Suggestion for SIE/CSE **Master/Semester project** at WSL:

Topic: Implementation of solvers for 1D Richards equation in Julia

Soil hydrology

We study the movement of water through the Swiss forest soils to understand how resilient the forest ecosystems are to prolonged summer drought periods. A process important for the survival of vegetation is the storage volume that soil provides to forests.

Richards equation

Unsaturated water transport in soils can be described by the Richards equation that is based on mass conservation. (For further information see chapter 5.2 in Soil Physics with Hydrus, or chapter 8.2 Soil Physics with Python). In 1D with *z* as the vertical dimension and *t* representing time, it reads:

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left(K(\psi)\left(\frac{\partial \psi}{\partial z} + \rho_w g\,\hat{z}\right)\right) - \mathrm{S}(z)$$

where θ(*z*) represents the volumetric soil moisture (m^3 water / m^3 soil volume) that ranges from 0 to the value of porosity (when all pores are filled), *ψ*(*z*) is the matric potential (Pa), i.e. the tension (negative pressure) under which water is hold and which drives the flow, K the hydraulic conductivity that depends on the water status of the soil and g the gravity. Finally sources or sinks such as water uptake by plant roots are represented by S(*z*). The hydraulic properties of the soil allow us to link the two representations of the soil water status: θ = *f(ψ)*.

Part of a mechanistic model of forest soil water balance

By means of a long-term network of 16 monitoring sites across Switzerland (see fig. 1) we collect measurements of the water stored in the soils and consumed by trees. A mechanistic model of the forest water balance *LWFBrook90.jl* (see fig.2), written in the Julia programming language, is calibrated to these measurements. The Richards equation is a central part of this simulation model.

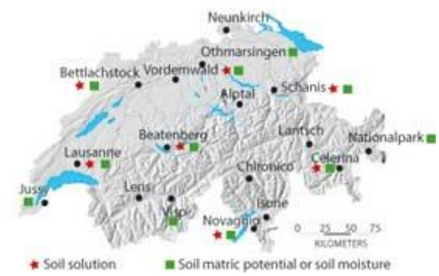

Figure 1 Sites of the forest monitoring network across Switzerland.

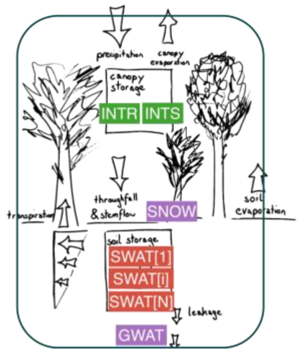

Figure 2 Schematic of modelled state variables (boxes) in the explicit water balance model LWFBrook90.jl

Objective

Progress on robust and accurate numerical methods for solving the Richards equation is still made. An overview of methods can be found in Maina and Ackerer (2017). An application showcasing simulations of another Julia-implemented water balance model is presented by Pollacco (2022). Our objective is to test alternative algorithms in the context of LWFBrook90.jl. Examples of these algorithms could be: (Celia, 1990), (Ross, 2003), (Tubini and Rigon, 2022), or (Li et al., 2021).

Expected work

- Get familier with the Julia package *DifferentialEquations.jl* (used internally by *LWFBrook90.jl)*
- Assess compatibility of the alternative algorithms with *DifferentialEquations.jl*
- Implement 1 or 2 algorithms in a sandbox example or directly into *LWFBrook90.jl*
- Run synthetic examples and/or real world simulations to evaluate computational efficiency
- Merge the developed code into the package *LWFBrook90.jl*

Contact me!

If you're interested in the topic for your Semester or Master project or if you have further questions, do not hesitate to contact me!

I'm looking forward to discussing with you!

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References

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