Cellular approximations of classifying spaces of compact Lie groups.

Alberto Gavira-Romero

Alberto Gavira-Romero (gavira@mat.uab.cat)
Universitat Autònoma de Barcelona

Abstract. Let A be a topological space. E. Dror-Farjoun introduces in 1995 the notion of A-homotopy, where A and its suspensions play the same role of the spheres in classical homotopy. So the A-homotopy groups of a space X are defined to be homotopy classes of pointed maps $\Sigma^i A \to X$. The idea of CW-complex space is replaced by the one of A-cellular space, i.e., a space built from A by means of pointed homotopy colimits. The concept of cellular approximation of X is replaced by the A-cellular approximation, this means, an A-cellular space $CW_A X$ called the A-cellularization of X, and a natural map $CW_A X \to X$ inducing an equivalence $\mathrm{map}_*(A, CW_A X) \simeq \mathrm{map}_*(A, X)$,and hence an isomorphism $\pi_i(CW_A X; A) \cong \pi_i(X; A)$.

In this work, given a prime p, we study the A-cellularization of the p-completion of classifying spaces BG, where G is a compact Lie group and A is a classifying space of type $B\mathbb{Z}/p^\infty \times B\mathbb{Z}/p^m$. In particular we show that if G is connected, then $CW_{B\mathbb{Z}/p^\infty \times B\mathbb{Z}/p^m}(BG^\wedge_p)$ is the homotopy fibre of $BG^\wedge_p \to (BG^\wedge_p)_\mathbb{Q}$ for a sufficiently large m, where $(\,\cdot\,)^\wedge_p$ is the Bousfield-Kan p-completion and $(\,\cdot\,)_\mathbb{Q}$ is the rational localization.