Geometric Computing Laboratory

Elastic Circle Packing

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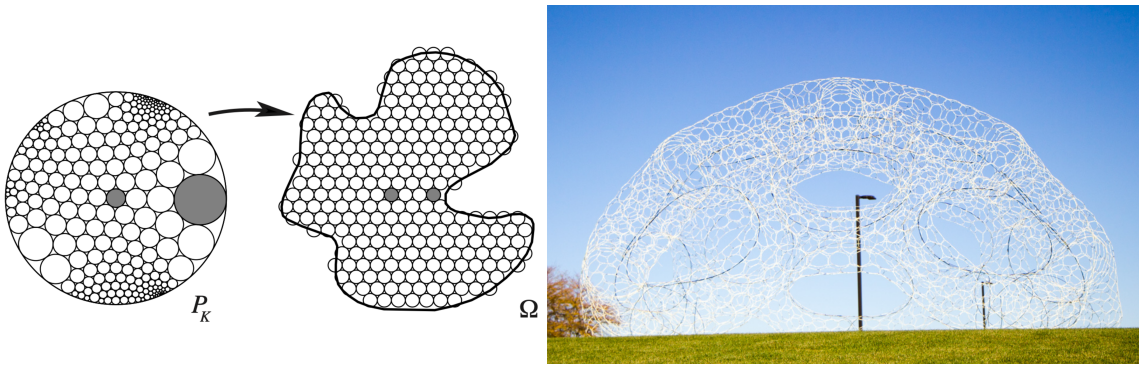

Figure 1: Left: Discrete Riemann mapping with circle packing [4]. Right: Dome structure built with thousands of woven circles of composite fibre [1].

Description

The goal of this project is to explore the space of structures built with circles made from elastic wires using circle packing. A circle packing is a configuration of circles tangent with each other where if we connect the centers of circles that are in tangent, the resulting pattern is a triangulation. It bridges combinatorics and geometry and leads to the definition of the discrete Riemann mapping theorem [5]. Schiftner et al. show in [3] how to compute circle packings on surfaces and apply the results for various structures in architectural geometry. However, the structures discussed in this work are all build with rigid elements. What happens when we use *elastic* elements instead?

To directly answer this question, we can replace each circle in a circle packing with an elastic wires and connect each tangent pair with external joint or internal link. In fact, a London based architectural design firm, Loop.pH [1], has built several pavilions using this method. They have shown structures with complex shapes and conducted preliminary experiment to add active elements to build transformable elastic circle packing using basic form finding approaches. In this project, we would like to study the connection between circle packing and elastic circles structure more systematically. Several interesting research questions and directions include:

- What are the mathematical conditions for a circle packing built with elastic wires to be in a 3D equilibrium?
- How does the geometric configuration of the connection at the tangent point influence the structural stability?
- Can we apply the discrete Riemann mapping theorem to build elastic circle packing structures that can transform from one shape to another by simply adjusting the size of the each elastic circle?

The starting point for this project is a physics-based simulation framework where we implemented the Discrete Elastic Rods model [2] and various ways of connecting rods (rigid joints, rotational joints, collision detection).

The project is designed for master students. Good knowledge of C++ and linear algebra is required.

References

[1] The sol dome. `https://loop.ph/portfolio/sol-dome/`.

[2] Bergou, M., M. Wardetzky, S. Robinson, B. Audoly, and E. Grinspun (2008). Discrete elastic rods. In *ACM SIGGRAPH 2008 Papers on - SIGGRAPH '08*, Los Angeles, California, pp. 1. ACM Press.

[3] Schiftner, A., M. Höbinger, J. Wallner, and H. Pottmann (2009). Packing circles and spheres on surfaces. In *ACM SIGGRAPH Asia 2009 papers*, pp. 1–8.

[4] Stephenson, K. (2003). Circle packing: a mathematical tale. *Notices of the AMS 50*(11), 1376–1388.

[5] Stephenson, K. (2005). *Introduction to circle packing: The theory of discrete analytic functions*. Cambridge University Press.