

# Algebraic $K$ -theory

## Spring 2020

### Syllabus

1. Introduction: motivations and relations with other fields
2.  $K_0$  and classification of modules
  - (a) Definition and elementary properties of  $K_0$ 
    - i. Group completion
    - ii. Elementary module theory
    - iii. Grothendieck groups
    - iv. Dévissage
    - v. The Resolution Theorem
    - vi. Stability
    - vii. Multiplicative structure
  - (b) Functoriality of  $K_0$ 
    - i. Exact functors
    - ii. Naturality of  $K_0(R)$
    - iii. Localization
3.  $K_1$  and classification of invertible matrices
  - (a) Elementary matrices and commutators
  - (b) Definition and elementary properties of  $K_1$
  - (c) Generalized determinants
  - (d)  $K_1$  as a Grothendieck group

## Bibliography

1. Bruce A. Magurn, *An Algebraic Introduction to K-theory*, Encyclopedia of Mathematics and its Applications **87**, Cambridge University Press, 2009.  
(The course will be based primarily on Chapters 3, 4, 5, 6, and 9 of this text.)
2. Joseph J. Rotman, *An Introduction to Homological Algebra*, Academic Press, 1979.  
(Chapters 2, 3, and 4 of this reference book should also prove very useful.)

## The exam

The exam for this course will be a written test, which will count for 70% of your grade.

Each week you will receive a set of exercises, of which one must be handed in the following week to be graded by the teaching assistant. Your average grade on the exercises will count for 30% of your final grade.

## The course wiki

All exercise sets, submitted solutions and other course information can be found here.

<https://www.epfl.ch/labs/hessbellwald-lab/teaching/2019-2020/>