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Student Project Proposal

Inverse Design of Elastic Springs

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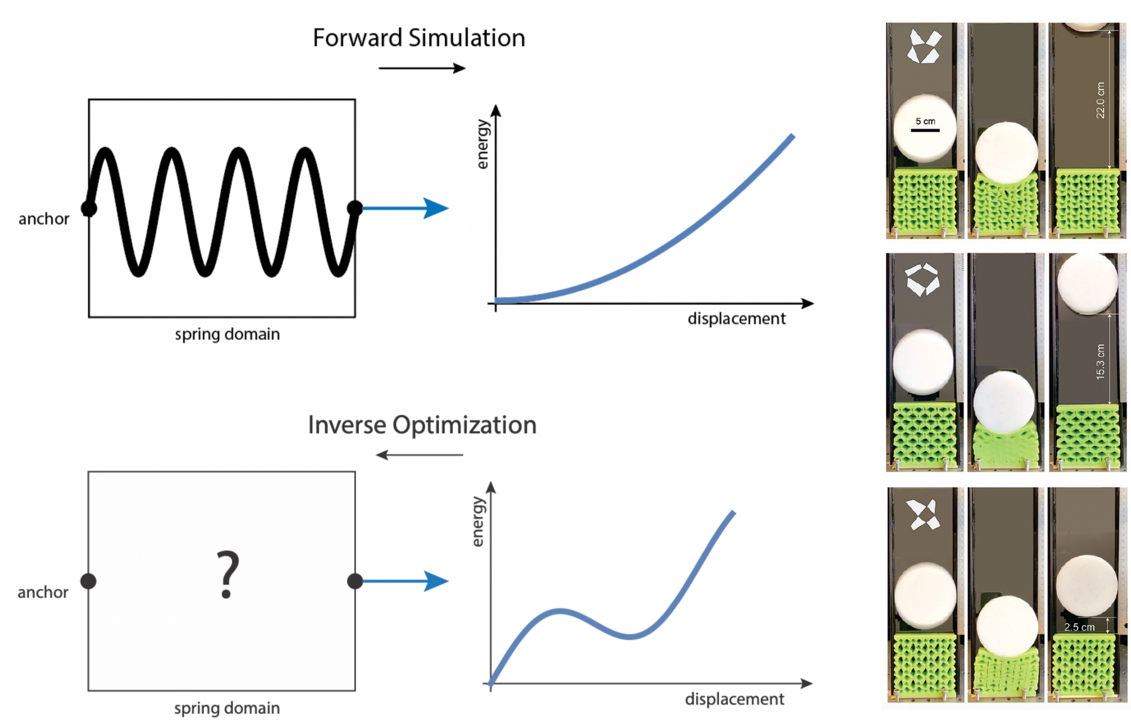

Figure 1: *Top left*: Existing forward simulation tools can compute the elastic energy of a spring at different elongations, and we can use them to plot the energy-displacement curve of the spring shown on the right. *Bottom left*: inverse optimization of a spring. The goal is to find the spring(s) matching a prescribed energy-displacement curve, if any exist. *Right*: a potential application, the design of metamaterials with different nonlinear elastic responses [2].

Description

The goal of this project is to develop and implement an inverse design algorithm for 1D springs, i.e. springs that are fixed at one point (anchor) and are tensioned along a linear path (see Fig. 1). Computing the energy vs. displacement curve of a given spring is straightforward, and can be done for example by simulating the spring elongation. The goal is to invert this process: Given a desired energy (or force) vs. displacement curve, is there a spring geometry that displays the prescribed elastic response? The final spring can then be 3D-printed [4] or, in a simplified 2D setting, laser cut from a flat sheet of material. Ensembles of elastic beams have been used, too [3].

One fundamental question is which kind of energy-displacement curves are even possible. For example, can we find a spring that is perfectly linear over a large displacement regime even for a non-linear base material? Can we design a logarithmic spring? What about more complex energy-displacement curves having multiple local minima, corresponding to meta-stable states?

The general problem is extremely challenging. A potential simplified way to address it is:

- Provide an initial design based on human intuition as a first guess;
- Sample the energy-displacement curve at discrete displacement points and solve a quasi-static problem to get the potential energy;
- Perform shape optimization, e.g. represent the initial design as a 2D mesh and solve for the displacements of the mesh vertices that best approximate the energy-displacement curve.

Depending on the application, this approach can give satisfactory results, but it has a number of limitations: the human-provided initial design, its topology being fixed, and self-intersections not being accounted for in the simulation. A more ambitious goal would be to have a fully automatic Topology Optimization (TO) pipeline. The idea of TO is to assign a value of 1 (material) or 0 (no material) to each point in the domain, with the constraint that there exists a continuous path of material from the anchor to the actuation point (see e.g. [5] and [6]). Given an energy-displacement curve, the pipeline would compute the topology and geometry of the spring without the need of a user-defined initial guess [1].

We provide a C++ code framework with Python interface to simulate elastic objects under load conditions (the forward simulation). The goal of the student would be to (1) get familiar with the code, (2) implement the simplified approach to address the inverse problem, and (3) tackle the problem from a TO perspective, if time permits.

Note that this document only depicts one of the many possible pathways approaching the problem. Students are encouraged to think about other attacking angles that interest them.

Prerequisites

Previous experience with C++ and Python is required. Knowledge of continuum mechanics and linear elasticity is a plus.

Remarks

This proposal is intended for a Master research project (8 or 12 credits) and can be adapted to fit the requirements of a Master's Thesis.

References

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[2] Bolei Deng, Ahmad Zareei, Xiaoxiao Ding, James C. Weaver, Chris H. Rycroft, and Katia Bertoldi. Inverse Design of Mechanical Metamaterials with Target Nonlinear Response via a Neural Accelerated Evolution Strategy. *Advanced Materials*, 34(41):2206238, October 2022.

[3] Christine Vehar Jutte and Sridhar Kota. Design of Nonlinear Springs for Prescribed Load-Displacement Functions. *Journal of Mechanical Design*, 130(8):081403, August 2008.

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[5] Jun Wu. Topology optimization. `https://www.youtube.com/playlist?list=PLhxzFmgOjqfW2zKF0DHckT3SyFHcY_-lU`.

[6] Jun Wu, Niels Aage, Rudiger Westermann, and Ole Sigmund. Infill optimization for additive manufacturing—approaching bone-like porous structures. *IEEE Transactions on Visualization and Computer Graphics*, 24(2):1127–1140, feb 2018.