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Student Project Proposal

Knot Classification using Language Models

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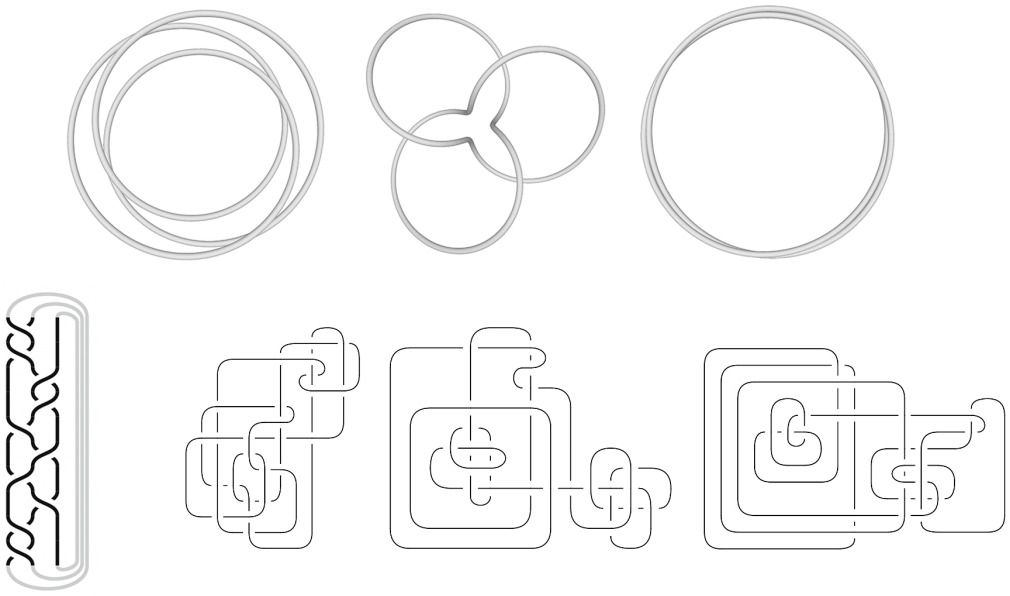

Figure 1: What knot? Top: three different embeddings of the trefoil knot. Bottom left: braid representation of a knot. Bottom right: three randomly generated knots to be classified [2].

Description

A knot in the mathematical sense can be imagined as a piece of string whose ends are connected together. This connection, and the impenetrability of the physical string, preserve the topology of the knot and prevent it from untying, even if deformed. A knot type is a set composed of all those closed curves that can be deformed one into the other without creating self-intersections.

A famous theorem by Alexander states that every knot can be represented as a closed braid [1] (see bottom left of Figure 1). A braid can be encoded by a braid word, which uses a finite set on symbols (letters) to represent over- and under-crossings between otherwise parallel strands. The mapping between knot type and braid word is one-to-many: we can exploit this redundancy to generate a set distinct braid words of known knot type to train a classifier.

Gukov and colleagues [2] show promising results using language models [5, 4] trained to distinguish trivial knots from non-trivial knots *i.e.*, to solve the Unknot Problem. In this project, we aim at approaching the Knot Classification Problem using language models on braids words. Kauffman *et al.* [3] use a different representation to solve the same classification problem.

Prerequisites

Good knowledge of at least one programming language is required, preferably Python. Good coding skills in PyTorch is expected. Familiarity with language models [5, 4] is a plus.

Remarks

The project is intended for Master students only.

References

[1] J. W. Alexander. A Lemma on Systems of Knotted Curves. *Proceedings of the National Academy of Sciences of the United States of America*, 9(3):93–95, March 1923.

[2] Sergei Gukov, James Halverson, Fabian Ruehle, and Piotr Sułkowski. Learning to unknot. *Machine Learning: Science and Technology*, 2(2):025035, 2021.

[3] LH Kauffman, NE Russkikh, and IA Taimanov. Rectangular knot diagrams classification with deep learning. *Journal of Knot Theory and Its Ramifications*, page 2250067, 2022.

[4] Nikita Kitaev, Łukasz Kaiser, and Anselm Levskaya. Reformer: The efficient transformer. *arXiv preprint arXiv:2001.04451*, 2020.

[5] Ashish Vaswani, Noam Shazeer, Niki Parmar, Jakob Uszkoreit, Llion Jones, Aidan N Gomez, Łukasz Kaiser, and Illia Polosukhin. Attention is all you need. *Advances in neural information processing systems*, 30, 2017.