}(x) + \mathcal{T}^T(x))$.

Energy behavior

We get consistent and explicit integrator by using discrete Lagrangian $L_d$ as

$L_d(x_i, x_{i+1}, t) = \frac{1}{2} \left[ \frac{(x_i - x_{i-1})^T}{h} \right] M \left[ \frac{(x_i - x_{i-1})^T}{h} \right] - M - M(x_i)$(10)

With $L_d$, let discrete-energy $E_d = E_{\mathbf{t}}$ as previously defined, for time step $\Delta t = t_i - t_{i-1}$ such that

$E_d = - \int L_d(x_i, x_{i+1}, t) \, dt = \frac{1}{2} \left[ \frac{(x_i - x_{i-1})^T}{h} \right] M \left[ \frac{(x_i - x_{i-1})^T}{h} \right] + V(x_i)$(11)

Since $E_d \neq E_{\mathbf{t}}$ for fixed time-steps, the difference between both sides of the inequality represents the variation of energy between successive integration steps, called the energy residue. And we note that the energy residue is smaller than twice the sets of magnitude in absolute value compared to the energy itself.

ONE EDGE AND TWO PLATES

We consider two thin-shells of same sizes, leaning against each other, so they form an edge. And, as previously, we get equilibrium position, by introducing a dissipative system.

REFERENCES


