YTM Juli 2018: (Orange in WORDS)

Intro Topological Quantum Field Theory (TQFT): See

Words: Witten '89; Jones...

Advert: Mark Penney 15:30 Today
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Def (Atiyah '89) A TQFT is

$$Z : (\mathrm{Bord}^{\mathcal{G}}_{\dots}, \amalg) \longrightarrow (\mathcal{C}\ \mathrm{Vect}_k, \otimes)$$

Here $\mathrm{Bord}^{\mathcal{G}}_{n,n+1,\dots,n+d}$ is the (∞,d)-category w/ $\mathcal{G}$-structure

obj. closed n-mfds

1-morphs. $(n+1)$-cobordisms ~~(up to $\mathcal{G}$-diffeo rel boundary)~~

WHY? * Interplay algebra & topology
* Manifold invariants
* Managable version of QFT.

BLUE↑ $\mathcal{C}$ - sym. mon. cat
$\mathcal{G}$ - geometric structure eg: * orientation
* framing
* spin structure.

2-morphs: $(n+2)$-cobordisms between $(n+1)$-cobordisms

⋮

d-morphs: $(n+d)$-cobords (up to $\mathcal{G}$-diffeo rel. boundary)

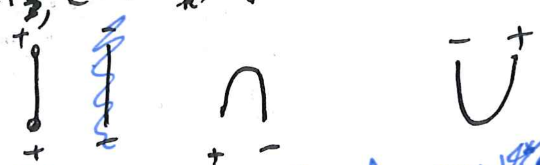
$d+1$-morphs: $\mathcal{G}$-diffeos of $(n+d)$-bords

⋮

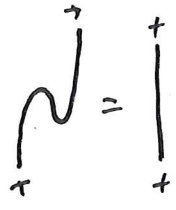
Classification Results:

Oriented 1dim'al: ($n=0$, $d=1$, $\mathcal{C} = \mathrm{Vect}_k$)

obj: $\overset{+}{\bullet}$, $\overset{-}{\bullet}$, $\emptyset$ morphs:



relation:



$Z(-)$: V V^* k Id_V ev: $V \otimes V^* \to k$ coev: $k \to V^* \otimes V$

i.e. 1d oriented $\overset{\text{TQFT}}{\longleftrightarrow}$ vectorspaces w/ choice of dual. "dualizable objects"

Cobordism Hypothesis (Lurie '09, ...) ∞-setting, $n=0$, $\mathcal{G}$ = framing:

TQFTs $\longleftrightarrow$ $\mathcal{C}^{\text{fully dualizable}}$

$Z \longmapsto Z(pt)$.

i.e. $Z(pt)$ is always fully dualizable & for every fd $\exists$ TQFT.

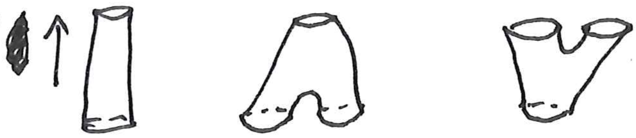
Mark: use this to find TQFTs w/ $\mathcal{G}$ = Oriented, ...

* 2d Oriented TQFT: ($n=1$, $d=1$, $\mathcal{C} = \mathrm{Vect}_k$)

[Idea: generators & relations ⟶ representation. WORPS]

Generators:

Objects  S^1

1-morphs 

$Z(-)$ V - vectorspace $\quad$ Id_V $\quad$ $\mu: V \otimes V \to V$ $\quad$ $\Delta: V \longrightarrow V \otimes V$

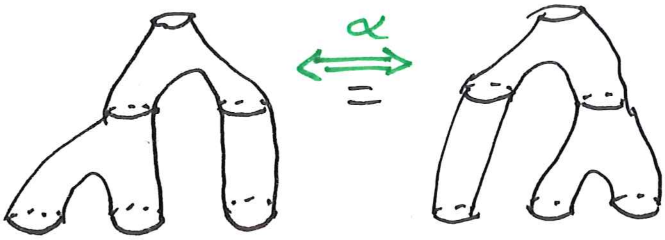
$Z(-)$ $\mathcal{A}$ - linear cat. $\quad$ $\mathrm{Id}_{\mathcal{A}}$ $\quad$ $\otimes: \mathcal{A} \boxtimes \mathcal{A} \to \mathcal{A}$

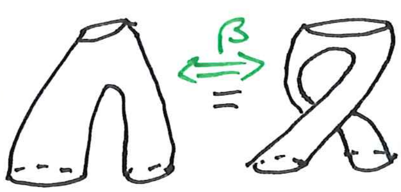
$k \to V$ $\quad$ $\varepsilon: V \to k$

$\mathrm{Vect} \to \mathcal{A}$

~~Subset of Relations:~~ 2-morphs:

(*) holds.





$Z(-)$ μ is associative.

$Z(-)$ $\otimes$ is associative up to iso.

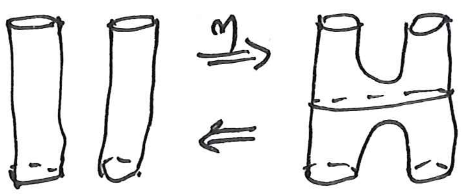
μ is commutative.

Braiding for $\otimes$.

LEAVE ROOM FOR:

Draw:

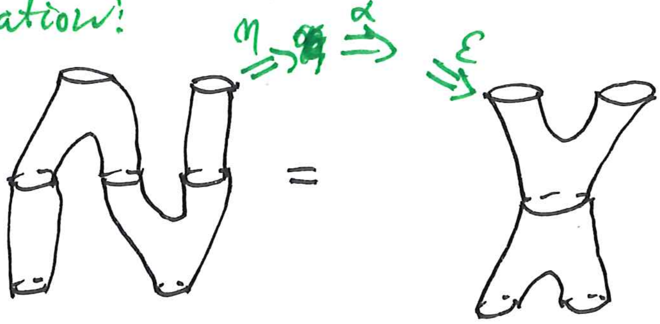
Non-invertible:



$Z(-)$ Witnesses for adjunctions between $Z(\text{copants})$ & $Z(\text{pants})$

Example of relation:

Frobenius:



is invertible.

⇝ 2d TQFTs $\overset{1:1}{\longleftrightarrow}$ (co)commutative Frobenius algs.

+ more!!!

~~1-2-3~~
<u>1-2-3 oriented TQFT</u>: $(n=1, d=2)$ $\mathcal{C} = \mathrm{LinCat}$: Vect_k-enriched cats., mon. str $\boxtimes$.
(Bartlett-Douglas-Schommer-Pries-Vicary ~~?~~, '14, '15, ...)
~~Classification of~~
Generators & relations for Bord^{or}_{123} (Morse 2-theory).

DO GREEN.

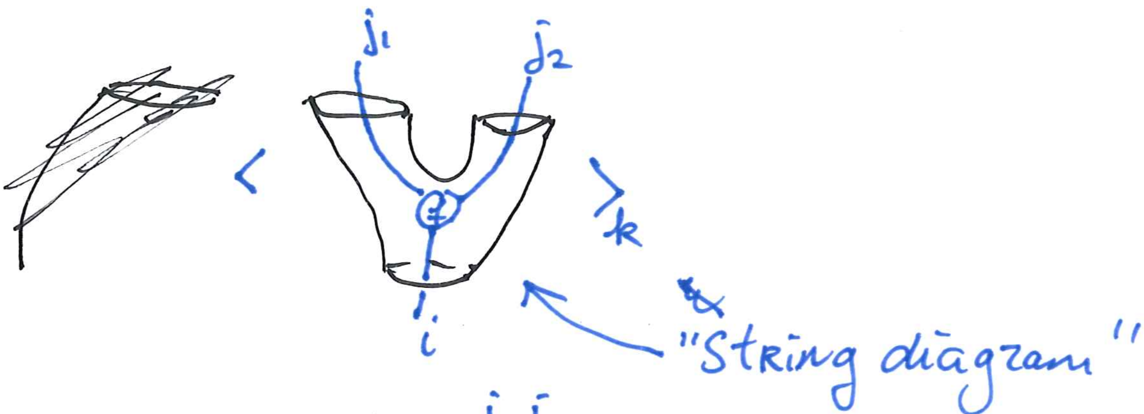
Interpretation:

$Z(S^1)$ is a modular fusion cat.

$\Rightarrow \mathrm{ob}(Z(S^1)) = \{$Vect-linear combis of simples$\}$

$\Rightarrow \Sigma$ surface, $Z(\Sigma)$ is determined by vector spaces encoding action on simples.

In existing picture: Yoneda-like trick:



$Z(-)^{j_1 j_2}_{i}$

* WING: saying that functors $\Leftrightarrow$ internal stringnets.

* Physicists: * "Moving Anyons"
  * Fermions? $\mathbb{Z}_2$!

Spin TQFT: Maths: * Reps of Free Fermion CNET (TMF)
* Fusion cats: Witt gps & pivo.

Topology:
Recall: $\pi_*(O)$: $\overset{0}{\mathbb{Z}_2}\ \overset{1}{\mathbb{Z}_2}\ \overset{\cdot\cdot}{0}\ \mathbb{Z}\ 0\ 0\ 0\ \mathbb{Z}$

$SO \leftarrow Spin$
$\uparrow$ double cover

Algebra: sVec: $\mathbb{Z}_2$-graded v.s.

Idea: mimic OV case. $Z: \mathrm{Bord}^{\mathrm{spin}}_{1,2,3} \longrightarrow \mathrm{sLinCat}$

Extras: * Spin involution $\xrightarrow{Z}$ Grading inv.

* Two circles: bounding & non-bounding.

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Thanks for listening!