

HW Puzzles: Week 4 (Vectors)

October 1, 2026

Puzzle 1. Let $A_1, \dots, A_n$ be sets of unit vectors in $\mathbb{R}^d$. Assume that for each i we have $\sum_{v \in A_i} v = 0$. Is it true that one can choose $v_i \in A_i$ for every i such that $|\sum_{i=1}^n v_i|_2 \leq \sqrt{d}$?

Puzzle 2. Let $A_1, \dots, A_n$ be sets of unit vectors in $\mathbb{R}^d$. Assume that for each i we only know that O lies in the convex hull of A_i . Is it true that one can choose $v_i \in A_i$ for every i such that $|\sum_{i=1}^n v_i|_2 \leq \sqrt{d}$?

Puzzle 3. Let P be a tetrahedron with vertices A_1, A_2, A_3 , and A_4 . For every $i = 1, 2, 3, 4$ let B_i be a point on the facet of P opposite to A_i . Show that the four lines $\overline{A_i B_i}$ intersect at a point iff it is possible to distribute a mass among the four vertices of P such that for every $i = 1, 2, 3, 4$ the point B_i is the center of mass of the three vertices on the facet opposite to A_i .

Puzzle 4. Let p be a prime and consider $\mathbb{F}_p^2$, the two dimensional plane over $\mathbb{F}_p$. What is the discrepancy of the set of all lines with respect to the set of p^2 points in this plane? In other words, we would like to color the points by red and blue so that for every line in this plane the difference between the number of red and blue points on it is small. What can we guarantee? (A line in $\mathbb{F}_p^2$ is a set of the form $\{(x, y) \mid ax + by = c \pmod{p}\}$ for some a, b, c such that $(a, b) \neq (0, 0)$.)

Puzzle 5. Explain why the discrepancy of n sets $A_1, \dots, A_n \in \{1, \dots, n\}$ cannot in general (in the sense of the worst case scenario) be better than roughly $\sqrt{n}$.