

HW Puzzles: Week 1 (Probability)

September 10, 2026

Puzzle 1. *How many times in average should we throw a dice until the first time each of the numbers $1, \dots, 6$ appears at least two times? k times?*

Puzzle 2. *What is the probability that two random numbers from $1, \dots, n$ are such that one divides the other?*

Puzzle 3. *We choose k points at random on a circle of radius 1. What is the expected area of their convex hull?*

Puzzle 4. *We choose 4 points at random in the unit disc. What is the probability that they are in convex position (meaning that they are the set of vertices of a convex 4-gon)? What if we replace 4 by k ?*

Puzzle 5. *We saw in class that if A is any set of integers, then one can find a subset of A of size at least $|A|/3$ such that the equation $x + y = z$ cannot be solved by numbers in A . Can one do better than $|A|/3$? Find a set A such that any subset of A of large enough cardinality (the smaller "large enough" is the better) does contain a solution to $x + y = z$.*